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序　　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了"中外物理学精品书系"，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，"中外物理学精品书系"力图完整呈现近现代世界和中国物理科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会　主任

中国科学院院士，北京大学教授

王恩哥

2010 年 5 月于燕园

Preface

This book draws from the graduate course '*Physics of Ultracold Atomic Gases*' at Renmin University of China (since 2010), and another similar course 'Cold Atom Physics' at the University of Science and Technology of China (since 2011), which, for the past few years, have served as elementary introductions of cold atomic gases for interested students. While the main textbook for both courses is the popular 'Bose-Einstein Condensation in Dilute Gases' by Pethick and Smith, we decide that it would be to the best of our interests, as well as to those of the students', to have a textbook of our own, so that students may have the convenience of a complementary reference, modest as it is, when confused in class. For that purpose, we have intended from the start to be pedagogical, and, while not compromising. Whether this is indeed the case still remains to be seen.

All the authors contribute extensively to the writing of the book: Part Ⅲ is written by Yongjian Han, the Introduction and Part Ⅱ are written by Wei Yi, and Part I is written by Wei Zhang. As such, the authors are listed in alphabetic order. We are grateful to Jiansong Pan for proof reading some of the chapters, and to Zhen Wang for making some of the figures in Part Ⅲ. We also acknowledge all the students who have taken the courses, whose brilliant feedbacks prove to be immensely helpful. We thank Xiaohong Chen from the Peking University Publishing House, whose continuous support makes this project possible. Finally, we apologize for the inevitable mistakes lurking out there, and hope that we may have the chance to improve in the future.

Beijing,
July 2014

Yongjian Han
Wei Yi
Wei Zhang

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Chapter 1

Introduction

Almost twenty years have passed since the first experimental observations of Bose-Einstein condensation (BEC) in ultracold atomic gases [1, 2, 3]. In these seminal experiments, up to $10^4 \sim 10^5$ neutral bosonic atoms of alkali were trapped using lasers and magnetic fields, and were cooled down to temperatures below the microkelvin range, at which point the collective behavior of the gas becomes quantum mechanical. The realization of BEC in dilute atomic gases turns a new page in our understanding of the Nature, and provides us with a versatile platform, on which previously unsolved physical problems can be studied and many novel ideas can be tested. Coupled with other recently developed techniques such as the Feshbach resonance, the optical lattice potentials, and the synthetic gauge fields, ultracold atomic gases, both bosonic and fermionic, are playing an increasingly important role in various fields of research, including quantum simulation, quantum computation, precision measurement, to name a few.

For a systematic introduction of BEC, we refer the readers to the excellent book by Pethick and Smith [4], where the authors also cover topics like cooling and trapping of cold atoms which are critical for the realization of BEC. The purpose of this book is to serve as a modest introduction to the more recent progresses such as Fermi condensate, polarized Fermi gas, synthetic gauge fields, and quantum simulation with optical lattice potentials, etc. We will also cover the basic theoretical framework under which physical processes such as two-body scattering, pairing superfluidity of fermions and so on, are modeled. Before doing so, let us first review, from a historical perspective, the realization of condensation of both bosonic and fermionic atoms in ultracold atomic gases.

The study of BEC dates back to 1924, when Bose introduced a new way of counting the microscopic states of the radiation field [5]. Using this method, Bose was able to re-derive Planck's formula for the black-body radiation in his seminal paper. Einstein later extended the approach to treat massive indistinguishable particles that obey the same statistics [6]. The resulting Bose-Einstein distribution leads to the striking conclusion that a majority of the particles would occupy a single quantum state with the lowest energy at low enough temperatures. Here, the occupation of the ground state is closely related to the system's ability to accommodate particles in the excited states, which increases with temperature. As a result, Bose-Einstein condensation should occur in a system with a low enough temperature and a high enough number density.

Physically, as the temperature is lowered, the thermal de Broglie wavelength of the particles in the system increases. Condensation occurs when the thermal de Broglie wavelength is on the order of the inter-particle separation. At this point, the wave packets of different particles overlap and interfere with each other to form a larger wave packet common to all particles in the condensate. A more detailed calculation shows that the condition for condensation in a free space of three dimensions is $\lambda n^{1/3} \sim 2.612$, which is consistent with the hand-waving argument outlined above. Here, n is the number density, λ is the thermal de Broglie wavelength. Considering the expression $\lambda = (2\pi\hbar^2/mk_BT)^{1/2}$, this argument is consistent with the previous analysis that condensate occurs with high density and low temperature. One often regards $\lambda n^{1/3}$ as the phase-space density [7]. To achieve condensate experimentally is to look for and implement a system whose phase-space density can go beyond 2.612.

Although the physics behind the theory is elegantly simple, it turned out to be quite difficult to realize a well-behaved BEC experimentally. To increase phase-space density often requires a high number density, which typically leads to the solidification of the system. The only exception is liquid ^{4}He, which remains to be a liquid at the lowest temperature. Regardless, in solids or in liquid helium, the interaction is typically quite strong and cannot be simply neglected. With interaction, Einstein's theory must be modified and it turns out that a strong interaction leads to the so-called depletion of the condensate, i.e., the process in which interaction-induced excitations make particles leave the condensate. In fact, although it has been suggested in 1935 that the superfluidity of liquid helium is related to BEC, the population of the ground state in a typical ^{4}He superfluid is only $\sim 10\%$ of the total particle number [8]. Furthermore, due to the strong interaction, the properties of the system cannot be characterized analytically. Hence it seems that the only way to achieve a weakly-interacting BEC is with a low number density. However, at the first sight, there are many questions to be answered: whether a system with such a low density is stable, how to trap the system in space, and how to cool the system to the required temperature, which, given the low density of the system, would be lower than any known temperatures in the whole Universe.

Soon, people realized that these problems can be solved in a dilute atomic gas. With a typical density of $n \approx 10^{12} \sim 10^{15}/\text{cm}^3$, these systems are usually metastable. Yet, so long as the lifetime of the dilute gas is much longer than the time required for thermal equilibrium, it is possible to realize and observe Bose-Einestein condenstates with weak interactions under typical experimental conditions. Due to the low particle density, the interatomic interaction is typically rather weak in these systems such that they can be well characterized in a perturbative fashion. The catch however, lies in the extremely low temperature required. In this case, for the phase-space density to reach unity, the temperature should be below the microkelvin range. There is also the question of how to hold the gas in space. Containers are obviously out of the question. Hence the key to BEC in a dilute atomic gas seems to lie in the cooling and trapping of the gas. Indeed, from hindsight, the three key developments that eventually paved the way for the realization of BEC in dilute gases are laser cooling, trapping with magnetic field or laser, and evaporative cooling [9].

Early experimental efforts towards BEC focused on dilute gases of spin polarized Hydrogen atoms, which feature weak attractive interaction and are metastable against molecule formation in a gaseous form. It was proposed that BEC as well as superfluidity can be realized in such a system [10]. Although the BEC of spin polarized hydrogen atoms was not realized until 1998, that is, after the BEC in alkali atoms, many important techniques had been developed first for hydrogen atoms, e.g., trapping with magnetic field, and evaporative cooling. For a dilute hydrogen atomic gas, the atoms are initially cooled cryogenically, which requires the gas to be in contact with a cold surface. This limits the achievable density of the gas owing to the interaction of atoms with the container wall. To avoid contact with the container, it is preferable to hold the atomic gas in place by a magnetic field. These ideas were entertained at MIT by Greytak and Kelppner, and led to the invention of magnetic trapping.

A revolutionary advance took place in the 1980s with the advent of laser cooling. Though not applicable to hydrogen atoms, laser cooling has the potential to cool a whole spectrum of atoms, e.g. alkali atoms, rare-earth atoms, alkaline-earth atoms etc., from room temperature down to hundreds of microkelvin. This is followed by the invention of magnetic-optical traps, in which the atoms can be cooled further via the Sisyphus cooling mechanism and reach the range of tens of microkelvin. There is however still a final gap to overcome before the condensation can take place. In the following years, there were many brilliant proposals for the so-called sub-recoil cooling, which aims to go below the temperature limit of previous laser cooling schemes. The practical limit of laser cooling is set by the scattering of atoms by laser, a necessary process in the cooling scheme. Thus the task of cooling further with laser seems insurmountable. The solution lies in the introduction of a second cooling stage, the evaporative cooling. Originally devised to cool spin polarized hydrogen atoms, the evaporative cooling selectively removes the hottest atoms in the trap and allows the rest thermalize. As a result, the number density increases while the temperature decreases. Thus, evaporative cooling is able to overcome the last several orders of magnitude in the final climb of the phase-space density, and is an essential step in realizing BEC in dilute atomic gases. In 1995, E. Cornell and C. Wieman at Boulder, R. G. Hulet at Rice, and W. Ketterle at MIT were the first to successfully combine laser cooling with evaporative cooling, and observe BEC in dilute gases of atoms. Decades of heroic experimental endeavor eventually paid off.

Soon after the realization of BEC in dilute gases of bosonic atoms, people started to think about bringing fermionic atoms into quantum degeneracy. While it is possible for bosonic atoms to occupy the same state, for fermionic atoms, a given quantum state can only accommodate at most one atom due to the Pauli blocking. As a result, the atoms in a Fermi gas at zero temperature occupy all available low energy states, thus forming a Fermi sea with a sharp edge in momentum space. However, in the presence of attractive interatomic interactions, the Fermi sea becomes unstable and fermions at the surface of the Fermi sea pair up to form the so-called Cooper pairs. The Cooper pairs are composite bosons and may condense to form a BEC at low temperatures [11]. This is the physical picture behind the famous Bardeen-Cooper-Schieffer (BCS) theory proposed in the 1950s to explain superconductivity [12] in metals. It is therefore interesting if one can also prepare such a Fermi condensate of composite bosons in an ultracold atomic Fermi gas.

The problem again lies in cooling. According to the BCS theory, the critical temperature at which the Fermi condensate emerges in an attractively interacting Fermi gas decreases exponentially with the interaction strength. In a dilute gas, the interaction is typically very weak, hence it seems impossible to reach the condensation temperature in the first place. In 1993, E. Tiesinga, B. Verhaar and H. Stoof proposed that the interatomic interactions can be tuned via the Feshbach resonance technique [13, 14], which was first discovered by H. Feshbach in 1958 in the context of nuclear physics [15]. In a Feshbach resonance, the two-body scattering length can be tuned by adjusting the external parameters. Microscopically, the effective s-wave scattering length can be smoothly adjusted from zero to infinity and from negative to positive by tuning the relative energy difference between a two-body quasi-bound state and the threshold of the two-body scattering continuum. In an ultracold atomic gas, such an energy difference can be tuned via either the external magnetic field or the laser field. The Feshbach resonance was observed in ultracold atomic gases in 1998 [16, 17, 18], and has since proved to be one of the most powerful tools in ultracold atoms, as it can lead to strong correlations in a dilute atomic gas.

For an ultracold Fermi gas, this implies a dramatically increased critical temperature, and the possibility of observing Fermi condensate in ultracold Fermi gases under practical experimental conditions. In 2003, condensation of fermion pairing states was observed in ultracold gases of ^{6}Li and ^{40}K [19, 20, 21]. Furthermore, by tuning the interaction strength, it was possible to demonstrate the crossover between a tightly bound molecular BEC of composite bosons and a condensate of fermion pairs with long-range correlation. This so-called BCS-BEC crossover had