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Stellar Structure and Evolution 2nd Edition

恒星结构与演化
第二版

(影印版)

〔德〕基彭汉 (R. Kippenhahn)

〔德〕魏格特 (A. Weigert) 著

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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子亲身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任
中国科学院院士，北京大学教授

王恩哥

2010年5月于燕园

Preface to the First Edition

The attempt to understand the physics of the structure of stars and their change in time – their evolution – has been bothering many physicists and astronomers ever since the last century. This long chain of successful research is well documented not only by numerous papers in the corresponding journals but also by a series of books. Some of them are so excellently written that despite their age they can still be recommended and not only as documents of the state of the art at that time. A few outstanding examples are the books of Emden (1907), Eddington (1926), Chandrasekhar (1939), and Schwarzschild (1958). But our science has rapidly expanded in the last few decades, and new aspects have emerged which could not even be anticipated, say, 30 years ago and which today have to be carefully explored.

This does not mean, however, that our ambition is to present a complete account of the latest and most refined numerical results. This can well be left to the large, and growing number of excellent review articles. This book is intended rather to be a textbook that will help students and teachers to understand these results as far as possible and present them in a simple and clear manner. We know how difficult this is since we ourselves have tried for the largest part of our scientific career to understand “how the stars work” – and then to make others believe it. In these attempts we have found that often enough a simplified analytical example can be more helpful than the discussion of an exceptionally beautiful numerical solution. Therefore we do not hesitate to include many simple considerations and estimates, if necessary, even at the expense of rigour and the latest results. The reader should also note that the list of references given in this book is not intended to represent a table of honour for the (known and unknown) heroes of the theory of stellar structure; it is merely designed to help the beginner to find a few first paths in the literature jungle and presents those papers from which we have more or less randomly chosen the numbers for figures and numerical examples (There are others of at least the same quality!).

The choice of topics for a book such as this is difficult and certainly subject to personal preferences. Completeness is neither possible nor desirable. Still, one may wonder why we did not include, for example, binary stars, although we are obviously interested in their evolution. The reason is that here one would have had

to include the physics of essentially non-spherical objects (such as disks), while we concentrate mainly on spherical configurations; even in the brief description of rotation the emphasis is on small deviations from spherical symmetry.

This book would never have been completed without the kind and competent help of many friends and colleagues. We mention particularly Wolfgang Duschl and Peter Schneider who read critically through the whole manuscript; Norman Baker, Gerhard Börner, Mounib El Eid, Wolfgang Hillebrandt, Helmuth Kahler, Ewald Müller, Henk Spruit, Joachim Wambsganß, and many others read through particular chapters and gave us their valuable advice. In fact it would probably be simpler to give a complete list of those of our colleagues who have *not* contributed than of those who helped us.

In addition we have to thank many secretaries at our institutes; several have left their jobs (for other reasons!) during the five years in which we kept them busy. Most of this work was done by Cornelia Rickl and Petra Berkemeyer in Munich and Christa Leppien and Heinke Heise in Hamburg, while Gisela Wimmersberger prepared all the graphs. We are grateful to them all.

Finally we wish to thank Springer-Verlag for their enthusiastic cooperation.

Munich and Hamburg
December 1989

Rudolf Kippenhahn
Alfred Weigert

Preface to the Second Edition

Twenty years after its first publication, this textbook is still a major reference for scientists and students interested in or working on problems of stellar structure and evolution. But with the incredible growth of computational power, the computation of stellar models has to large extent become a standard tool for astrophysics. While the early computations were restricted to single choices for mass, compositions and possibly evolutionary stage, by now models for the whole parameter space exist. The first edition of this book was restricted to a few examples for low- and intermediate-mass star evolution and lacked the broader view now being possible. There are even semi-automatic stellar evolution codes that may be used remotely via the Internet.

However, stellar evolution programs should not be used without a thorough understanding of the stellar physics. Therefore, a textbook concentrating on the foundations of the theory and explaining in detail specific phases and events in the life of a star is very much needed to allow scientifically solid modelling of stars. This is the reason why this book deserved a second edition.

Much to our regret, A. Weigert passed away two years after publication of the first edition. He left a gap that cannot be filled. Given the above mentioned need for a second edition and the requirement to add up-to-date stellar models, it was decided to have A. Weiss join R. Kippenhahn in preparing the new edition.

The two authors of this book came to discriminate between the *eternal truth* and the *mutable* parts. The latter ones refer to the current state of modelling and knowledge obtained from numerical models and their comparison to observations. Such chapters were updated, extended, or added. As far as possible, the stellar models shown were specifically calculated for this purpose, with the present, much evolved version of the original code by Kippenhahn, Weigert, and Hofmeister. The numerical results are therefore much more homogeneous and consistent than in the first edition.

The *eternal truth* concerns the aforementioned basic physics and their understanding. These parts of the book have been left almost untouched, since the authors (and those readers who were consulted) did not see any reason to change them.

The authors are indebted to many friends and colleagues who gave their advice or comments, with respect to both necessary changes and the new text passages.

The support of Santi Cassisi, Jørgen Christensen-Dalsgaard, Wolfgang Hillebrandt, Thomas Janka, Ralf Klessen, Ewald Müller, Hans Ritter, Maurizio Salaris, and Helmut Schlattl was essential for us.

We are also very grateful to all those colleagues who very generously provided their own data to help filling gaps that we could not fill with our own models. They were (again in alphabetical order) Leandro Althaus, Isabelle Baraffe, Raphael Hirschi, Marco Limongi, Marcelo Miller Bertolami, Aldo Serenelli, and Lionel Siess. Needless to say, their data also came with much wanted and helpful advice and sometimes fruitful scientific discussions about details of the models.

Norbert Grüner's help in the difficult task of generating a useful index is acknowledged, too.

Last, but not least, we thank Mrs. Rosmarie Mayr-Ihbe, who designed, corrected, and improved the many figures that we added to this second edition.

Garching
February 2012

Achim Weiss

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