Mechanical Science

G W Taylor,

ST(P) Technology Today Series

Mechanical Science

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Preface

This book conforms with the general approach which has been used in other volumes in the ST(P) Technology Today Series. The contents will be central to the interests of students following Technician Education Council schemes, and in particular to the Level III standard Mechanical Science Unit No U75/058.

Reflecting the philosophy advocated by the TEC, progress through the work is based on objectives for each topic of the course. Numerous worked examples are included in all the sections, and a plentiful supply of exercises is provided at the end of each topic these having been carefully graded to provide controlled practice for the student user.

The author wishes to record his appreciation for the valuable help given in the preparation of the script by Mr R Brayne and by other colleagues.

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Symbols and SI Units

SYMBOLS FOR PHYSICAL QUANTITIES

The following is a list of symbols used in this book, which are generally in accordance with *British Standard* 1991:

- A area, amplitude
- a acceleration
- b breadth
- D diameter
- d diameter, depth
- E modulus of elasticity or Young's modulus
- F force
- f frequency
- G modulus of rigidity or shear modulus
- g acceleration of a freely falling body at Earth's surface ($9.81\ m/s^2$)
- h height, head of liquid
- H difference of heads of liquid
- I second moment of area, moment of inertia
- J polar second moment of area
- k radius of gyration
- l length
- M moment, bending moment, mass
- m mass
- $\dot{m}$ mass flow rate
- N revolutions per minute
- p pressure
- $\dot{Q}$ volume flow rate
- R reaction (force), indicates a radius on a diagram
- r radius
- s displacement, distance travelled
- T torque, period
- t time
- V volume
- v velocity
- x displacement or distance in a horizontal direction
- y displacement or distance in a vertical direction
- α (alpha) angular acceleration, coefficient of linear expansion
- ϵ (epsilon) direct strain
- θ (theta) angle turned through, angle of twist
- μ (mu) coefficient of friction
- ν (nu) Poisson's ratio
- π (pi) ratio of circumference to diameter of a circle (3.142)
- ρ (rho) density
- σ (sigma) direct stress
- τ (tau) shear stress
- ϕ (phi) indicates a diameter on a diagram
- ω (omega) angular velocity

SYMBOLS FOR SI UNITS

SI stands for the Systeme International d'Unites, or International System of Units, which has been adopted by British industry for engineering calculations.

The following is a list of symbols used in this book which are also in accordance with British Standard 1991:

bar	bar (= 10^5 N/m^2)
°C	degree Celsius
g	gram(me)
h	hour
Hz	hertz (cycle per second)
J	joule
kg	kilogram(me)
ℓ	litre
m	metre
min	minute
N	newton
Pa	pascal (= N/m^2)
rad	radian
rev	revolution
s	second
t	tonne (= 1000 kg)
W	watt

To express multiples and subdivisions of SI units concisely, the following prefixes are used with them:

Multiplication Factor		*Prefix*	*Symbol*
1 000 000 000 000	$= 10^{12}$	tera	T
1 000 000 000	$= 10^{9}$	giga	G
1 000 000	$= 10^{6}$	mega	M
1 000	$= 10^{3}$	kilo	k
100	$= 10^{2}$	hecto	h
10	$= 10^{1}$	deca	da
0.1	$= 10^{-1}$	deci	d
0.01	$= 10^{-2}$	centi	c
0.001	$= 10^{-3}$	milli	m
0.000 001	$= 10^{-6}$	micro	μ
0.000 000 001	$= 10^{-9}$	nano	n
0.000 000 000 001	$= 10^{-12}$	pico	p
0.000 000 000 000 001	$= 10^{-15}$	femto	f
0.000 000 000 000 000 001	$= 10^{-18}$	atto	a

The multiplication factors preferred in SI are those increasing in steps of 10^3. Thus the use of hecto, deca, deci and centi should be avoided if possible.

Contents

Chapter 1 Stress, Strain and Elasticity

After reaching the end of this chapter you should be able to:

1) Solve from first principles problems involving composite bars under the action of uni-axial loads only, at uniform temperature.

2) Solve problems as in 1) but including the effect of temperature change.

3) State that total direct strain is the sum of the strain due to uni-axial loading and temperature change.

4) Define shear stress, shear strain, modulus of rigidity (shear modulus) and solve associated problems.

5) Define Poisson's ratio.

6) Apply the Poisson's ratio effect in stress—strain relationships to solve associated problems in two dimensions (excluding shear stress action).

STRESS

Stress is the name given to forces acting within a material which enable it to resist external loading. However it is more usual to use the term 'stress' to denote the magnitude, or intensity of stress. Stress is generally classified according to the type of loading.

DIRECT STRESS

This is the simplest form of stress which results from an external force being applied at right angles to the plane on which the stress occurs.

Fig. 1.1 shows bars loaded in tension and in compression and in each case the bars have been separated at section AA in order that the stress may be shown acting on the cross-sectional area.

Now

$$\text{Direct stress is measured by the ratio } \frac{\text{load}}{\text{cross-sectional area}}$$

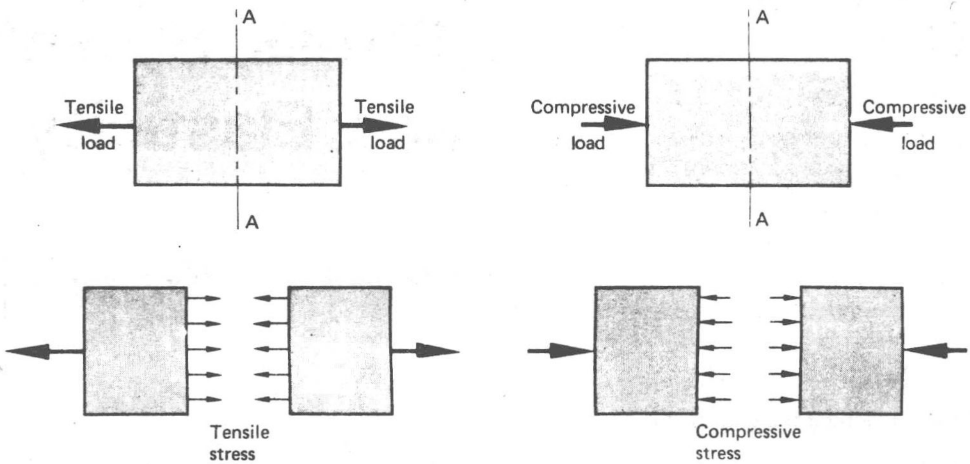

Fig. 1.1

DIRECT STRAIN

Strain is concerned with the deformation of a material when in a state of stress. Strain is classified in a similar manner to stresses. Hence direct strain occurs in the direction of loading as shown in Fig. 1.2.

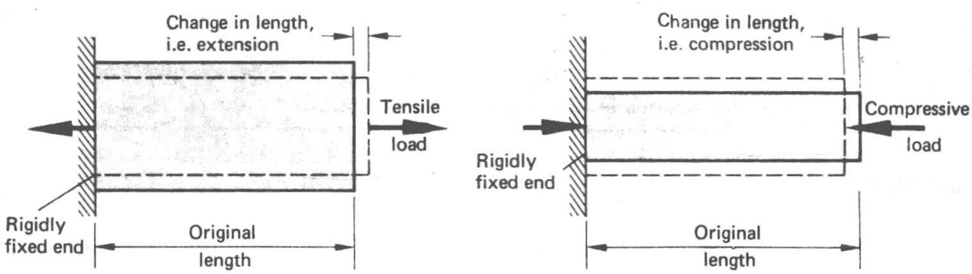

Fig. 1.2

Now

Direct strain is measured by the ratio $\dfrac{\text{change in length}}{\text{original length}}$

You should note that deformation of the material occurs in directions other than parallel to the loading. Fig. 1.2 shows that the bar loaded in tension becomes thinner, whilst the bar loaded in compression becomes thicker. This will be examined more fully later in this chapter.

ELASTICITY

A material is said to be elastic if after being deformed by a load it returns to its original shape when unloaded.

To help you appreciate this let us look at part of a stress–strain graph for a typical engineering material as shown in Fig. 1.3.

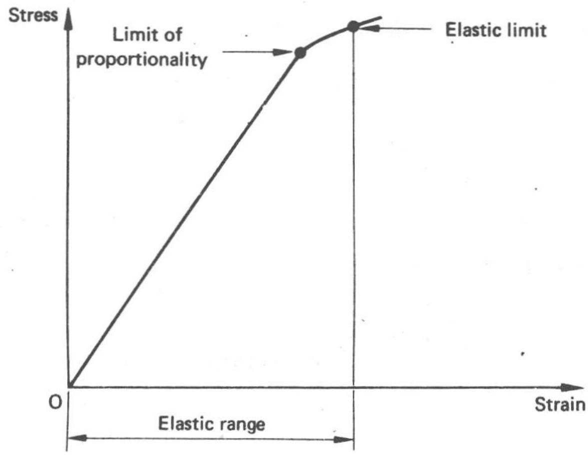

Fig. 1.3

In almost all engineering design problems we cannot tolerate permanent deformation so we keep within the elastic range. Within this range you will see that the graph is a straight line up to the 'limit of proportionality'. Our calculations are nearly always based on the straight portion of the graph to which Hooke's law refers.

HOOKE'S LAW AND MODULUS OF ELASTICITY

Hooke's law states that:

> For an elastic material, up to the limit of proportionality, the stress is directly proportional to the strain it produces.

In equation form this may be stated as: $\text{stress} \propto \text{strain}$

or: $\text{stress} = (\text{a constant}) \times \text{strain}$

from which: $$\frac{\text{stress}}{\text{strain}} = \text{a constant}$$

For direct stress and strain the equation becomes:

$$\frac{\text{direct stress}}{\text{direct strain}} = E$$

where E is a constant called 'the modulus of elasticity' or 'Young's modulus'.

SYMBOLS AND UNITS

STRESS

The direct load (or force) is denoted by F N

The cross-sectional area is denoted by A m^2

The direct stress is denoted by σ $\frac{N}{m^2}$

(σ is the Greek letter 'sigma')

Now we have said previously that:

$$\text{direct stress} = \frac{\text{load}}{\text{cross-sectional area}}$$

$$\therefore \qquad \sigma = \frac{F}{A}$$

In SI the basic unit of force is the newton (N), and the basic unit of area is the square metre (m^2). Hence the basic unit of stress is the newton per square metre (N/m^2). This basic unit of stress is also called a pascal (Pa).

For most practical purposes this basic unit of stress is far too small and multiples have to be used. The standard SI multiples are the kilonewton per square metre (kN/m^2), the meganewton per square metre (MN/m^2), and the giganewton per square metre (GN/m^2).

We generally recommend the use of basic SI units when solving problems in engineering. However when stress is involved the arithmetic is made easier by using mm units for length in the calculations, the units for stress being N/mm^2.

Stress data may well be given in MN/m^2 and we will need to convert to N/mm^2.

Now $\quad 1\ MN/m^2 = 1 \times 10^6\ N/m^2$

$$= \frac{1 \times 10^6}{10^6}\ N/mm^2 \quad \text{since } 1\ m = 1000\ mm$$

$\therefore \quad 1\ MN/m^2 = 1\ N/mm^2$

This means that the numerical values are the same,

e.g. $\quad 85\ MN/m^2 = 85\ N/mm^2$

STRAIN

The original length is denoted by l m

Extension or compression is denoted by x m

Direct strain is denoted by ϵ and has no units

(ϵ is the Greek letter 'epsilon')

Now we have said previously that:

$$\text{direct strain} = \frac{\text{change in length}}{\text{original length}}$$

$$\therefore \qquad = \frac{x}{l}$$

Since strain is a ratio of two units, i.e. $\frac{\text{m}}{\text{m}}$, it will have no units.

MODULUS OF ELASTICITY (*E*)

Now $$E = \frac{\text{direct stress}}{\text{direct strain}}$$

$$\text{units of } E = \frac{\text{units of stress}}{\text{units of strain}}$$

and since strain has no units, then the units of E are the same as those of stress.

Values of E are usually given in units of GN/m^2, and hence we need to know that:

$$1\ \text{GN/m}^2 = 10^3\ \text{MN/m}^2 = 10^3\ \text{N/mm}^2$$

EXAMPLE 1.1

A rectangular steel bar 16 mm x 10 mm x 200 mm long extends 0.12 mm under a tensile load of 20 kN. Find:

(a) the stress;
(b) the strain;
(c) the modulus of elasticity, E, of the bar material.

(a) Now $$\text{tensile stress} = \frac{\text{tensile load}}{\text{cross-sectional area}}$$

Also tensile load $= 20\ \text{kN} = 20 \times 10^3\ \text{N}$,

and cross-sectional area $= 16 \times 10 = 160\ \text{mm}^2$, and substituting, we have

$$\text{tensile stress} = \frac{20 \times 10^3}{160} \frac{\text{N}}{\text{mm}^2}$$

$$= 125\ \text{N/mm}^2$$

$$= 125\ \text{MN/m}^2$$

(b) Now $$\text{strain} = \frac{\text{extension}}{\text{original length}}$$

Also extension = 0.12 mm, and original length = 200 mm

Therefore substituting gives $$\text{strain} = \frac{0.12}{200}\frac{\text{mm}}{\text{mm}}$$
$$= 0.0006$$

(c) Modulus of elasticity $$E = \frac{\text{stress}}{\text{strain}}$$

and if we substitute stress = 125 N/mm², and strain = 0.0006, then

$$E = \frac{125}{0.0006}\ \text{N/mm}^2$$
$$= 208 \times 10^3\ \text{N/mm}^2$$
$$= 208\ \text{GN/m}^2$$

FACTOR OF SAFETY

The ultimate stress, as shown in Fig. 1.4, is the nominal maximum stress which occurs in a material before eventual failure.

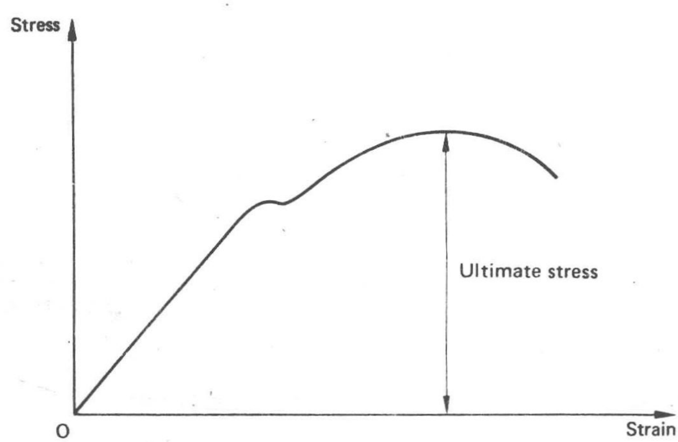

Fig. 1.4

When we are designing engineering structures and components the maximum stress we permit is considerably lower than the ultimate stress and is called the working stress or maximum permissible stress. We make use of the 'factor of safety' such that:

$$\text{factor of safety} = \frac{\text{ultimate stress}}{\text{working stress}}$$

or $$\text{working stress} = \frac{\text{ultimate stress}}{\text{factor of safety}}$$

The numerical value of the factor of safety varies for any particular application since it allows for factors occurring which cannot be known exactly. These may include type of load (e.g. steady or fluctuating), unforeseen change in dimensions, unknown imperfections in component materials, and abnormal use of the component after manufacture.

An alternative way of using a factor of safety comes from the idea of keeping the working stress within the elastic range. It is then defined as the ratio of the elastic limit stress to the working stress. However, this will give a different numerical value for the factor of safety and it follows that if this alternative definition is used it must be clearly stated, otherwise the previous definition should be assumed.

EXAMPLE 1.2

A copper cylinder is 122 mm long and 30 mm diameter. If the ultimate compressive stress of copper is 300 MN/m² find the maximum load which the cylinder can support if the factor of safety is 6. Find also the compression of the cylinder if the modulus of elasticity for copper $E_c = 96\ \text{GN/m}^2$.

We have $$\text{working stress} = \frac{\text{ultimate stress}}{\text{factor of safety}}$$

and we are given that ultimate stress = $300\ \text{MN/m}^2$

and factor of safety = 6

Hence $$\text{working stress} = \frac{300}{6}$$

$$= 50\ \text{MN/m}^2$$

$$= 50\ \text{N/mm}^2$$

Now we have $$\text{working stress} = \frac{\text{load}}{\text{cross-sectional area}}$$

or $$\text{load} = (\text{cross-sectional area}) \times (\text{working stress})$$

but $$\text{cross-sectional area} = \frac{\pi}{4}(30)^2 = 707\ \text{mm}^2 \quad \text{and}$$

$$\text{working stress} = 50\ \text{N/mm}^2,$$

and substituting these values we have:

$$\text{load} = 707 \times 50 \quad \text{mm}^2\ \text{N/mm}^2$$
$$= 35\,400\ \text{N}$$
$$= 35.4\ \text{kN}$$

Now
$$E = \frac{\text{stress}}{\text{strain}}$$

∴
$$\text{strain} = \frac{\text{stress}}{E} \quad \text{(no units for strain)}$$

and substituting the values stress $= 50\ \text{N/mm}^2$ and
$E = 96\ \text{GN/m}^2 = 96 \times 10^3\ \text{N/mm}^2$,

we have:
$$\text{strain} = \frac{50}{96 \times 10^3} \quad \frac{\text{N/mm}^2}{\text{N/mm}^2}$$
$$= 0.000\,521$$

Now
$$\text{strain} = \frac{\text{compression}}{\text{original length}}$$

∴
$$\text{compression} = (\text{original length}) \times (\text{strain})$$

If we substitute original length = 122 mm and strain = 0.000 521:

$$\text{compression} = (122) \times (0.000\,521)$$
$$= 0.0636\ \text{mm}$$

EXERCISE 1.1

1) A steel rod carries an axial load of 100 kN. If the allowable tensile stress is 75 MN/m², find the minimum diameter of the rod.

2) An aluminium rod 10 mm diameter and 4 m long carries an axial load of 0.9 kN. If for aluminium $E_a = 80\ \text{GN/m}^2$, find:
(a) the stress, (b) the elongation.

3) (a) A steel wire 2 mm diameter is made from material whose ultimate tensile strength is 500 MN/m². Using a factor of safety of 5, find the maximum load which can be supported by the wire.
(b) If the wire is 7 m long and the extension is found to be 3.33 mm, find the value of the modulus of elasticity E.

4) A rectangular bar is 40 mm by 20 mm by 6 m long. Find the axial load carried by the bar if its extension is 2.00 mm. Take $E = 100\ \text{GN/m}^2$.

5) A metal bar 25 mm diameter and 200 mm long extends 0.15 mm under an axial load of 80 kN. Find:
(a) the value of E for the metal;
(b) the extension when the load is 100 kN.

6) A 0.175 m long steel bar whose diameter is 20 mm is compressed 0.075 mm by a compressive force of 35 kN. Find:

(a) the value of the modulus of elasticity for the steel;
(b) the extension under a pull of 20 kN.

7) A hollow copper cylinder 30 mm outside diameter and 18 mm inside diameter supports an axial compressive load of 50 kN. The axial compression of the cylinder is 0.21 mm and the value of E for copper is $95\ \text{GN/m}^2$. Find:

(a) the stress in the copper;
(b) the original length of the cylinder.

8) Fig. 1.5 shows the piston and piston rod of a hydraulic machine. Find the stress in the piston rod and the amount by which it is compressed if E is $200\ \text{GN/m}^2$.

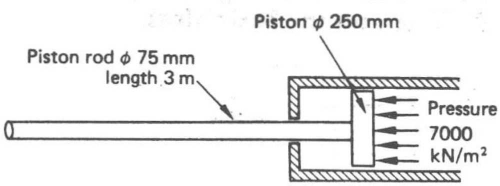

Fig. 1.5

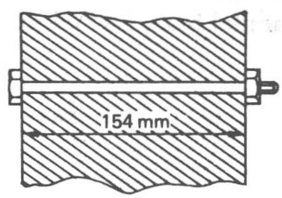

Fig. 1.6

9) The bolt shown in Fig. 1.6 has a diameter of 8 mm and a metric fine thread of 1 mm pitch. If the nut is originally tight, and neglecting any compression in the material through which the bolt passes, find the increase in the stress in the bolt due to a tightening of the nut by rotating it one eighth of a turn. Take E as $200\ \text{GN/m}^2$.

10) A turned bar is as shown in Fig. 1.7. If the stress in the centre portion is $100\ \text{MN/m}^2$, find the diameter d mm. If the modulus of elasticity for the bar material is $210\ \text{GN/m}^2$, find the total extension of the whole bar.

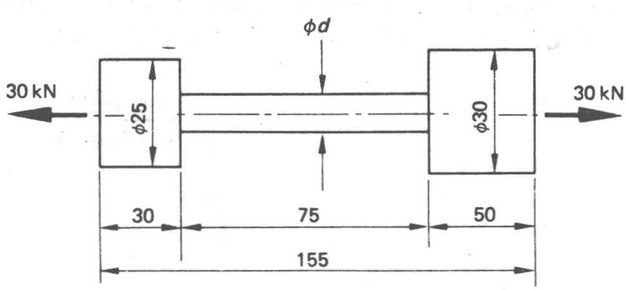

Fig. 1.7