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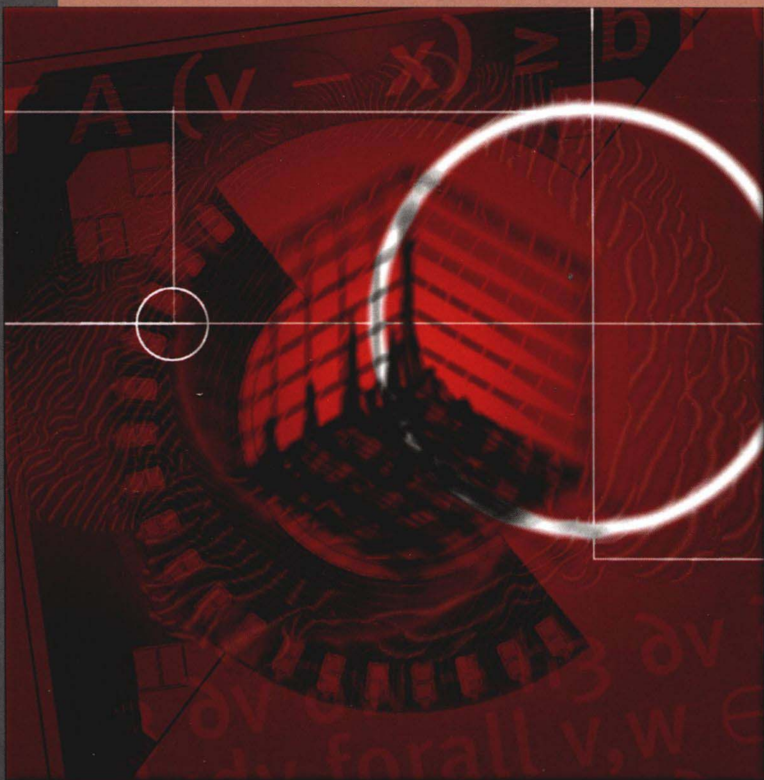
A Math Primer for Engineers



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A Math Primer for Engineers

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A MATH PRIMER FOR ENGINEERS

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Preface

The purpose of this Math Primer is to provide a brief introduction to those parts of mathematics which are, or could be, useful in engineering, especially bioengineering. A wide range of topics is covered and in each area an attempt is made to summarize the **ideas** involved without going into **details**. The pace is varied. In the earlier sections there is a relatively leisurely description of simple topics. Later, the tempo increases. Sometimes, the speed is quite hair-raising. Nevertheless, it is hoped that the reader may still catch a glimpse of ideas which may spark interest.

It is possible to describe mathematical ideas using few or no formulas and equations, and several well-known books do just this. This is rather like describing a rocket in words - one knows what it does but has no chance of building one. Here formulas and equations have not been avoided. In fact the text is littered with them, but every effort has been made to keep them simple in the hope of persuading the reader that they are not only useful but also accessible to engineers.

Mathematics and engineering are inevitably interrelated, and this interaction will steadily increase as the use of mathematical modelling grows. The interaction is not one-sided and there are many examples of cases where engineers have contributed to mathematics. As a young man, the author read, and was impressed by, the notes of the engineer and physicist **Oliver Heaviside** who did pioneering work in the application of complex numbers in engineering, in the solution of differential equations using symbolic methods, and in vector calculus. His achievements are recalled by the **Heaviside function**[1] in mathematics and the **Heaviside layer** in the atmosphere.

Although mathematicians and engineers often misunderstand one another, their basic approach is quite similar. Consider the problem of designing steam boilers. One of the worst maritime disasters in the history of the USA occurred on April 27, 1865 when a steam boiler exploded on the steamboat *Sultana*; more than 1500 passengers and crew died. Boiler explosions continued to occur frequently: alone from 1880 to 1890 more than 2000 steam boilers exploded. In response, the ASME (American Society for Mechanical Engineers) drew up its very first standard entitled *Code for the Conduct of Trials of Steam Boilers* in the year 1884, and in 1914 the ASME issued the first edition of the *ASME Boiler Code, Rules for the Construction of Stationary Boilers and for Allowable Working Pressure* which set standards for the design of boilers; this code has evolved through the years and is still an industry standard. Every major accident is investigated by the

[1] Concepts in **bold** type are referenced in the Index of Concepts.

engineering community to determine whether the appropriate industrial codes need to be amended. The stakes are high, as shown by the literature on **forensic engineering** (see e.g. Peter R. Lewis *Safety First?* [Lew10]) and the recent disasters in the Gulf of Mexico and Fukushima. In contrast, the mathematical approach is that of fail-safe design. Every possible boiler under every possible condition would be analysed. If successful, this would be formulated as a theorem: Under conditions A, B, ... a boiler with this design will be safe. There is of course a slight snag with this approach - the theorem may never be proved in which case boilers may never be built!

Another point of similarity between mathematics and engineering is the historical development of each subject. The design of bridges has slowly evolved over the centuries as new ideas and concepts were introduced and new materials became available. In a very similar fashion, the mathematical techniques described below have slowly evolved, starting from the simple concept of a number and expanding step by step. Non-mathematicians often believe that the development of mathematics has more-or-less stopped, whereas in fact the subject continues to develop rapidly. In an attempt to convey this dynamic development, which has accelerated in recent years, the dates when concepts were first introduced are frequently cited.

A recurring theme is that of **modelling** or, in American usage **modeling**, by which is meant the formulation of a mathematical model for a physical (or biological) system, which can be used to explain and predict the behaviour of the physical system. Examples of modelling are scattered throughout the text and some general remarks will be found in Chapter 16. The *Index of Applications* contains a list of the applications discussed in the text.

In a collaboration between mathematicians and engineers, it is usually easier for the mathematicians to learn the engineering jargon and background, rather than the opposite. In this connection one can not do better than quote John R. Womersley who made a significant contribution to the analysis of arterial blood flow: John R. Womersley *An elastic tube theory of pulse transmission and oscillatory flow in mammalian arteries* [Wom57, page 4]:

> It is a particular pleasure to record the friendly collaboration and criticism of Dr. D. A. McDonald, Reader in Physiology at Bart's, who not only first introduced the author to this fascinating problem, but also helped him to acquire some of the experimentalist's "feel" for the physical conditions in many small ways, hard to define, but nonetheless real. This work itself, will, it is hoped, be regarded as a successful result of an experiment in interdisciplinary cooperation. It demonstrates that the successful application of mathematics to biological problems is not to be found in the study or the library. An essential condition for success is that the mathematician must get the "feel" of the problem by personal contact and discussion with the physiologist, and must at the same time have sufficient professional standing to maintain a critical attitude. If the mathematician is merely a junior hired "to do the mathematics" there will be no progress. If he is a Professor in another department to whose Olympian presence problems are brought, some interesting mathematics will be done, but it will lack point and substance from

> the physiologist's point of view. The history of past attempts at the solution of this very problem is full of such examples. It would seem that progress is likely to depend on the occurrence of happy accidents of the sort that led to the work described here. [Bart's is a London hospital]

In such a collaboration, some of the legitimate concerns of the mathematician may become lost. Here, an attempt has been made to show, using simple examples, that questions such as those involving the existence and uniqueness of solutions to problems are often not just pedantic but reflect real practical difficulties.

Over the past half century, the mathematical literature has become much more terse, and older famous textbooks by well-known mathematicians now seem very leisurely and imprecise. Much has been gained by the increased rigour, but for the non-specialist and beginner the older literature has advantages and will sometimes be referred to here.

As long ago as 1908 the exceptionally creative French mathematician, physicist, and engineer [2] **Henri Poincaré** complained about what he called the advance of the "logicians" and wrote: (Henri Poincaré *Science and Method* [Poi03, page 129]):

> The engineer must receive a complete mathematical education, but of what use is it to be to him, except to enable him to see the different aspects of things and to see them quickly? He has no time to split hairs. In the complex physical objects that present themselves to him he must promptly recognize the point where he can apply the mathematical instruments we have put in his hands.

In this spirit the author hopes that the reader will become acquainted with some new *mathematical instruments* which he or she can apply.

During the past ten years the author has been fortunate to be able to collaborate with a physiologist, **Paul Lunkenheimer**, and a biomechanical engineer, **Peter Niederer**, and this book could not have been written without their encouragement. The author would also like to thank his colleagues in the Institute for Numerical and Applied Mathematics, Westfälische Wilhelms-Universität, Münster, for making it possible for the author to work on this text after his retirement. Thanks are due to **Andrew D. McCulloch**, Professor of Biomechanics at the University of California San Diego, who generously provided help and access to his computer software "Continuity" during a stimulating visit to San Diego many years ago. Thanks are also due to **Dr. Randas Batista**, who made it possible for the author to watch the Batista operation many times and thereby gain an appreciation of the real-life complications of heart surgery.

A special thank you is due to my colleague Frank Wübbeling who, on numerous occasions, provided advice on coping with the idiosyncrasies of ever-changing computer systems.

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[2] For many years Poincaré (1854-1912) simultaneously held posts as a professor of mathematics and as a senior mining engineer!

The staff of IOS Press - in particular Kairi Look and Maureen Twaig - provided helpful advice and patient support.

Finally, I thank my wife Gabriele for her help and encouragement during the long gestation period of this book.

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