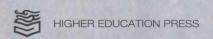
ALM 33
Advanced Lectures in Mathematics

# **Introduction to Modern Mathematics**

现代数学导论

Editors: Shiu-Yuen Cheng · Lizhen Ji · Yat-Sun Poon Jie Xiao · Lo Yang · Shing-Tung Yau





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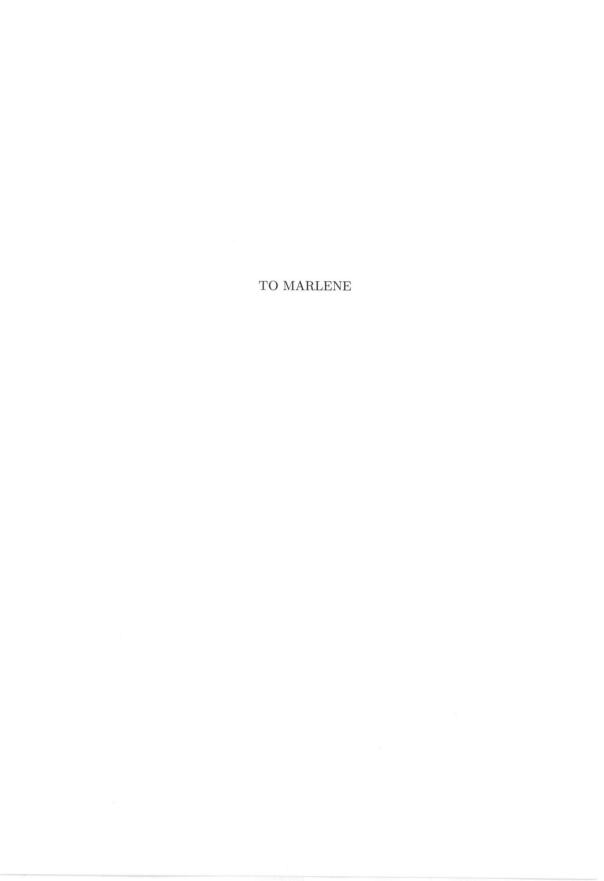
# Introductory Lectures on Surgery Theory

Thomas Farrell\* and Somnath Basu†

\*Department of Mathematical Sciences Binghamton University, NY 13902-6000, USA Email: farrell@math.binghamton.edu

> †Department of Mathematics Rkm Vivekananda University Howrah, WB 711202, India Email: somnath@maths.rkmvu.ac.in







### **Preface**

This article is closely based on a graduate level topics course in topology given in the fall of 2012. The course had a large and enthusiastic audience which consisted of mathematicians with many varying interests ranging from graduate students looking for or working on theses to researchers in dynamical systems, topology and geometry. Because of this the lectures were designed to accommodate the backgrounds and interests of the audience. These lectures eventually focussed on the early work of Kervaire and Milnor [30] and ended with a long discussion of Kervaire's paper [29] which constructed the first example of a topological manifold that does not support a smooth structure. Thus these lectures are only meant to be an introduction to Surgery Theory, not a systematic account of this field. Hopefully they will whet the reader's appetite to delve deeper into this fascinating subject. Such a reader can pursue it further in the two classic textbooks [9] by W. Browder and [56] by C.T.C. Wall. The monograph [31] by R. Kirby and L. Siebenmann extends the reach of surgery theory to encompass topological manifolds while that by M. Freedman and F. Quinn [17] extends it to include 4-dimensional manifolds. For the more recent algebraic point of view on this subject, the reader should see A. Ranicki's monograph [45]. Finally, to obtain a taste of the current research in the area, we recommend the following sources [15], [11], [12] and [14].

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