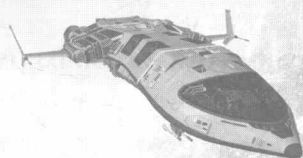


# PRINCIPLES OF QUANTUM ARTIFICIAL INTELLIGENCE



Andreas Wichert

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PRINCIPLES OF  
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ARTIFICIAL  
INTELLIGENCE

for André

# Preface

Artificial intelligence and quantum computation divide the subject into many major areas. Each of these areas are now so extensive and huge, that a major understanding of the core concepts that unite them is extremely difficult. This book is about the core ideas of artificial intelligence and quantum computation. They are united in new subarea of artificial intelligence: “Quantum Artificial Intelligence”.

The book is composed of two sections: the first is on classical computation and the second section is on quantum computation. In the first section, we introduce the basic principles of computation, representation and problem solving. In the second section, we introduce the principles of quantum computation and their relation to the core ideas of artificial intelligence, such as search and problem solving. We illustrate their use with several examples.

The notes on which the book is based evolved in the course “Information and Computation for Artificial Intelligence” in the years 2008 – 2012 at Department of Computer Science and Engineering, Instituto Superior Técnico, Technical University of Lisbon. Thanks to Technical University of Lisbon for rewarding me a sabbatical leave in the 2012-2013 academic year, which has given me the time to finish this book. My research in recent years has benefited from many discussions with Ana Paiva, Luís Tarrataca, Ângelo Cardoso, João Sacramento and Catarina Moreira. Especially I would like to thank Luís Tarrataca and offer all of him deepest gratitude. The chapter about “Quantum Problem-Solving” is mainly based on his work. Finally, I would like to thank my loving wife *Manuela*, without her encouragement the book would be never finished.

*Andreas Wichert*

# Contents

<i>Preface</i>	vii
1. Introduction	1
1.1 Artificial Intelligence . . . . .	1
1.2 Motivation and Goals . . . . .	2
1.3 Guide to the Reader . . . . .	4
1.4 Content . . . . .	4
1.4.1 Classical computation . . . . .	4
1.4.2 Quantum computation . . . . .	6
2. Computation	9
2.1 Entscheidungsproblem . . . . .	9
2.1.1 Cantor's diagonal argument . . . . .	11
2.1.2 Reductio ad absurdum . . . . .	11
2.2 Complexity Theory . . . . .	12
2.2.1 Decision problems . . . . .	13
2.2.2 P and NP . . . . .	13
2.3 Church–Turing Thesis . . . . .	14
2.3.1 Church–Turing–Deutsch principle . . . . .	15
2.4 Computers . . . . .	15
2.4.1 Analog computers . . . . .	15
2.4.2 Digital computers . . . . .	16
2.4.3 Von Neumann architecture . . . . .	16
3. Problem Solving	19
3.1 Knowledge Representation . . . . .	19

3.1.1	Rules . . . . .	19
3.1.2	Logic-based operators . . . . .	20
3.1.3	Frames . . . . .	23
3.1.4	Categorical representation . . . . .	23
3.1.5	Binary vector representation . . . . .	24
3.2	Production System . . . . .	25
3.2.1	Deduction systems . . . . .	26
3.2.2	Reaction systems . . . . .	28
3.2.3	Conflict resolution . . . . .	28
3.2.4	Human problem-solving . . . . .	29
3.2.5	Example . . . . .	29
3.3	Sub-Symbolic Models of Problem-Solving . . . . .	30
3.3.1	Proto logic . . . . .	31
3.3.2	Binding problem . . . . .	32
3.3.3	Icons . . . . .	32
3.3.4	Euclidian geometry of the world . . . . .	35
4.	Information . . . . .	37
4.1	Information and Thermodynamics . . . . .	37
4.1.1	Dice model . . . . .	39
4.1.2	Entropy . . . . .	40
4.1.3	Maxwell paradox and information . . . . .	41
4.1.4	Information theory . . . . .	42
4.2	Hierarchical Structures . . . . .	47
4.2.1	Example of a taxonomy . . . . .	49
4.3	Information and Measurement . . . . .	50
4.3.1	Information measure $I$ . . . . .	52
4.3.2	Nature of information measure . . . . .	55
4.3.3	Measurement of angle . . . . .	55
4.3.4	Information and contour . . . . .	56
4.4	Information and Memory . . . . .	57
4.5	Sparse code for Sub-symbols . . . . .	67
4.5.1	Sparsification based on unary sub-vectors . . . . .	68
4.6	Deduction Systems and Associative Memory . . . . .	68
4.6.1	Taxonomic knowledge organization . . . . .	74
5.	Reversible Algorithms . . . . .	75
5.1	Reversible Computation . . . . .	75



5.2	Reversible Circuits . . . . .	76
5.2.1	Boolean gates . . . . .	76
5.2.2	Reversible Boolean gates . . . . .	76
5.2.3	Toffoli gate . . . . .	77
5.2.4	Circuit . . . . .	78
6.	Probability . . . . .	79
6.1	Kolmogorovs Probabilities . . . . .	79
6.1.1	Conditional probability . . . . .	80
6.1.2	Bayes's rule . . . . .	81
6.1.3	Joint distribution . . . . .	82
6.1.4	Naïve Bayes and counting . . . . .	84
6.1.5	Counting and categorization . . . . .	85
6.1.6	Bayesian networks . . . . .	86
6.2	Mixed Distribution . . . . .	90
6.3	Markov Chains . . . . .	91
7.	Introduction to Quantum Physics . . . . .	95
7.1	Unitary Evolution . . . . .	95
7.1.1	Schrödinger's cat paradox . . . . .	96
7.1.2	Interpretations of quantum mechanics . . . . .	96
7.2	Quantum Mechanics . . . . .	97
7.2.1	Stochastic Markov evolution and unitary evolution . . . . .	98
7.3	Hilbert Space . . . . .	99
7.3.1	Spectral representation* . . . . .	101
7.4	Quantum Time Evolution . . . . .	103
7.5	Compound Systems . . . . .	105
7.6	Von Neumann Entropy . . . . .	108
7.7	Measurement . . . . .	109
7.7.1	Observables . . . . .	110
7.7.2	Measuring a compound system . . . . .	111
7.7.3	Heisenberg's uncertainty principle* . . . . .	112
7.8	Randomness . . . . .	114
7.8.1	Deterministic chaos . . . . .	114
7.8.2	Kolmogorov complexity . . . . .	114
7.8.3	Humans and random numbers . . . . .	116
7.8.4	Randomness in quantum physics . . . . .	116
8.	Computation with Qubits . . . . .	119

8.1	Computation with one Qubit . . . . .	119
8.2	Computation with $m$ Qubit . . . . .	121
8.3	Matrix Representation of Serial and Parallel Operations . . . . .	123
8.4	Entanglement . . . . .	125
8.5	Quantum Boolean Circuits . . . . .	127
8.6	Deutsch Algorithm . . . . .	130
8.7	Deutsch Jozsa Algorithm . . . . .	132
8.8	Amplitude Distribution . . . . .	135
8.8.1	Cloning . . . . .	136
8.8.2	Teleportation . . . . .	137
8.9	Geometric Operations . . . . .	139
9.	Periodicity . . . . .	145
9.1	Fourier Transform . . . . .	145
9.2	Discrete Fourier Transform . . . . .	147
9.2.1	Example . . . . .	149
9.3	Quantum Fourier Transform . . . . .	150
9.4	FFT . . . . .	153
9.5	QFT Decomposition . . . . .	154
9.5.1	QFT quantum circuit* . . . . .	155
9.6	QFT Properties . . . . .	159
9.7	The QFT Period Algorithm . . . . .	161
9.8	Factorization . . . . .	164
9.8.1	Example . . . . .	164
9.9	Kitaev's Phase Estimation Algorithm* . . . . .	168
9.9.1	Order finding . . . . .	170
9.10	Unitary Transforms . . . . .	171
10.	Search . . . . .	173
10.1	Search and Quantum Oracle . . . . .	173
10.2	Lower Bound $\Omega(\sqrt{n})$ for $U_f$ -based Search* . . . . .	175
10.2.1	Lower bound of $a_t$ . . . . .	176
10.2.2	Upper bound of $a_t$ . . . . .	178
10.2.3	$\Omega(\sqrt{n})$ . . . . .	179
10.3	Grover's Amplification . . . . .	180
10.3.1	Householder reflection . . . . .	180
10.3.2	Householder reflection and the mean value . . . . .	181
10.3.3	Amplification . . . . .	182

10.3.4	Iterative amplification . . . . .	184
10.3.5	Number of iterations . . . . .	191
10.3.6	Quantum counting . . . . .	192
10.4	Circuit Representation . . . . .	194
10.5	Speeding up the Traveling Salesman Problem . . . . .	195
10.6	The Generate-and-Test Method . . . . .	196
11.	Quantum Problem-Solving . . . . .	199
11.1	Symbols and Quantum Reality . . . . .	199
11.2	Uninformed Tree Search . . . . .	200
11.3	Heuristic Search . . . . .	203
11.3.1	Heuristic functions . . . . .	205
11.3.2	Invention of heuristic functions . . . . .	205
11.3.3	Quality of heuristic . . . . .	207
11.4	Quantum Tree Search . . . . .	208
11.4.1	Principles of quantum tree search . . . . .	208
11.4.2	Iterative quantum tree search . . . . .	210
11.4.3	No constant branching factor . . . . .	211
11.5	Quantum Production System . . . . .	212
11.6	Tarrataca's Quantum Production System . . . . .	213
11.6.1	3-puzzle . . . . .	213
11.6.2	Extending for any $n$ -puzzle . . . . .	217
11.6.3	Pure production system . . . . .	218
11.6.4	Unitary control strategy . . . . .	219
11.7	A General Model of a Quantum Computer . . . . .	220
11.7.1	Cognitive architecture . . . . .	220
11.7.2	Representation . . . . .	221
12.	Quantum Cognition . . . . .	223
12.1	Quantum Probability . . . . .	223
12.2	Decision Making . . . . .	226
12.2.1	Interference . . . . .	231
12.3	Unpacking Effects . . . . .	232
12.4	Conclusion . . . . .	233
13.	Related Approaches . . . . .	235
13.1	Quantum Walk . . . . .	235
13.1.1	Random walk . . . . .	235

13.1.2	Quantum insect . . . . .	235
13.1.3	Quantum walk on a graph . . . . .	237
13.1.4	Quantum walk on one dimensional lattice . . . . .	238
13.1.5	Quantum walk and search . . . . .	239
13.1.6	Quantum walk for formula evaluation . . . . .	239
13.2	Adiabatic Computation . . . . .	240
13.2.1	Quantum annealing . . . . .	241
13.3	Quantum Neural Computation . . . . .	243
13.4	Epilogue . . . . .	245
<i>Bibliography</i>		247
<i>Index</i>		259

## Chapter 1

# Introduction

Symbolical artificial intelligence is a field of computer science that is highly related to quantum computation. At first glance, this statement appears to be a contradiction. However, the artificial intelligence framework, such as search and production system theory, allows an elegant description of a quantum computer model that is capable of quickly executing programs.

### 1.1 Artificial Intelligence

Artificial intelligence (AI) is a subfield of computer science that models the mechanisms of intelligent human behavior (intelligence). This approach is accomplished via simulation with the help of artificial artifacts, typically with computer programs on a machine that performs calculations. It should be noted that the machine does not need to be electronic. Indeed, Charles Babbage (1791-1871) sketched the first mechanical machine (a difference engine) for the calculation of certain values of polynomial functions [Hyman (1985)]. With the goal of mechanizing calculation steps, Babbage sketched the first model of a mechanical universal computer and called it an analytical engine. At the same time, Lady Ada Lovelance (1815-1852) thought about the computing power of such a machine. She argued that such a machine could only perform what it was told to do; such a machine could not generate new knowledge.

The term “artificial intelligence” itself was invented by the American computer scientist John McCarthy. It was used in the title of a conference that took place in the year 1956 at Dartmouth College in the USA. During this meeting, programs were presented that played chess and checkers, proved theorems and interpreted texts. The programs were thought to simulate human intelligent behavior. However, the terms “intelligence”

and “intelligent human behavior” are not very well defined and understood. The definition of artificial intelligence leads to the paradox of a discipline whose principal purpose is its own definition.

A.M. Turing (1912-1954), in 1950, wrote the essay “Computing Machinery and Intelligence”, in which he poses the question of how to determine whether a program is intelligent or not [Turing (1950)]. He defines intelligence as the reaction of an intelligent being to certain questions. This behavior can be tested by the so-called Turing test. A subject communicates over a computer terminal with two non-visible partners, a program and a human. If the subject cannot differentiate between the human and the program, the program is called intelligent. The questions posed can originate from any domain. However, if the domain is restricted, then the test is called a restricted Turing test. A restricted domain could be, for example, a medical diagnosis or the game of chess.

Human problem-solving algorithms are studied in Artificial Intelligence. The key idea behind these algorithms is the symbolic representation of the domain in which the problems are solved. Symbols are used to denote or refer to something other than themselves, namely other things in the world (according to the, pioneering work of Tarski [Tarski (1944, 1956, 1995)]). They are defined by their occurrence in a structure and by a formal language which manipulates these structures [Simon (1991); Newell (1990)] (see Figure 1.1). In this context, symbols do not, by themselves, represent any utilizable knowledge. For example, they cannot be used for a definition of similarity criteria between themselves. The use of symbols in algorithms which imitate human intelligent behavior led to the famous physical symbol system hypothesis by Newell and Simon (1976) [Newell and Simon (1976)]: “The necessary and sufficient condition for a physical system to exhibit intelligence is that it be a physical symbol system.” Symbols are not present in the world; they are the constructs of a human mind and simplify the process of representation used in communication and problem solving.

## 1.2 Motivation and Goals

Traditional AI is built around abstract algorithms and data structures that manipulate symbols. One of the important algorithms is the tree or graph search. Common forms of knowledge representation are symbolic rules and semantic nets. Traditional AI attempts to imitate human behavior without

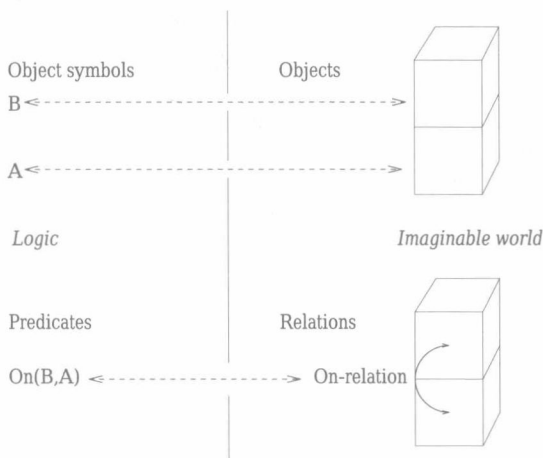


Fig. 1.1 Object represented by symbols and relation represented by predicate.

any relationship to physical reality, for example, in biological hardware. Sub-Symbolical processing, on the other hand, belongs to biology-inspired AI, which involves methods such as neural networks or behavioral systems. Could the physical nature, as described by quantum physics, also lead to algorithms that imitate human behavior? What are the possibilities for the realization of artificial intelligence by means of quantum computation? Computational algorithms that are inspired by this physical reality are described by quantum computation. We will answer questions such as why and how to use quantum algorithms in artificial intelligence.

Questions that appear to be quite simple, such as: what are random numbers and how can we generate them, cannot be answered by traditional computer science. The widely used pseudo random generators are based on deterministic procedures and do not generate randomness; instead, they generate pseudo-randomness. Pseudo random generators are related to deterministic chaos sequences, which are described by mathematical chaos theory. Chaotic patterns can arise from very simple mathematics. While the results can be similar to true randomness, the output patterns are generated by deterministic rules. Chaotic patterns differ from most deterministic systems because any small change made to their variables can result in unpredictable changes to the system behavior.

Recently, quantum algorithms for AI were proposed, including a quantum tree search algorithm and a quantum production system [Tarrataca and Wichert (2011b,a, 2012b, 2013b)]. In this book, we introduce quantum computation and its application to AI. Based on information science, we will illustrate the general principles that govern information processing and information structures.

### 1.3 Guide to the Reader

This book is about some core ideas of artificial intelligence and quantum computation and is composed of two sections: the first is on classical computation and the second section is on quantum computation. In the first section, we introduce the basic principles of computation, representation and problem solving. Quantum physics is based on information theory and probability theory. We present both theories and indicate their relationships to artificial intelligence by associative memory and Bayesian networks. In the second section, we introduce the principles of quantum computation and its mathematical framework. We present two principles on which quantum computation is based, the discrete Fourier transform and Grover's algorithm. Based on these principles, we introduce the iterative quantum tree search algorithm that speeds up the search. In the next step we introduce a quantum production system on which a universal quantum computer model is based. Finally, related topics such as quantum cognition and quantum random walk are presented. Readers who want to develop a general understanding of the quantum computation mathematical framework should read the second section, beginning with the chapter "Introduction to Quantum Physics". Sections that go more into detail are marked by a star "\*" and can be skipped on the first reading.

### 1.4 Content

#### 1.4.1 *Classical computation*

**Computation - Chapter 2** The Entscheidungsproblem is presented, and the Turing machine is introduced. The proof of the Entscheidungsproblem is based on Cantor's diagonal argument and Gödelization. The Universal Turing machine is an abstract model of a computer. Computational complexity theory addresses questions regarding which problems can be



solved in a finite amount of time on a computer. The Church–Turing thesis states that any algorithmic process can be simulated on a Turing machine. Two classes of practical computers are presented: analog and digital computers.

**Problem Solving - Chapter 3** In the first step, the knowledge representation framework is introduced: rules, logic-based operators, frames and categorical representations. In the next step, production systems are introduced. A production system is a model of human problem solving. It is composed of long-term memory and working memory, which is also called short-term memory. We distinguish between deduction systems and reaction systems. Planning can be performed more easily by reaction systems in which the premise specifies the conditions that must be satisfied; in this way, the condition that specifies an action can be undertaken. An 8-puzzle example is presented. There is an assumption that the distance between states in the problem space is related to the distance between the sub-symbols that represent the states in sub-symbolical problem-solving. Sub-symbolical problem-solving takes advantage of the geometric nature of the world.

**Information - Chapter 4** Information theory is highly related to mathematical probability theory and thermodynamics. Entropy is a measure of the disorder of the configuration of states and can be described by a dice model. The Maxwell paradox identifies information with a negative measure of entropy. The ideal entropy represents the minimal number of optimal questions that must be addressed to know the result of an experiment. We will indicate the relationships between information and hierarchical structures and measurement. In the section on information and memory, we will introduce a biologically inspired model of associative memory. The Information and storage capacity of the model is high, given that the binary representation is sparse. Finally, a deduction system based on associative memory is presented.

**Reversible Algorithms - Chapter 5** Bennett (1973) showed that irreversible overwriting of one bit causes at least  $k \cdot T \cdot \log 2$  joules of energy dissipation, where  $k$  is Boltzmann's constant and  $T$  is the absolute temperature. Bennett also indicated that this lower bound can be ignored when using reversible computation. Reversible computing is a model of computing in which the computational process is reversible. Reversible Boolean gates and circuits are described.