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Advanced Microeconomic Theory

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ADDISON-WESLEY

An imprint of Addison Wesley Longman, Inc.

Reading, Massachusetts • Menlo Park, California • New York • Harlow, England
Don Mills, Ontario • Sydney • Mexico City • Madrid • Amsterdam

Advanced Microeconomic Theory
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Jehle. Geoffrey Alexander.
Advanced microeconomic theory / Geoffrey A. Jehle, Philip J. Reny.
p. cm.
Includes bibliographical references (p.485-487) and index.
ISBN 0-321-01436-7
1. Microeconomics. 2. Economics, Mathematical. I. Reny, Philip J.
II. Title
HB172.J44 1998
97-25251
338.5--dc21
CIP

2 3 4 5 6 7 8 9 10—MA—01009998

Preface

This book is for first-year graduate students and advanced undergraduates. It will help to bridge the very wide gap between a good undergraduate preparation in economics and the preparation in mathematics and modern microeconomic theory that students are expected to acquire quickly at the graduate level.

Part I provides a lengthy and largely self-contained development of the set theory, real analysis, topology, calculus, and modern optimization theory which are indispensable tools of modern microeconomics. The exposition is formal but presumes nothing more than a good grounding in single-variable calculus and simple linear algebra as a starting point. A student with good college-level mathematics will be able to learn most of what is needed by carefully reading and working exercises. Those with more extensive mathematics training will find this part useful as a reference source for most of the principal mathematical theorems that arise in microeconomic theory.

Part II is devoted to modern consumer and producer theories. We treat consumer theory first, and at some length—both because of its intrinsic importance, and because its methods and results are paradigms for many other topic areas. Producer theory is next, and we draw attention to the many formal similarities between these two important building blocks of modern microeconomics.

In Part III we examine the behavior of economic agents when they come together on markets. We develop neoclassical models of competitive and noncompetitive market structures, and we take a first look at how market structure, market equilibrium, and economic efficiency are related. This is followed by the theory of general equilibrium, where we consider some of those same questions at the level of the economic system. The chapter on social choice and welfare provides an accessible introduction to systematic normative analysis.

Strategic behavior is the subject of Part IV. A lengthy chapter on game theory provides careful and detailed instruction on both strategic and extensive form games. This is followed by a final chapter on the economics of information, where we adopt a game-theoretic approach to this important area of current activity and interest.

In every chapter of the book, our focus is on the modern core of its respective area. In this way, the careful reader can build a deep understanding of the principal pillars of modern microeconomics, and can see the connections between them. There are exercises at the end of every chapter, and working through as many of these as possible is the surest way to master the material. Hints and answers to selected exercises are provided at the back of the book.

We would like to thank Jack Greenman for supporting this project from the beginning. It has been a pleasure to work with him. We are also very grateful to Eric Howe, Marc Lieberman, David Pearce, Douglass Shaw, and Eugene Silberberg for many constructive comments.

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PART I

Language and Methods

C H A P T E R 1

Sets and Mappings

1.1 ELEMENTS OF LOGIC

Important ideas in the economics literature are often stated in the form of **theorems**. A theorem is simply a statement deduced from other statements, and you should be familiar with them from your courses in mathematics. Theorems provide a compact and precise format for presenting the assumptions and important conclusions of sometimes lengthy arguments, and so help to identify immediately the scope and limitations of the result presented. Theorems must be proved, however, and a proof consists of establishing the validity of the statement in the theorem in a way that is consistent with the rules of logic. We begin with some of the language and simple rules of logic that you will often encounter and soon begin using yourself.

1.1.1 Necessity and Sufficiency

Necessity and **sufficiency** are fundamental logical notions. In ordinary language, when you make the statement, "A is necessary for B," what do you mean? When you make the statement, "A is sufficient for B," what do you mean? It is worth a moment's reflection on the different meaning of these two statements.

Consider any two statements, A and B. When we say, "A is necessary for B," we mean that A must hold or be true in order for B to hold or be true. For B to be true requires A to be true, so whenever B is true, we know that A must also be true. So we might have said, instead, that "A is true *if* B is true," or simply that "A *is implied by* B" (A $\Leftarrow$ B). There is, therefore, an equivalence in ordinary language and in logic between the phrases "A is necessary for B," "A if B," and "A is implied by B."

Suppose we know that "A $\Leftarrow$ B" is a true statement. What if A is not true? Because A is necessary for B, when A is not true, then B cannot be true either. But doesn't this just say that "B not true" is necessary for "A not true"? Or that "not-B is implied by not-A" ($\sim$ B $\Leftarrow$ $\sim$ A)? This latter form of the original statement is called the **contrapositive** form. Contraposition of the arguments in the statement *reverses* the direction of implication for a true statement.

Let's consider a simple illustration of these ideas. Let A be the phrase "x is an integer less than 10." Let B be the statement, "x is an integer less than 8." Clearly, A is necessary for B because "x is an integer less than 10" is implied by the statement "x is an integer less than 8." If we form the contrapositive of these two statements, the statement $\sim$ A becomes "x is not an integer less than 10," and the statement $\sim$ B becomes "x is not an integer less than 8." Beware that the statement $\sim \text{A} \Leftarrow \sim \text{B}$ is *false*. The value of x could well be 9. We must *reverse* the direction of implication to obtain a contrapositive statement that is also true. The proper contrapositive statement, therefore, would be: "x is not an integer less than 8" is implied by "x is not an integer less than 10," or $\sim \text{B} \Leftarrow \sim \text{A}$.

The notion of necessity is distinct from that of *sufficiency*. When we say, "A is sufficient for B," we mean that whenever A holds, B must hold. We can say, "A is true *only if* B is true," or that "A *implies* B" ($\text{A} \Rightarrow \text{B}$). Once again, whenever the statement $\text{A} \Rightarrow \text{B}$ is true, the contrapositive statement, $\sim \text{B} \Rightarrow \sim \text{A}$, is also true.

Two implications, "$\text{A} \Rightarrow \text{B}$" and "$\text{A} \Leftarrow \text{B}$," can both be true. When this is so, we say that "A is necessary and sufficient for B," or that "A is true if and only if B is true," or "A iff B." When A is necessary and sufficient for B, we say that the statements A and B are **equivalent** and write "$\text{A} \Longleftrightarrow \text{B}$."

To illustrate briefly, suppose that A and B are the following statements: A $\equiv$ "X is yellow," B $\equiv$ "X is a lemon." Certainly, if X is a lemon, then X is yellow. Here, A is necessary for B. At the same time, just because X is yellow does not mean that it must be a lemon. It could be a banana. So A is *not* sufficient for B. Suppose instead that the statements are: A $\equiv$ "X is a sour, yellow-skinned fruit" and B $\equiv$ "X is a lemon." Here, A is implied by B, or A is necessary for B. If X is a lemon, then it must be a yellow and sour fruit. At the same time, A implies B, or A is sufficient for B, because if X is a yellow and sour fruit, it must be a lemon. Because A is necessary and sufficient for B, there must be an *equivalence* between lemons and sour, yellow-skinned fruit.

1.1.2 Theorems and Proofs

Mathematical theorems usually have the form of an implication or an equivalence, where one or more statements are alleged to be related in particular ways. Suppose we have the theorem "$\text{A} \Rightarrow \text{B}$." Here, A is called the **premise** and B the **conclusion.** To prove a theorem is to establish the validity of its conclusion given the truth of its premise, and several methods can be used to do that.

In a **constructive proof**, we assume that A is true, deduce various consequences of that, and use them to show that B must also hold. This is also sometimes called a **direct proof**, for obvious reasons. There is also the **contrapositive proof**. In a contrapositive proof, we assume that B does *not* hold, then show that A cannot hold. This approach takes advantage of the logical equivalence between the claims "$\text{A} \Rightarrow \text{B}$" and "$\sim \text{B} \Rightarrow \sim \text{A}$" noted earlier, and essentially involves a constructive proof of the contrapositive to the original statement. Finally, there is a **proof by contradiction, or reductio ad absurdum.** Here, the strategy is to assume that A is true, assume that B is *not* true, and attempt to derive a logical contradiction. This approach relies on the fact that if $\text{A} \Rightarrow \sim \text{B}$ is false, then $\text{A} \Rightarrow \text{B}$ must be true. Sometimes, proofs by contradiction can get the job done very efficiently, yet because they involve no constructive chain of reasoning between A and B as the other two do, they very seldom illuminate the relationship between the premise and the conclusion.

If we assert that A is *necessary and sufficient* for B, or that A $\iff$ B, we must give a proof in "both directions." That is, both A $\Rightarrow$ B *and* B $\Rightarrow$ A must be established before a complete proof of the assertion has been achieved.

It never hurts to keep in mind the old saying that goes, "Proof by example is no proof." Suppose the statements A $\equiv$ "x is a student" and B $\equiv$ "x has red hair" are given, and we make the assertion A $\Rightarrow$ B. Then clearly finding one student with red hair and pointing him out to you is not going to convince you of anything. Examples are good for illustrating, but typically not for proving. In this book, we will never pretend to have proven something by providing an example that merely illustrates the point. Occasionally, however, it will be necessary to state some theorems without proof. It is hoped that being asked to accept things on faith will just encourage you to explore beyond the scope of this book.

Finally, a sort of converse to the old saying about examples and proofs should be noted. Whereas citing a hundred examples can never prove a certain property *always* holds, citing one solitary *counterexample* can *disprove* that the property always holds. For instance, to disprove the assertion about the color of students' hair, you need simply point out one student with brown hair. A counterexample proves that the claim cannot *always* be true because you have found at least one case where it is not.

1.2 ELEMENTS OF SET THEORY

1.2.1 Notation and Basic Concepts

The language and methods of set theory have thoroughly infiltrated microeconomic theory. No doubt the notion of a set and many basic rules of manipulating sets are familiar. Nonetheless, because we will encounter some of them repeatedly, it is best to review a few of the basics.

A **set** is any collection of elements. Sets can be defined by *enumeration* of their elements, e.g., $S = \{2, 4, 6, 8\}$, or by *description* of their elements, e.g., $S = \{\, x \mid x$ is a positive even integer greater than zero and less than $10\}$. When we wish to denote membership or inclusion in a set, we use the symbol $\in$. For example, if $S = \{2, 5, 7\}$, we say that $5 \in S$.

A set S is a **subset** of another set T if every element of S is also an element of T. We write $S \subset T$ (S is contained in T) or $T \supset S$ (T contains S). If $S \subset T$, then $x \in S \Rightarrow x \in T$.

Two sets are **equal sets** if they each contain exactly the same elements. We write $S = T$ whenever $x \in S \Rightarrow x \in T$ and $x \in T \Rightarrow x \in S$. Thus, S and T are equal sets if and only if $S \subset T$ *and* $T \subset S$. For example, if $S = \{\text{integers}, x \mid x^2 = 1\}$ and $T = \{-1, 1\}$, then $S = T$.

A set S is **empty** or is an **empty set** if it contains no elements at all. For example, if $A = \{x \mid x^2 = 0 \text{ and } x > 1\}$, then A is empty. We denote the empty set by the symbol $\emptyset$ and write $A = \emptyset$.

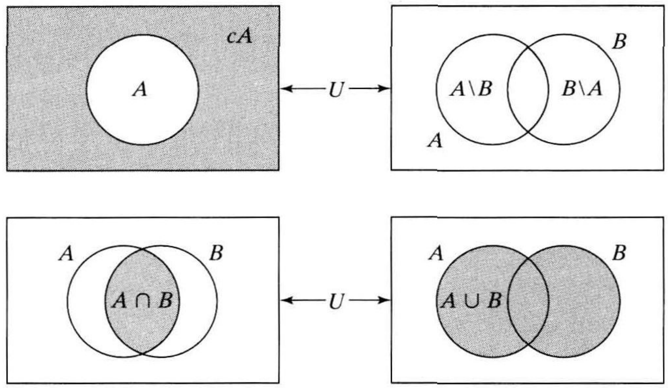

Figure 1.1. Venn diagrams.

The **complement** of a set S in a universal set U is the set of all elements in U that are not in S and is denoted S^c. If $U = \{2, 4, 6, 8\}$ and $S = \{4, 6\}$, then $S^c = \{2, 8\}$. More generally, for any two sets S and T in a universal set U, we define the **set difference** denoted $S \setminus T$, or $S - T$, as all elements in the set S that are not elements of T. Thus, we can think of the complement of the set S in U as the set difference $S^c = U \setminus S$.

The basic operations on sets are **union** and **intersection**. They correspond to the logical notions of "or" and "and," respectively.[1] For two sets S and T, we define the *union* of S and T as the set $S \cup T \equiv \{x \mid x \in S \text{ or } x \in T\}$. We define the *intersection* of S and T as the set $S \cap T \equiv \{x \mid x \in S \text{ and } x \in T\}$. Some of these sets are illustrated in Figure 1.1.

Sometimes we want to examine sets constructed from an arbitrary number of other sets. We could use some notation such as $\{S_1, S_2, S_3, \ldots\}$ to denote the set of all sets that concern us, but it is more common to collect the necessary (possibly infinite) number of integers starting with 1 into a set, $I \equiv \{1, 2, 3, \ldots\}$, called an **index set,** and denote the collection of sets more simply as $\{S_i\}_{i \in I}$. We would denote the union of all sets in the collection by $\cup_{i \in I} S_i$, and the intersection of all sets in the collection as $\cap_{i \in I} S_i$.

The product of two sets S and T is the set of "ordered pairs" in the form (s, t), where the first element in the pair is a member of S and the second is a member of T. The product of S and T is denoted

$$S \times T \equiv \{(s, t) \mid s \in S, t \in T\}.$$

One familiar set product is the "Cartesian plane." This is the plane in which you commonly graph things. It is the visual representation of a set product constructed from the set of real numbers. The set of real numbers is denoted by the special symbol $\mathbf{R}$ and is defined as

$$\mathbf{R} \equiv \{x \mid -\infty < x < \infty\}.$$

If we form the set product

[1]In everyday language, the word "or" can be used in two senses. One, called the *exclusive* "or," carries the meaning "either, but not both." In mathematics, the word "or" is used in the *inclusive* sense. The *inclusive* "or" carries the meaning "either or both."