Copulae and Multivariate Probability Distributions in Finance

Edited by
Alexandra Dias, **Mark Salmon** and
Chris Adcock

ROUTLEDGE

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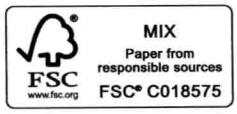

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Christian Genest, Michel Gendron and Michaël Bourdeau-Brien
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Testing for structural changes in exchange rates' dependence beyond linear correlation
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Chris Adcock is Professor of Financial Econometrics at the University of Sheffield, UK, and Visiting Professor of Quantitative Finance at the University of Southampton, UK. He is the founding Editor of *The European Journal of Finance* and is one of the founding Associate Editors of the *Journal of Mathematical Finance*. Chris has acted as an Advisor to a number of international investment managers, and algorithms he has designed have been used by Citibank and DSI International Investment Management (now part of UBS), as well as several other asset management groups. His current research interests are centred around the development of portfolio selection and asset pricing theory.

Preface

In 2007, Alexandra Dias and Mark Salmon organised a two-day workshop at Warwick Business School with the title Copulae and Multivariate Probability Distributions in Finance. The meeting took place in September 2007 and was a great success. Subsequently, *The European Journal of Finance* published a special issue with a selection of papers from the workshop. In addition, the special issue included a *tour d'horizon* of the subject area by Christian Genest, Michel Gendron and Michaël Bourdeau-Brien. Their paper served as an excellent introduction to the special issue.

After the special issue appeared, we were approached by Taylor & Francis to turn the special issue into this edited volume. The aim of the publication of this book is to further disseminate the material in the special issue. The three of us hope that this will both reach a wider audience than just those who read *The European Journal of Finance* and serve as a reminder of the intrinsic interest to be found in the subject matter and of its importance, not only to modern finance but also to other application areas.

As noted by the editor of another volume in this series, books are not created in a vacuum. We are grateful to Emily Ross of Taylor & Francis for the opportunity to publish this book and particularly grateful too for her editorial work. It is also another opportunity to thank Christian and his two colleagues, the other authors and the many referees for their hard work. All concerned have contributed greatly to this publication.

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Contents

The Advent of Copulas in Finance

Christian Genest[a], Michel Gendron[b] and Michaël Bourdeau-Brien[b]

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The authors provide bibliometric evidence to illustrate the development of copula theory in mathematics, statistics, actuarial science and finance. They identify the main contributors to the field, and the most important areas of application in finance. They also describe some of the remaining methodological challenges.

1. Introduction

This paper is the lead article in a special issue of *The European Journal of Finance* that gives an account of the September 2007 Warwick conference on 'Copulas and multivariate probability distributions in finance'. Its contents illustrate the potential of copula modeling techniques in financial contexts.

Copula theory and applications have developed considerably in recent years. In an article that criticizes its overuse, Mikosch (2006) mentions that between 2003 and 2005 the number of Google hits for the word 'copula' jumped from 10,000 to 650,000. As these lines are being written, the count is well over 750,000. Although it is clear that the most common use of the term 'copula' on the web has nothing to do with science, signs that copula methodology is developing are numerous and beyond dispute. For example, in 1999, MathSciNet listed 60 articles with the word 'copula' in the title; the count had risen to 187 by 2005, and to 295 by August 2008.

What about the field of finance? This question prompted us to survey the academic literature on copulas in June 2006. This was done using 26 bibliographic databases such as Google Scholar, Web of Science, Synergy, Proquest, MathSciNet, Elsevier Science Direct, etc. We identified over 4200 papers that mentioned the word 'copula'. After detailed examination, we eliminated duplicate entries and papers where the word 'copula' either did not refer to the mathematical concept or appeared only in the bibliography. At the end of the process, we were left with 871 documents that we then classified according to various criteria: journal, author, author's institution, area of research, field of application, etc.

This database contains a great deal of information about the advent of copulas in finance which we would like to share with you. The growth of the copula literature is quantified in Section 2 and a breakdown by field of study is provided in Section 3. The means through which copula theory is spreading in finance are described in Section 4. The most active contributors, both outside and within the financial/actuarial/econometric sector, are identified in Sections 5 and 6, respectively.

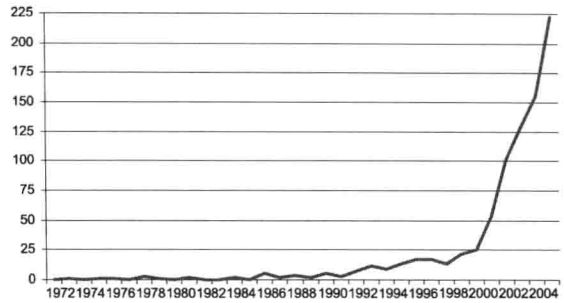

Figure 1. Number of documents on copula theory, 1971–2005.

The main areas of application in finance are listed in Section 7, and some of the current challenges are outlined in Section 8. Concluding comments are given in Section 9.

2. Changes over time

Our database contains 871 documents contributing to copula theory and its applications in different contexts. This literature encompasses 893 authors from 49 different countries and 418 institutions around the world. Overall, 77% of these authors are affiliated to universities; most of the others work in banks, insurance companies and financial institutions. Their writings appeared in some 165 journals and conference proceedings.

The most striking feature of the data set is the rapid growth in the annual number of contributions to the subject. This is illustrated in Figure 1. A more detailed examination reveals that the growth falls into three periods.

(1) Before 1986, the literature was sparse and mostly mathematical. The concept of copula can be traced back at least to the work of Wassily Hoeffding and Maurice Fréchet, though the term itself was coined by Sklar (1959). Many contributions were related to the study of probabilistic metric spaces, as described in the book by Schweizer and Sklar (1983).
(2) Beginning in 1986, one can see a slow, systematic rise in the number of publications. Growth was largely due to the emergence of the concept of copula in statistics and to three conferences devoted to the subject: Rome (1990), Seattle (1993) and Prague (1996).
(3) From 1999 on, the number of contributions grew considerably. The books by Joe (1997) and Nelsen (1999) were influential in disseminating copula theory; the book by Drouet-Mari and Kotz (2001), which focusses on correlation and dependence, is also noteworthy. Actuarial and financial applications were fuelled by Frees and Valdez (1998) and Embrechts, McNeil, and Straumann (1999), who illustrated the potential for copula modeling in these fields.

3. Breakdown by field of study

What is the part of finance to the spectacular growth of copula methodology in the past few years? To investigate this issue, we subjectively grouped the 871 documents in our database into nine mutually exclusive categories: mathematics; statistics; biostatistics; operations research; natural sciences; engineering; actuarial science; economics and finance. We achieved this classification by carefully examining the contents of each document. About 1% did not match any of the categories and were left unclassified.

Figure 2 shows the results of the grouping. Even though people in finance have been interested in copulas only since 2000, they produced the largest proportion of documents, *i.e.* 41%. Next come

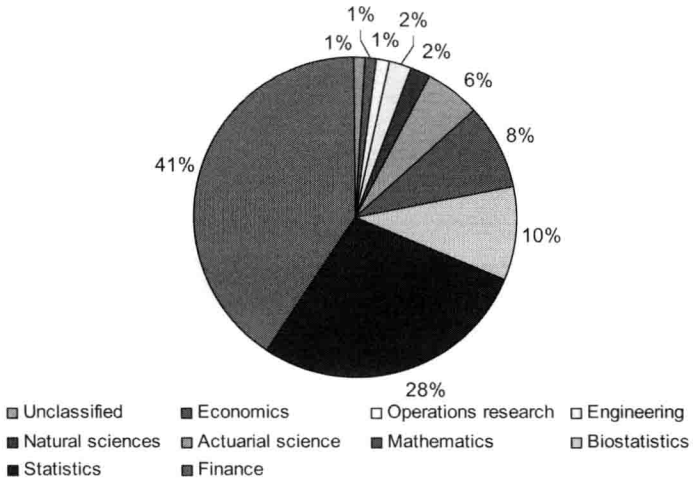

Figure 2. Breakdown by discipline of the 871 documents in the database.

Table 1. List of journals that published the largest number of copula-related articles.

Rank	Journal	Papers published
1	*Journal of Multivariate Analysis*	29
2	*Statistics & Probability Letters*	26
3	*Insurance: Mathematics and Economics*	23
4	*Communications in Statistics: Theory and Methods*	19
5	*Biometrika*	14
6	*Risk Magazine*	14
7	*The Canadian Journal of Statistics*	12
8	*Biometrics*	12
9	*Quantitative Finance*	11
10	*Journal of Nonparametric Statistics*	10

statistics (28%), biostatistics (10%), mathematics (8%) and actuarial science (6%). Interestingly, in June 2006, finance and actuarial science together contributed 47% of the literature, whereas mathematics, statistics and biostatistics together accounted for 46%. No doubt finance-related documents now account for over half of the literature on the subject. We will later discuss the nature of these contributions.

The level of activity in each discipline is also reflected by Table 1, which lists the peer-review journals that carried the largest number of articles concerned with copulas. As of June 2006, statistics continued to lead the rooster. This is not surprising, given that copulas have a long history in this area. Remarkably, *Risk Magazine* and *Quantitative Finance* make the list, even though the earliest papers on the topic appeared there in 2001. A fair proportion of copula-related articles in *Insurance: Mathematics and Economics* also pertain to finance.

4. Types of contribution in finance

We identified 353 finance-related documents in the database. As illustrated by Figure 3, these contributions to copula theory are varied in nature. They include lecture notes, books, theses,

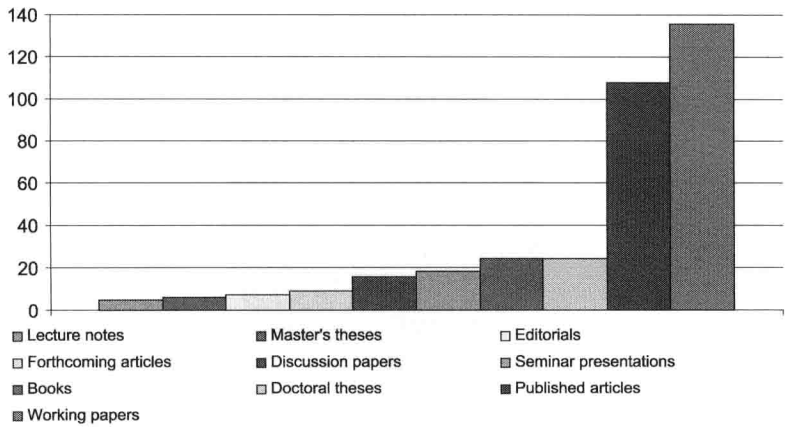

Figure 3. Breakdown by type of the 353 finance-related documents in the database.

editorials, seminar presentations and so on, but the bulk of the contributions consists of working papers or articles that are either published or forthcoming in peer-review journals.

Looking at Figure 3, one can see that working papers account for approximately one-third of the total. This is a larger proportion than published manuscripts, even when forthcoming articles are included. This is the result of a combination of factors.

(1) Many working papers seem to be intended for publication but had not reached that state yet as of June 2006.
(2) Several of them are fairly old and either may have failed to meet the publication standards or encountered resistance from the main-stream financial community.
(3) Many reports are authored by financial analysts working for banks, insurance companies and regulatory agencies faced, *e.g.* with the requirements of the Basel II Accords. These people typically have little time to devote to the lengthy academic publishing process.
(4) Clearly, some working papers were never intended for publication but were rather designed as surveys of a widely scattered literature and aimed at practitioners.

The work of the 'Groupe de recherche du Crédit Lyonnais' is a prime example of web-based dissemination of copula methodology in a financial context. The pedagogical documents written between 2002 and 2004 by Thierry Roncalli and his associates (Éric Bouyé, Valdo Durrleman, Jean-Frédéric Jouanin, Ashkan Nikeghbali, Gaël Riboulet, etc.) have contributed much to the popularity of copulas in finance. To this date, these reports remain freely available at http://gro.creditlyonnais.fr/content/rd/home_copulas.htm.

Some of the accumulated wisdom transpires in the book by Roncalli (2004).

5. Main contributors outside finance

The 871 documents in our database make it possible to identify the most prolific contributors to copula theory and its applications in different disciplines. In this section, we describe the main research groups in the fields of mathematics, statistics and biostatistics. Some of their work has influenced subsequent contributions in actuarial science, finance and econometrics. The latter developments are reviewed in Section 6.

According to our database, Roger Nelsen is the most prolific author among mathematicians. His book (Nelsen, 1999; re-edited in 2006) is very frequently cited. His American colleagues (Jerry Frank, Greg Fredricks, Berthold Schweizer, etc.) and his collaborators in Spain (Juan Quesada-Molina, José Antonio Rodríguez-Lallena, Manuel Úbeda-Flores, etc.) and in Italy (Carlo Sempi, Fabrizio Durante, etc.) have played a major role, notably in the construction of copula families and in the derivation of bounds that have found applications in finance.

In statistics, the number of contributors is much larger. The 'Québec group' (the first author and his colleagues, viz. Bruno Rémillard, Louis-Paul Rivest, Philippe Capéraà, Belkacem Abdous, etc.) has been by far the most prolific, not only in terms of publications, but also in training graduate students and postdoctoral fellows (Anne-Laure Fougères, Kilani Ghoudi, Mhamed Mesfioui, Jean-François Quessy, Lajmi Lakhal Chaieb, David Beaudoin, etc.) who continue to generate copula-related research. Rank-based methods have been central in the group's contributions to inference for copula models. This nonparametric approach, rooted in the work of Rüschendorf (1976) and Deheuvels (1979), is summarized by Genest and Favre (2007).

Further to the copula conferences held in Barcelona (2000), Québec (2004) and Tartu (2007), research papers have proliferated on statistics-related aspects of copulas and their applications in extreme-value theory, hydrology and so on. Arthur Charpentier, Anne-Catherine Favre, Jean-David Fermanian, Johanna Nešlehová, Olivier Scaillet, Rafael Schmidt and Johan Segers are prominent figures in this new wave of researchers with an interest in the subject.

In biostatistics, the forerunner is clearly David Oakes. As early as the late 1980s, his papers were instrumental in applying copula methodology to Cox's proportional hazards models; see, *e.g.* Oakes (1989) and the book by Hougaard (2001) for the connection between frailty models and the pervasive Archimedean copulas. Truncation and censoring, which are common issues in survival analysis, have been central concerns of biostatisticians interested in copulas, such as Jason Fine, Philippe Lambert or Joanna Shih.

6. Main contributors in actuarial science, finance and econometrics

In actuarial science, the popularity of copulas owes much to the review article by Frees and Valdez (1998), published in the *North American Actuarial Journal*. Few people know, however, that Carrière and Chan (1986) had already used copulas to compute the bounds of an annuity's value to the last survivor. Although he did not couch problems in terms of copulas, Floriaan De Vylder was also an early contributor to optimization problems on Fréchet spaces in an actuarial context; in particular, see Part II of De Vylder (1996) or Denuit and Charpentier (2004).

Insurance: Mathematics and Economics has published the overwhelming majority of copula-related actuarial papers. It has featured many contributions to the field by Michel Denuit, Jan Dhaene, Étienne Marceau, Mhamed Mesfioui, Shaun Wang, Mario Wüthrich, etc. Their concerns focus mainly on quantifying dependence between claims in a portfolio and its impact on pricing. In addition, Marco Scarsini and Alfred Müller have played a key role in studying copula-based dependence concepts and stochastic orderings used, *e.g.* in the analysis of worst-case scenarios (see, *e.g.* the book by Müller and Stoyan, 2002). This literature is surveyed in the book by Denuit et al. (2005).

In the field of finance, the first and oldest research group is that of Paul Embrechts (ETH Zürich). As early as 1999, Embrechts, McNeil and Straumann were using the concept of copula to alert readers of *Risk Magazine* to the pitfalls of correlation. The papers by Embrechts and his collaborators on the use of copulas in managing financial risks are by far the most numerous and oft cited. They culminated in 2005 with the publication of the book by McNeil, Frey, and

Embrechts (2005). This text is unique in applying both copula theory and extreme value theory to an eclectic mix of subjects: credit risk, market risk, operational risk and insurance. Members of this group include Wolfgang Breymann, Valérie Chavez-Demoulin, Alexandra Dias, Rüdiger Frey, Filip Lindskog, Alexander J. McNeil, Johanna Nešlehová, Philipp Schönbucher, Giovanni Puccetti. Although not direct collaborators, Claudia Czado, Claudia Klüppelberg and Werner Hürlimann can be associated to this group.

In 2000, an independent research group on copulas in finance emerged in Italy, around Umberto Cherubini and Elisa Luciano. They co-authored over 10 papers on option pricing and hedging of credit derivatives. Their work has led them to write, with Walter Vecchiato, a book on copula methods in finance (Cherubini, Luciano, and Vecchiato, 2004). Also worth mentioning is the book on extreme financial risks by Malevergne and Sornette (2006), which places strong emphasis on the theory of copulas and their empirical testing and calibration.

In econometrics, papers in which copula methodology is used explicitly are still relatively rare. Noteworthy contributions have been made, *e.g.* by Xiaohong Chen, Yanqin Fan, Andrew Patton, Mark Salmon, Murray Smith and Bas Werker. Additional applications of copulas in econometrics are mentioned in the review paper by Trivedi and Zimmer (2005). Although it is clear that the flexibility provided by copulas is welcome in problems involving dependence between variables and over time, the jury is still out on how best this can be accomplished, particularly when the underlyings are measured in continuous time.

7. Major areas of application in finance

Two major phenomena account for the rise of copula modeling in finance: the lack of normality in (log) returns and the dependence between extreme values of various assets. Our database shows that these themes are recurrent in financial applications of copulas. Broadly speaking, contributions to the latter can be grouped into the following four categories.

(1) *Risk management*: Topics included here are those covered in the book by McNeil, Frey, and Embrechts (2005), *i.e.* credit, market, operational risk and risk aggregation. Developments in this area were stimulated by the Basel II Accords and the influential contributions of Embrechts, McNeil, and Straumann (1999) and Li (2000).
(2) *Portfolio management*: Included here are papers dealing with dependence between international financial markets, different classes of assets and currencies. For example, the paper by Patton (2004) on bivariate equity portfolio management falls in this category.
(3) *Pricing of derivatives*: This broad category comprises work on the pricing of exotic options, collateralized debt obligations and credit default swaps. Beginning with their 2002 paper, Cherubini and Luciano have been among the main contributors to this area, which is discussed in detail in the book by Cherubini, Luciano, and Vecchiato (2004).
(4) *Risk measurement*: We have merged in this category the papers discussing value-at-risk, expected shortfall and financial contagion. The paper of Embrechts, Höing, and Juri (2003) provides an early example of VaR studies using copulas.

The 353 documents related to finance were classified according to these four categories. For presentation purposes, 13 documents from the first half of 2006 were excluded. In addition, 23 documents were left unclassified, either because they were general introductions to copulas or because they pertained to other topics such as market microstructure or monetary flows.

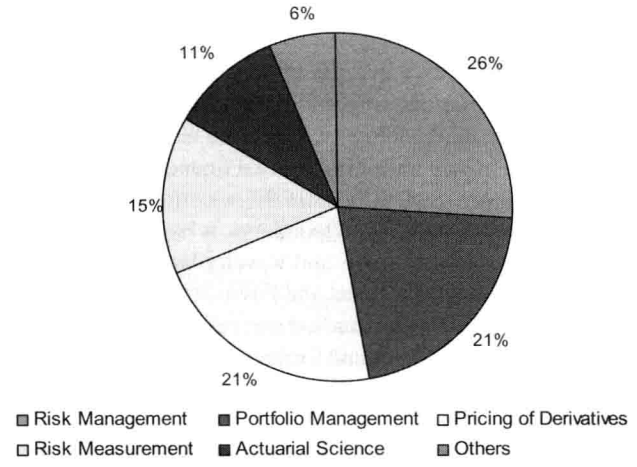

Figure 4. Breakdown by topic of 381 documents related to finance or actuarial science from 1999 to 2005.

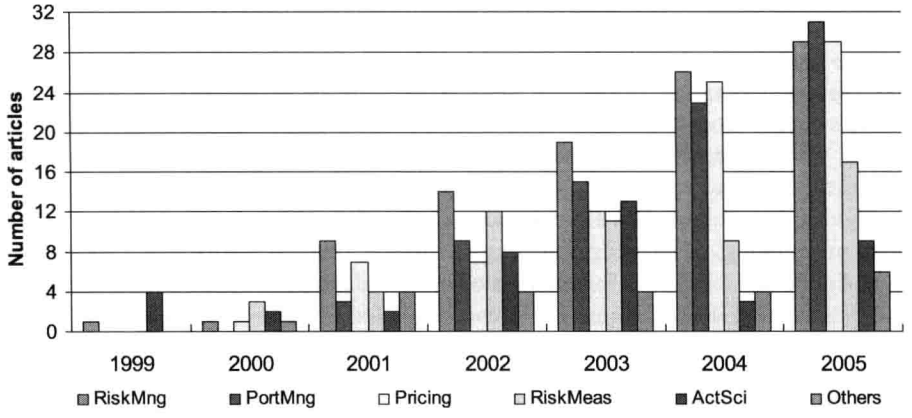

Figure 5. Distribution over time of 381 documents related to finance or actuarial science from 1999 to 2005.

Figure 4 shows the level of activity in each of these areas. Because many actuarial contributions have a close connection to finance, 41 documents relating to that field were included as well. The distribution in time of the same 381 documents is displayed in Figure 5.

Three observations can be made from the graphs.

(1) Copula modeling seems to be spreading more or less evenly in various areas of finance.
(2) The growth rate is roughly linear and similar in portfolio management, risk management and the pricing of derivatives; the use of copulas in risk measurement is roughly constant.
(3) Applications of copulas are more numerous in finance than in actuarial science, despite a late start. This being said, finance is a broader field of research than actuarial science, and the boundary between the two areas is somewhat arbitrary.

8. What does the future hold?

The data we collected make it obvious that the interest of the financial community for copula modeling is blooming and will continue to grow in the foreseeable future. If this approach is to be adopted widely, however, the methodology will need to be expanded to face the challenges of abundant, time-dependent and highly multidimensional financial data.

First, there is a need to develop graphical tools for the selection and validation of copula models. At present, the detection and visualization of dependence is based on chi-plots, K-plots and rank-rank plots for small sample sizes or on kernel- and wavelet-based estimates of the copula density when data are sufficiently abundant (cf. Genest and Favre 2007; Genest, Masiello, and Tribouley 2009). These techniques should be further studied and refined. In the spirit of residual plots for linear regression, graphical diagnostic tools and formal tests also need to be built to check whether data can be represented by specific dependence structures such as Archimedean, meta-elliptical or extreme-value copulas.

Second, further research is needed to assess the added value of copulas in time-series modeling. In recent years, they have been used to capture serial dependence in a univariate context. They have also been employed to characterize the dependence between multivariate residuals from time series that have been modeled either jointly or marginally. See Patton (2009) and references therein for some of the pros and cons of these various approaches. Many technical challenges will need to be met to adapt rank-based inference tools to the treatment of model residuals, and more generally to multivariate data that are not independent.

Third, great opportunities for financial applications lie in the development of high-dimensional multivariate copula models and associated inference techniques. Hierarchical models based on pair-copula vine decompositions seem particularly promising. This approach is described, for example, in the book by Kurowicka and Cooke (2006). See Aas et al. (2009) for a nice introduction and an application involving Norwegian stocks and bonds.

Further methodological challenges bearing on finance – but not limited to it – concern the construction of goodness-of-fit tests for copula models, the adaptation of copula modeling techniques to count data and the development of inference tools for extreme-value structures. For reviews of these topics, see Genest, Rémillard, and Beaudoin (2009), Genest and Nešlehová (2007) and McNeil, Frey, and Embrechts (2005), respectively.

Beyond technical issues, the most pressing challenge for the copula community is the inclusion of their modeling techniques in commercial software packages. Of all factors, the availability of a user-friendly copula toolkit is probably most likely to stimulate the widespread adoption and use of this methodology, particularly among financial analysts and engineers.

9. Conclusion

The bibliometric study presented here documents the advent and spectacular growth of copula theory. As we saw, this explosion of interest is stronger in finance than in any other field but to this date, relatively few contributions have found their way in finance peer-review journals. The initiative of *The European Journal of Finance* thus seems timely, particularly in the wake of the subprime crisis that led analysts, banks and regulators to question the old, Gaussian paradigm.

While our study identifies the main actors and a few key writings in the early development of copula theory, we avoided crowding the paper with references to the literature. We thought it would be more useful to provide the beginner with a list of books that cover the material from various perspectives: actuarial, financial, mathematical, statistical, etc. The bibliography also features a

few surveys, viz. Frees and Valdez (1998), Owzar and Sen (2004), Trivedi and Zimmer (2005), Kolev, dos Anjos, and Mendes (2006) and Genest and Favre (2007).

We encourage people to sieve through this material before they start using copulas or contribute to the field. In the footsteps of Mikosch (2006), who used a tale of Hans Christian Andersen to qualify the copula fashion, we suggest that the reader acts as the hero of Charles Perrault's *Le petit poucet* (Hop o'my Thumb) and drops little white pebbles through his or her readings to avoid getting lost in the densest, darkest part of the forest.

Acknowledgements

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