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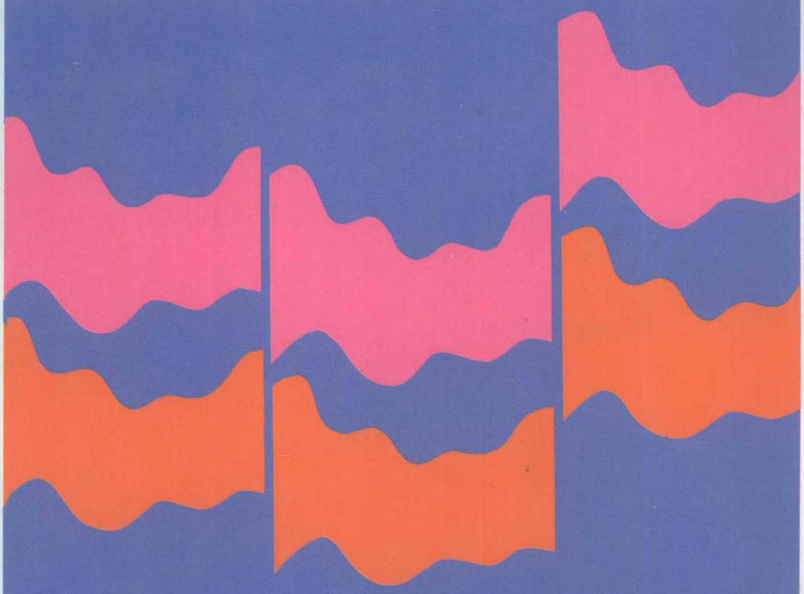

MATHEMATICAL ANALYSIS

SECOND EDITION

APOSTOL

(美) Tom M. Apostol 著

加州理工学院

机械工业出版社

China Machine Press

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PREFACE

A glance at the table of contents will reveal that this textbook treats topics in analysis at the "Advanced Calculus" level. The aim has been to provide a development of the subject which is honest, rigorous, up to date, and, at the same time, not too pedantic. The book provides a transition from elementary calculus to advanced courses in real and complex function theory, and it introduces the reader to some of the abstract thinking that pervades modern analysis.

The second edition differs from the first in many respects. Point set topology is developed in the setting of general metric spaces as well as in Euclidean n-space, and two new chapters have been added on Lebesgue integration. The material on line integrals, vector analysis, and surface integrals has been deleted. The order of some chapters has been rearranged, many sections have been completely rewritten, and several new exercises have been added.

The development of Lebesgue integration follows the Riesz-Nagy approach which focuses directly on functions and their integrals and does not depend on measure theory. The treatment here is simplified, spread out, and somewhat rearranged for presentation at the undergraduate level.

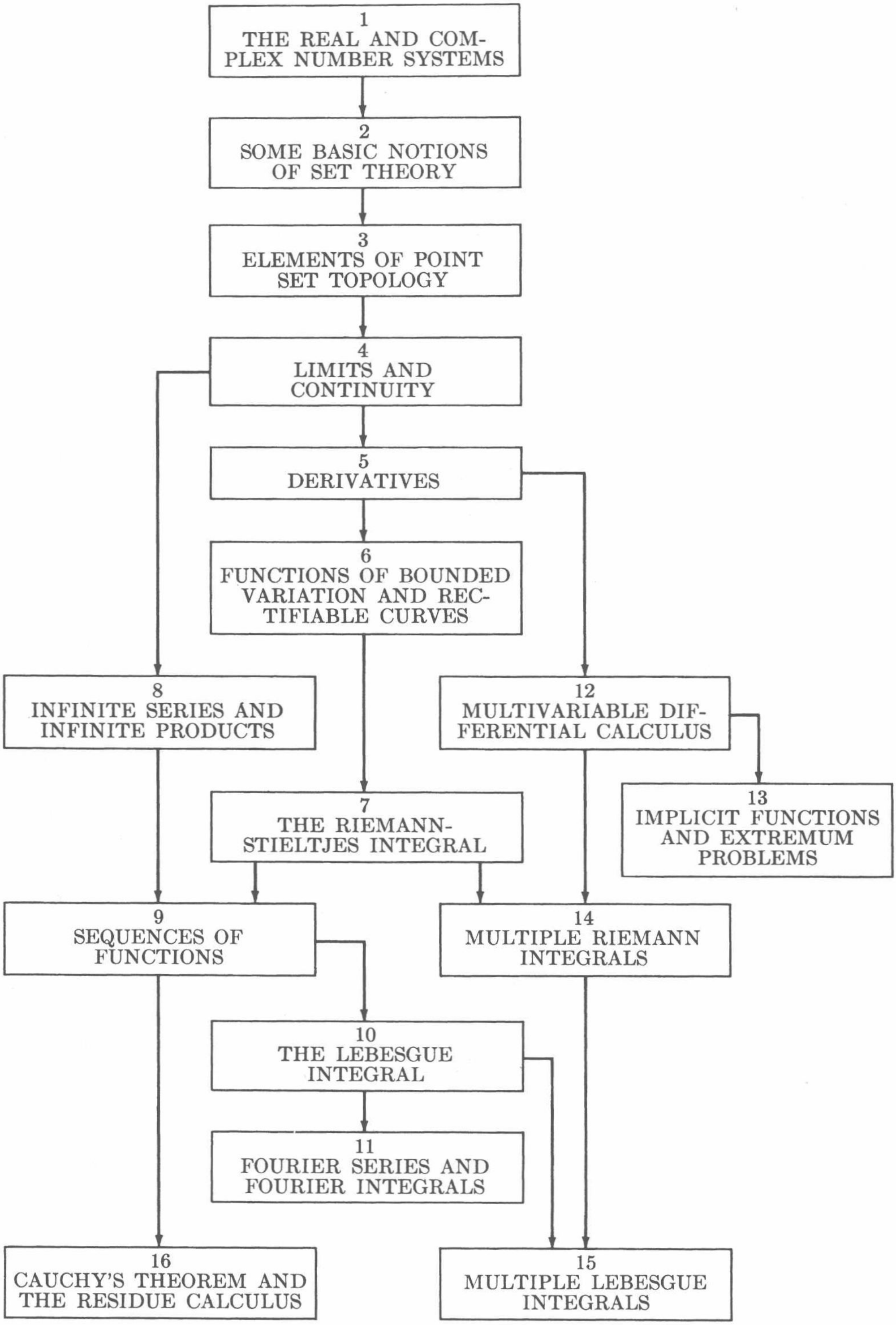
The first edition has been used in mathematics courses at a variety of levels, from first-year undergraduate to first-year graduate, both as a text and as supplementary reference. The second edition preserves this flexibility. For example, Chapters 1 through 5, 12, and 13 provide a course in differential calculus of functions of one or more variables. Chapters 6 through 11, 14, and 15 provide a course in integration theory. Many other combinations are possible; individual instructors can choose topics to suit their needs by consulting the diagram on the next page, which displays the logical interdependence of the chapters.

I would like to express my gratitude to the many people who have taken the trouble to write me about the first edition. Their comments and suggestions influenced the preparation of the second edition. Special thanks are due Dr. Charalambos Aliprantis who carefully read the entire manuscript and made numerous helpful suggestions. He also provided some of the new exercises. Finally, I would like to acknowledge my debt to the undergraduate students of Caltech whose enthusiasm for mathematics provided the original incentive for this work.

Pasadena
September 1973

T.M.A.

LOGICAL INTERDEPENDENCE OF THE CHAPTERS



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Chapter 1 The Real and Complex Number Systems

Chapter 2 Some Basic Notions of Set Theory

Chapter 3 Elements of Point Set Topology

Chapter 4 Limits and Continuity

CHAPTER 1

THE REAL AND COMPLEX NUMBER SYSTEMS

1.1 INTRODUCTION

Mathematical analysis studies concepts related in some way to real numbers, so we begin our study of analysis with a discussion of the real-number system.

Several methods are used to introduce real numbers. One method starts with the positive integers 1, 2, 3, ... as undefined concepts and uses them to build a larger system, the positive *rational numbers* (quotients of positive integers), their negatives, and zero. The rational numbers, in turn, are then used to construct the *irrational numbers*, real numbers like $\sqrt{2}$ and π which are not rational. The rational and irrational numbers together constitute the real-number system.

Although these matters are an important part of the foundations of mathematics, they will not be described in detail here. As a matter of fact, in most phases of analysis it is only the *properties* of real numbers that concern us, rather than the methods used to construct them. Therefore, we shall take the real numbers themselves as undefined objects satisfying certain axioms from which further properties will be derived. Since the reader is probably familiar with most of the properties of real numbers discussed in the next few pages, the presentation will be rather brief. Its purpose is to review the important features and persuade the reader that, if it were necessary to do so, all the properties could be traced back to the axioms. More detailed treatments can be found in the references at the end of this chapter.

For convenience we use some elementary set notation and terminology. Let S denote a set (a collection of objects). The notation $x \in S$ means that the object x is in the set S, and we write $x \notin S$ to indicate that x is not in S.

A set S is said to be a *subset* of T, and we write $S \subseteq T$, if every object in S is also in T. A set is called *nonempty* if it contains at least one object.

We assume there exists a nonempty set $\mathbf{R}$ of objects, called real numbers, which satisfy the ten axioms listed below. The axioms fall in a natural way into three groups which we refer to as the *field axioms*, the *order axioms*, and the *completeness axiom* (also called the *least-upper-bound axiom* or the *axiom of continuity*).

1.2 THE FIELD AXIOMS

Along with the set $\mathbf{R}$ of real numbers we assume the existence of two operations, called *addition* and *multiplication*, such that for every pair of real numbers x and y

the *sum* $x + y$ and the *product* xy are real numbers uniquely determined by x and y satisfying the following axioms. (In the axioms that appear below, x, y, z represent arbitrary real numbers unless something is said to the contrary.)

Axiom 1. $x + y = y + x, \ xy = yx$ *(commutative laws).*

Axiom 2. $x + (y + z) = (x + y) + z, \ x(yz) = (xy)z$ *(associative laws).*

Axiom 3. $x(y + z) = xy + xz$ *(distributive law).*

Axiom 4. *Given any two real numbers x and y, there exists a real number z such that $x + z = y$. This z is denoted by $y - x$; the number $x - x$ is denoted by 0. (It can be proved that 0 is independent of x.) We write $-x$ for $0 - x$ and call $-x$ the negative of x.*

Axiom 5. *There exists at least one real number $x \neq 0$. If x and y are two real numbers with $x \neq 0$, then there exists a real number z such that $xz = y$. This z is denoted by y/x; the number x/x is denoted by 1 and can be shown to be independent of x. We write x^{-1} for $1/x$ if $x \neq 0$ and call x^{-1} the reciprocal of x.*

From these axioms all the usual laws of arithmetic can be derived; for example, $-(-x) = x$, $(x^{-1})^{-1} = x$, $-(x - y) = y - x$, $x - y = x + (-y)$, etc. (For a more detailed explanation, see Reference 1.1.)

1.3 THE ORDER AXIOMS

We also assume the existence of a relation $<$ which establishes an ordering among the real numbers and which satisfies the following axioms:

Axiom 6. *Exactly one of the relations $x = y$, $x < y$, $x > y$ holds.*

NOTE. $x > y$ means the same as $y < x$.

Axiom 7. *If $x < y$, then for every z we have $x + z < y + z$.*

Axiom 8. *If $x > 0$ and $y > 0$, then $xy > 0$.*

Axiom 9. *If $x > y$ and $y > z$, then $x > z$.*

NOTE. A real number x is called *positive* if $x > 0$, and *negative* if $x < 0$. We denote by $\mathbf{R}^+$ the set of all positive real numbers, and by $\mathbf{R}^-$ the set of all negative real numbers.

From these axioms we can derive the usual rules for operating with inequalities. For example, if we have $x < y$, then $xz < yz$ if z is positive, whereas $xz > yz$ if z is negative. Also, if $x > y$ and $z > w$ where both y and w are positive, then $xz > yw$. (For a complete discussion of these rules see Reference 1.1.)

NOTE. The symbolism $x \leq y$ is used as an abbreviation for the statement:

$$\text{“}x < y \quad \text{or} \quad x = y.\text{”}$$

Thus we have $2 \leq 3$ since $2 < 3$; and $2 \leq 2$ since $2 = 2$. The symbol $\geq$ is similarly used. A real number x is called *nonnegative* if $x \geq 0$. A pair of simultaneous inequalities such as $x < y$, $y < z$ is usually written more briefly as $x < y < z$.

The following theorem, which is a simple consequence of the foregoing axioms, is often used in proofs in analysis.

Theorem 1.1. *Given real numbers a and b such that*

$$a \leq b + \varepsilon \qquad \text{for every } \varepsilon > 0. \tag{1}$$

Then $a \leq b$.

Proof. If $b < a$, then inequality (1) is violated for $\varepsilon = (a - b)/2$ because

$$b + \varepsilon = b + \frac{a - b}{2} = \frac{a + b}{2} < \frac{a + a}{2} = a.$$

Therefore, by Axiom 6 we must have $a \leq b$.

Axiom 10, the completeness axiom, will be described in Section 1.11.

1.4 GEOMETRIC REPRESENTATION OF REAL NUMBERS

The real numbers are often represented geometrically as points on a line (called the *real line* or the *real axis*). A point is selected to represent 0 and another to represent 1, as shown in Fig. 1.1. This choice determines the scale. Under an appropriate set of axioms for Euclidean geometry, each point on the real line corresponds to one and only one real number and, conversely, each real number is represented by one and only one point on the line. It is customary to refer to the *point x* rather than the point representing the real number x.

Figure 1.1

The order relation has a simple geometric interpretation. If $x < y$, the point x lies to the left of the point y, as shown in Fig. 1.1. Positive numbers lie to the right of 0, and negative numbers to the left of 0. If $a < b$, a point x satisfies the inequalities $a < x < b$ if and only if x is *between* a and b.

1.5 INTERVALS

The set of all points between a and b is called an *interval*. Sometimes it is important to distinguish between intervals which include their endpoints and intervals which do not.

NOTATION. The notation $\{x : x \text{ satisfies } P\}$ will be used to designate the set of all real numbers x which satisfy property P.