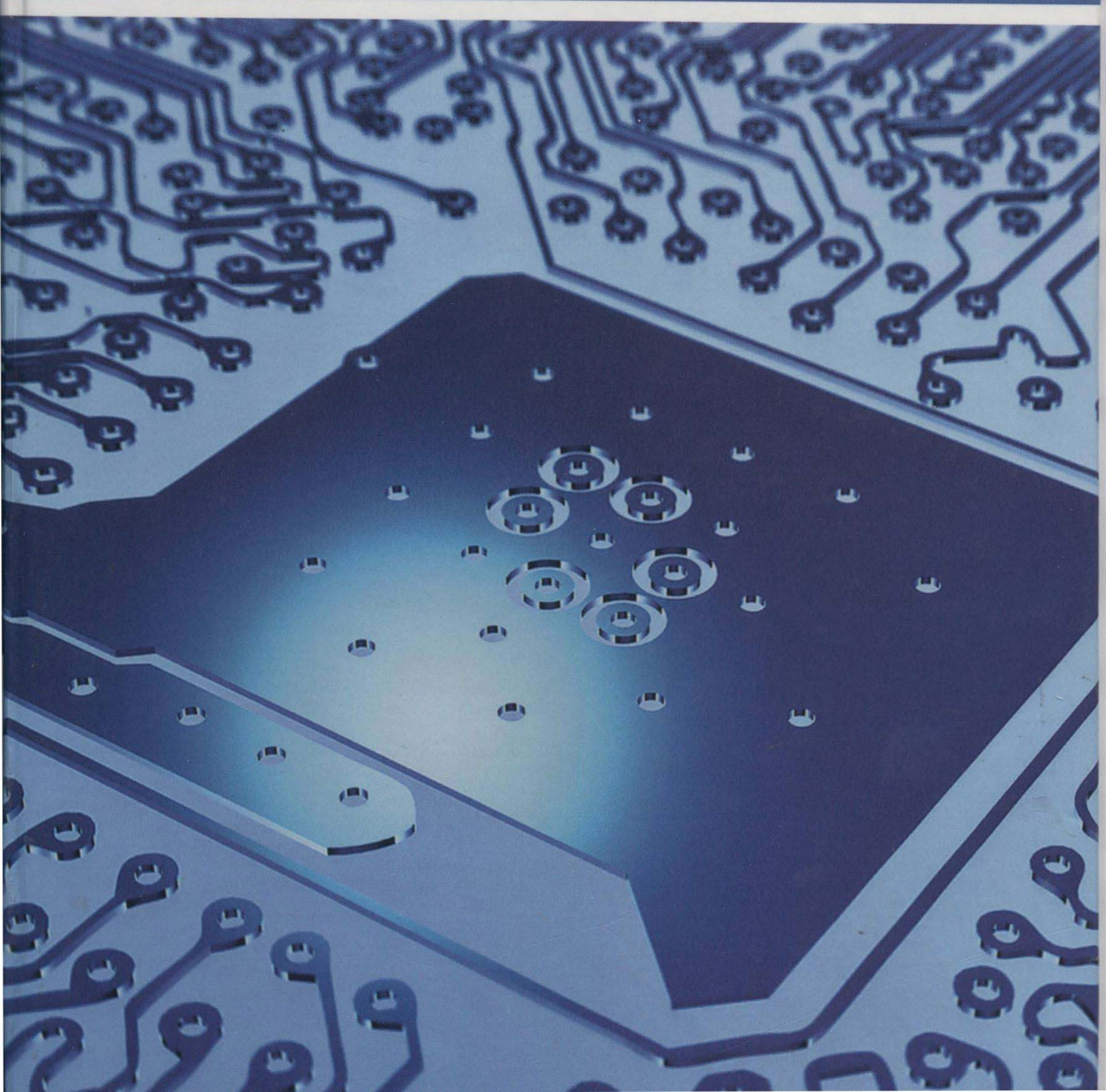
Design Aspects of PID Controllers

Ashley Potter



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Preface

The aim of this book is to educate the readers regarding the various design aspects of PID controllers. The design of PID controllers were first introduced in the market in 1939 and is still considered as a challenging field that needs novel approaches for the formulation of solutions for PID tuning complications while capturing the effects of noise and process variations. The intensified complexity of novel applications in fields like microsystems technology, dc motors, automotive applications, industry procedures, pneumatic mechanisms, needs controllers that embody significant characteristics of the systems into their design like system's nonlinearities, disturbance rejection needs, model uncertainties, time delays and performance criteria among others. This book aims to present distinct PID controller designs for several contemporary technology applications in order to satisfy the requirements of a wide audience of researchers, professionals and scholars interested in studying about the progresses in PID controllers and associated topics.

All of the data presented henceforth, was collaborated in the wake of recent advancements in the field. The aim of this book is to present the diversified developments from across the globe in a comprehensible manner. The opinions expressed in each chapter belong solely to the contributing authors. Their interpretations of the topics are the integral part of this book, which I have carefully compiled for a better understanding of the readers.

At the end, I would like to thank all those who dedicated their time and efforts for the successful completion of this book. I also wish to convey my gratitude towards my friends and family who supported me at every step.

Editor

Contents

Part 1

Tuning Methods for 3 Terms Controllers – Classical Approach

Wavelet PID and Wavenet PID: Theory and Applications

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1. Introduction

We introduce in this chapter a new area in PID controllers, which is called multiresolution PID (MRPID). Basically, a MRPID controller uses wavelet theory for the decomposition of the tracking error signal. We present a general error function in terms of partial errors which gives us the various frequencies appearing in the general errors. Once we obtain the spectrum of the error signal, we divide the error at frequencies that are weighted by gains proposed by the designer. We can say that the MRPID is a generalization of conventional PID controller in the sense that the error decomposition is not only limited to three terms.

The PID is the main controller used in the control process. However, the linear PID algorithm might be difficult to be used with processes with complex dynamics such as those with large dead time and highly nonlinear characteristics. The PID controller operation is based on acting proportionally, integrally and derivative way over the error signal $e(t)$, defined it as the difference between the reference signal y_{ref} and the process output signal $y(t)$, for generating the control signal $u(t)$ that manipulates the output of the process as desired, as shown in the Fig. 2, where the constants k_P k_I and k_D are the controller gains. There are several analytical and experimental techniques to tune these gains (Aström & Hägglund, 2007). One alternative is auto-tuning online the gains as in (Cruz et al., 2010; O. Islas Gómez, 2011a; Sedighizadeh & Rezazadeh, 2008a) where they use a wavelet neural networks to identify the plant and compute these gain values, this approach has been applied in this chapter.

The chapter is organized as follows: a general overview of the wavelets and multiresolution decomposition is given in Section 2. In Section 3 we preset some experimental results of the close-loop system with the MRPID controller. The PID controller based on wavelet neural network and experimental is given in Section 4, while the experimental results are given in Section 5. Finally, the conclusions of the contribution about wavelet PID and wavenet controllers are presented in Section 6.

2. PID controller based on wavelet theory and multiresolution analysis

2.1 Wavelet theory and multiresolution analysis

Here, we briefly summarize some results from the wavelet theory that are relevant to this work, for it we use the notation presented in the Table 1. For more comprehensive discussion of wavelets and their applications in control, signal processing, see e.g., (Daubechies, 1992; Hans, 2005; Mallat, 1989a;b; Parvez, 2003; Parvez & Gao, 2005; Vetterli & Kovačević, 1995).

$\psi(t)$	Mother wavelet function
$\psi_{a,b}$	Daughter wavelet function
$W_f(a,b)$	Continuous wavelet transform
$W_f[a,b]$	Discrete wavelet transform
$< f, g >$	Inner product between f and g
$\bigoplus$	Direct sum of subspaces
$V \perp W$	V is orthogonal to W
$L^2(\mathbb{R})$	Vector space of all measurable, square integrable functions
$\mathbb{R}$	Vector space of the real numbers
$\mathbb{Z}$	Set of all integers

Table 1. Notation

A wavelet is defined as an oscillatory wave ψ of very short duration and satisfy the admissibility condition (Daubechies, 1992), given by

$$\Psi(0) = \int_{-\infty}^{\infty} \psi(t)dt = 0, \tag{1}$$

where Ψ is the Fourier transform of wavelet function ψ, the latter also called wavelet mother function, the mathematical representation of some mother wavelet are shown in Table 2 and their graphs are plotted in Fig. 1. Wavelet function ψ is called the "mother wavelet" because different wavelets generated from the expansion or contraction, and translation, they are called "daughter wavelets", which have the mathematical representation given by:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a \neq 0;\ a, b \in \mathbb{R}, \tag{2}$$

where a is the dilation variable that allows for the expansions and contractions of the ψ and b is the translation variable and allows translate in time.

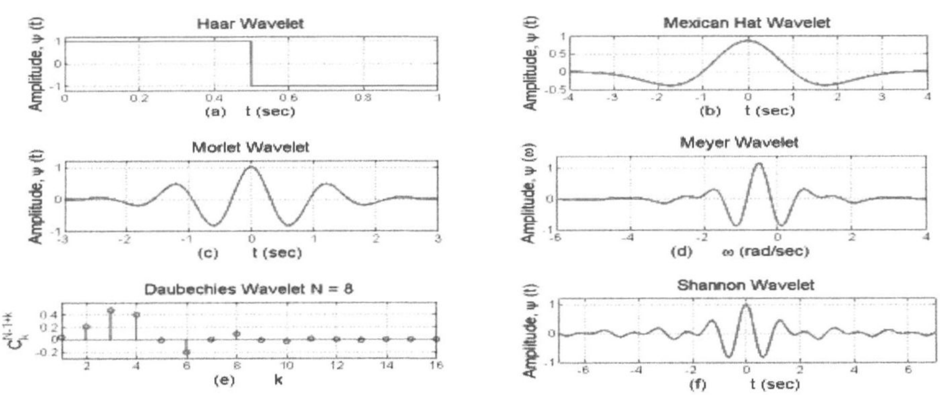

Fig. 1. Graphics of the mother wavelets showed in Table 2.

Haar	$\psi(t) = \begin{cases} 1, \text{ if } t \in [0, \frac{1}{2}] \\ -1, \text{ if } t \in (\frac{1}{2}, 1] \\ 0, \text{ otherwise} \end{cases}$
Mexican hat	$\psi(t) = \frac{2}{\sqrt{3}} \pi^{-\frac{1}{4}} (1 - t^2) e^{(-\frac{1}{2} t^2)}$
Morlet	$\psi(t) = e^{-\frac{t^2}{2}} \, cos(5t)$
Shannon	$\psi(t) = \frac{sin(\frac{\pi}{2} t)}{\frac{\pi}{2} t} \, cos(\frac{3\pi}{2}) t$
Daubechies	$P(y) = \sum_{k=0}^{N-1} C_k^{N-1+k} \, y^k$; C_k^{N-1+k} are binomial coefficients, N is the order of the wavelet
Meyer	$\psi(\omega) = \begin{cases} \frac{e^{\frac{i\omega}{2}}}{\sqrt{2\pi}} \sin(\frac{\pi}{2} \, v \, (\frac{3}{2\pi} \lvert\omega\rvert - 1)), \\ \quad \text{if } \frac{2\pi}{3} \leq \lvert\omega\rvert \leq \frac{4\pi}{3} \\ \frac{e^{\frac{i\omega}{2}}}{\sqrt{2\pi}} \cos(\frac{\pi}{2} \, v \, (\frac{3}{4\pi} \lvert\omega\rvert - 1)), \\ \quad \text{if } \frac{4\pi}{3} \leq \lvert\omega\rvert \leq \frac{8\pi}{3} \\ 0, \text{ otherwise} \end{cases}$ $v = a^4 (35 - 84a + 70a^2 - 20a^3)$, $a \in [0, 1]$

Table 2. Some examples of common mother wavelets

There are two types of wavelet transform: continuous wavelet transform (CWT) and discrete wavelet transform (DWT), whose mathematical definition are given by (3) and (4), respectively (Daubechies, 1992):

$$W_f(a, b) = \langle f, \psi_{a,b} \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi \left(\frac{t - b}{a} \right) dt, \tag{3}$$

$$W_f[a, b] = \frac{1}{\sqrt{a_0^m}} \int_{-\infty}^{\infty} f(t) \, \psi \left(\frac{t}{a_0^m} - kb_0 \right) dt, \tag{4}$$

for CWT, the expansion parameters a and translation b vary continuously on $\mathbb{R}$, with the restriction $a > 0$. For DWT, the parameters a and b are only discrete values: $a = a_0^m$, $b = kb_0 a_0^m$, where $a_0 > 1$, b_0 and are fixed values. In both cases $f \in L^2(\mathbb{R})$, i.e., a function that belongs to the space of all square integrable functions.

In DWT, one of the most important feature is the multiresolution analysis (Mallat, 1989a;b). Multiresolution analysis with a function $f \in L^2(\mathbb{R})$, can be decomposed in the form of successive approximations, using wavelet basis functions. The multiresolution analysis consists of a sequence successive approximations of enclosed spaces, nested spaces $\{V_N : N \in \mathbb{Z}\}$ with the following properties (Daubechies, 1992):

1. Nesting: $V_N \subset V_{N+1}, \ \forall \, N \in \mathbb{Z}$.
2. Closure: $clos \left(\bigcup_{N \in \mathbb{Z}} V_N \right) = L^2(\mathbb{R})$.
3. Shrinking: $\bigcap_{N \in \mathbb{Z}} V_N = \{0\}$.
4. Multiresolution: $f[n] \in V_N \Longleftrightarrow f[2n] \in V_{N+1} \ \forall \, N \in \mathbb{Z}$.
5. Shifting: $f[n] \in V_N \Longleftrightarrow f[n - 2^{-N} k] \in V_N \ \forall \, N \in \mathbb{Z}$.

6. There exists a *scaling function* $\phi \in V_0$ such that the integer shifts of ϕ form an orthonormal basis for V_0, i.e.,

$$V_0 = span\{\phi_{N,k}[n], \ \ N,k \in \mathbb{Z}\},$$

where

$$\phi_{N,k}[n] = 2^{-\frac{N}{2}}\phi[2^{-N}n - k], \tag{5}$$

forming an orthogonal basis of V_0. Then for each V_N exists additional space W_N that meets the following conditions (Daubechies, 1992)

$$V_{N+1} = V_N \oplus W_N, \tag{6}$$

$$V_N \perp W_N = 0, \ \ \forall\, N \in \mathbb{Z}, \tag{7}$$

and is

$$\psi_{N,k}[n] = 2^{-\frac{N}{2}}\psi[2^{-N}n - k], \ \ \forall\, N,k \in \mathbb{Z}, \tag{8}$$

forming an orthogonal basis for W_N, i.e. at $\psi[n]$ can generate the space W_N.

From the above we can say that the purpose of analysis multiresolution is to determine a function $f[n]$ by successive approximations, as

$$f[n] = \sum_{k=-\infty}^{\infty} c_{N,k}\phi_{N,k}[n] + \sum_{m=1}^{N}\sum_{k=-\infty}^{\infty} d_{m,k}\psi_{m,k}[n], \tag{9}$$

with

$$c_{m,k} = \sum_{k=-\infty}^{\infty} f[n]\overline{\phi_{m,k}[n]},$$

$$\tag{10}$$

$$d_{m,k} = \sum_{k=-\infty}^{\infty} f[n]\overline{\psi_{m,k}[n]}.$$

Where N is the level at which decomposes $f[n]$ and $\overline{\phi[n]}$, $\overline{\psi[n]}$ are conjugate functions for $\phi[n]$ and $\psi[n]$, respectively. Multiresolution analysis, in addition to being intuitive and useful in practice, form the basis of a mathematical framework for wavelets. One can decompose a function a soft version and a residual, as we can see from (9), where the wavelet transform decomposes a signal $f[n]$ in one approach or trend coefficients c and detail coefficients d which, together with $\phi[n]$ and $\psi[n]$, are the smoothed version and the residue, respectively.

The important thing here is that the decomposition of the $f[n]$ for large enough value of N can be approximated arbitrarily close to V_N. This is that $\exists$ some $\epsilon > 0$ such that

$$\|f[n] - \sum_{k=-\infty}^{\infty} c_{N,k}\phi_{N,k}[n]\| < \epsilon. \tag{11}$$

The approach by the truncation of the wavelet decomposition can be approximated as:

$$f[n] \approx \sum_{k=-\infty}^{\infty} c_{N,k}\phi_{N,k}[n]. \tag{12}$$

This expression indicates that some fine components (high frequency) belonging to the wavelet space W_N for the $f[n]$ are removed and the components belonging to the coarse scale space V_N are preserved to approximate the original function at a scale N. Then (12) tells us that any function $f[n] \in L^2(\mathbb{R})$ can be approximated by a finite linear combination.

2.2 Wavelet PID controller design

A classic control scheme consists of three basic blocks as shown in Fig. 2: the plant can be affected by external perturbation P, the sensor measures, the variable of interest y, and finally the controller makes the plant behaves in a predetermined manner, y_{ref}. One of the most employed controller in the modern industry is a classical control Proportional, Integral and Derivative, PID because its easy of implementation, requiring only basics testing for tuning gains k_P k_I and k_D (Aström & Hägglund, 2007).

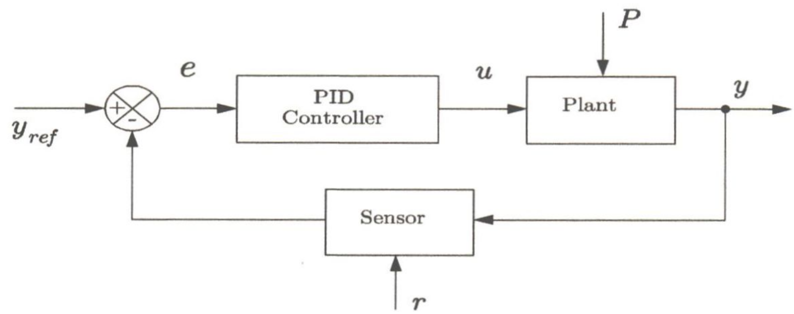

Fig. 2. Scheme of a SISO system with a PID controller.

In general, a PID controller takes as input the error signal e and acts on it to generate an output control signal u, as

$$u = k_P e + k_I \int_0^t e dt + k_D \frac{de}{dt}, \tag{13}$$

where k_P, k_I y k_D are the PID gains to be tuned, and e is the error signal which is defined as

$$e = y_{ref} - y, \tag{14}$$

The form of a discrete PID is (Visioli, 2006):

$$u(k) = u(k-1) + k_P\left[e(k) - e(k-1)\right] + k_I e(k) + k_D\left[e(k) - 2e(k-1) + e(k-2)\right], \tag{15}$$

whose transfer function is given by

$$\frac{u(z)}{e(z)} = k_P + k_I \frac{T}{2}\frac{z+1}{(z-1)} + k_D \frac{1}{T}\frac{(z-1)}{z}, \tag{16}$$

and its operation is the same way that the continuous PID.

Taking the parameters k_P, k_I and k_D of the PID, as adjustment variables, then (15) can be described as

$$u(k) = u(k-1) + \sum_{i=0}^{2} k_i e(k-i), \tag{17}$$

or equivalently

$$\Delta u(k) = \sum_{i=0}^{2} k_i e(k-i), \tag{18}$$

where $k_0 = k_P + k_I + k_D$, $k_1 = -k_P - 2k_D$ y $k_2 = k_D$. From (18), we see that the control law of a classic PID is a linear decomposition of the error, only that this decomposition is fixed, that is, always has three terms, this makes the difference between the classic PID and the MRPID, where here the number of decompositions can be infinite and even more than each one is different scales of time-frequency, this means that the MRPID controller decomposes the signal error e for high, low and intermediate frequencies, making use of multiresolution analysis for the decomposition. Where the components of the error signal are computed using (9) through a scheme of sub-band coding, as shown in Fig. 3.

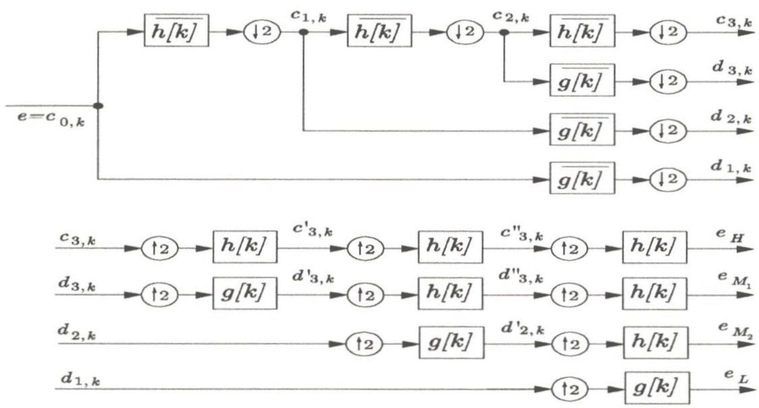

Fig. 3. Sub-band coding scheme for decomposition of the error signal e for N=3.

Thus each of these components are scaled with their respective gains and added together to generate the control signal u, as follows:

$$u = K_H e_H + K_{M_1} e_{M_1} + \cdots + k_i e_i + \cdots + K_{M_{N-1}} e_{M_{N-1}} + K_L e_L, \tag{19}$$

$$u(k) = \mathbf{K}\,\mathbf{E_m}(k), \tag{20}$$

where

$$\mathbf{K} = [K_H \; K_{M_1} \; \cdots \; K_i \; \cdots \; K_{M_{N-1}} \; K_L], \tag{21}$$

$$\mathbf{E_m}(k) = [e_H(k) \; e_{M_1}(k) \; \cdots \; e_i(k) \; \cdots \; e_{M_{N-1}}(k) \; e_L(k)]^T, \tag{22}$$

where N is the level of the MRPID controller.

While a classical PID control has three parameters to be tuned k_P, k_I and k_D, the MRPID control has two or more parameters and the number of parameters depends on the level of decomposition is applied to the signal error e. The schematic diagram of a plant using a MRPID control is shown in Fig. 4.

As shown in Table 2, there are a number of different wavelets, the wavelet selection affects the operation of the controller. Therefore, there are characteristics that should be taken into account, such as:

- The type of system representation (continuous or discrete).