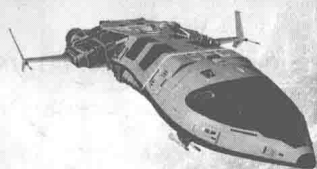


PRINCIPLES OF QUANTUM ARTIFICIAL INTELLIGENCE

Andreas Wichert

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for André

Preface

Artificial intelligence and quantum computation divide the subject into many major areas. Each of these areas are now so extensive and huge, that a major understanding of the core concepts that unite them is extremely difficult. This book is about the core ideas of artificial intelligence and quantum computation. They are united in new subarea of artificial intelligence: “Quantum Artificial Intelligence”.

The book is composed of two sections: the first is on classical computation and the second section is on quantum computation. In the first section, we introduce the basic principles of computation, representation and problem solving. In the second section, we introduce the principles of quantum computation and their relation to the core ideas of artificial intelligence, such as search and problem solving. We illustrate their use with several examples.

The notes on which the book is based evolved in the course “Information and Computation for Artificial Intelligence” in the years 2008 – 2012 at Department of Computer Science and Engineering, Instituto Superior Técnico, Technical University of Lisbon. Thanks to Technical University of Lisbon for rewarding me a sabbatical leave in the 2012-2013 academic year, which has given me the time to finish this book. My research in recent years has benefited from many discussions with Ana Paiva, Luís Tarrataca, Ângelo Cardoso, João Sacramento and Catarina Moreira. Especially I would like to thank Luís Tarrataca and offer all of him deepest gratitude. The chapter about “Quantum Problem-Solving” is mainly based on his work. Finally, I would like to thank my loving wife *Manuela*, without her encouragement the book would be never finished.

Andreas Wichert

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Chapter 1

Introduction

Symbolical artificial intelligence is a field of computer science that is highly related to quantum computation. At first glance, this statement appears to be a contradiction. However, the artificial intelligence framework, such as search and production system theory, allows an elegant description of a quantum computer model that is capable of quickly executing programs.

1.1 Artificial Intelligence

Artificial intelligence (AI) is a subfield of computer science that models the mechanisms of intelligent human behavior (intelligence). This approach is accomplished via simulation with the help of artificial artifacts, typically with computer programs on a machine that performs calculations. It should be noted that the machine does not need to be electronic. Indeed, Charles Babbage (1791-1871) sketched the first mechanical machine (a difference engine) for the calculation of certain values of polynomial functions [Hyman (1985)]. With the goal of mechanizing calculation steps, Babbage sketched the first model of a mechanical universal computer and called it an analytical engine. At the same time, Lady Ada Lovelance (1815-1852) thought about the computing power of such a machine. She argued that such a machine could only perform what it was told to do; such a machine could not generate new knowledge.

The term “artificial intelligence” itself was invented by the American computer scientist John McCarthy. It was used in the title of a conference that took place in the year 1956 at Dartmouth College in the USA. During this meeting, programs were presented that played chess and checkers, proved theorems and interpreted texts. The programs were thought to simulate human intelligent behavior. However, the terms “intelligence”

and “intelligent human behavior” are not very well defined and understood. The definition of artificial intelligence leads to the paradox of a discipline whose principal purpose is its own definition.

A.M. Turing (1912-1954), in 1950, wrote the essay “Computing Machinery and Intelligence”, in which he poses the question of how to determine whether a program is intelligent or not [Turing (1950)]. He defines intelligence as the reaction of an intelligent being to certain questions. This behavior can be tested by the so-called Turing test. A subject communicates over a computer terminal with two non-visible partners, a program and a human. If the subject cannot differentiate between the human and the program, the program is called intelligent. The questions posed can originate from any domain. However, if the domain is restricted, then the test is called a restricted Turing test. A restricted domain could be, for example, a medical diagnosis or the game of chess.

Human problem-solving algorithms are studied in Artificial Intelligence. The key idea behind these algorithms is the symbolic representation of the domain in which the problems are solved. Symbols are used to denote or refer to something other than themselves, namely other things in the world (according to the, pioneering work of Tarski [Tarski (1944, 1956, 1995)]). They are defined by their occurrence in a structure and by a formal language which manipulates these structures [Simon (1991); Newell (1990)] (see Figure 1.1). In this context, symbols do not, by themselves, represent any utilizable knowledge. For example, they cannot be used for a definition of similarity criteria between themselves. The use of symbols in algorithms which imitate human intelligent behavior led to the famous physical symbol system hypothesis by Newell and Simon (1976) [Newell and Simon (1976)]: “The necessary and sufficient condition for a physical system to exhibit intelligence is that it be a physical symbol system.” Symbols are not present in the world; they are the constructs of a human mind and simplify the process of representation used in communication and problem solving.

1.2 Motivation and Goals

Traditional AI is built around abstract algorithms and data structures that manipulate symbols. One of the important algorithms is the tree or graph search. Common forms of knowledge representation are symbolic rules and semantic nets. Traditional AI attempts to imitate human behavior without