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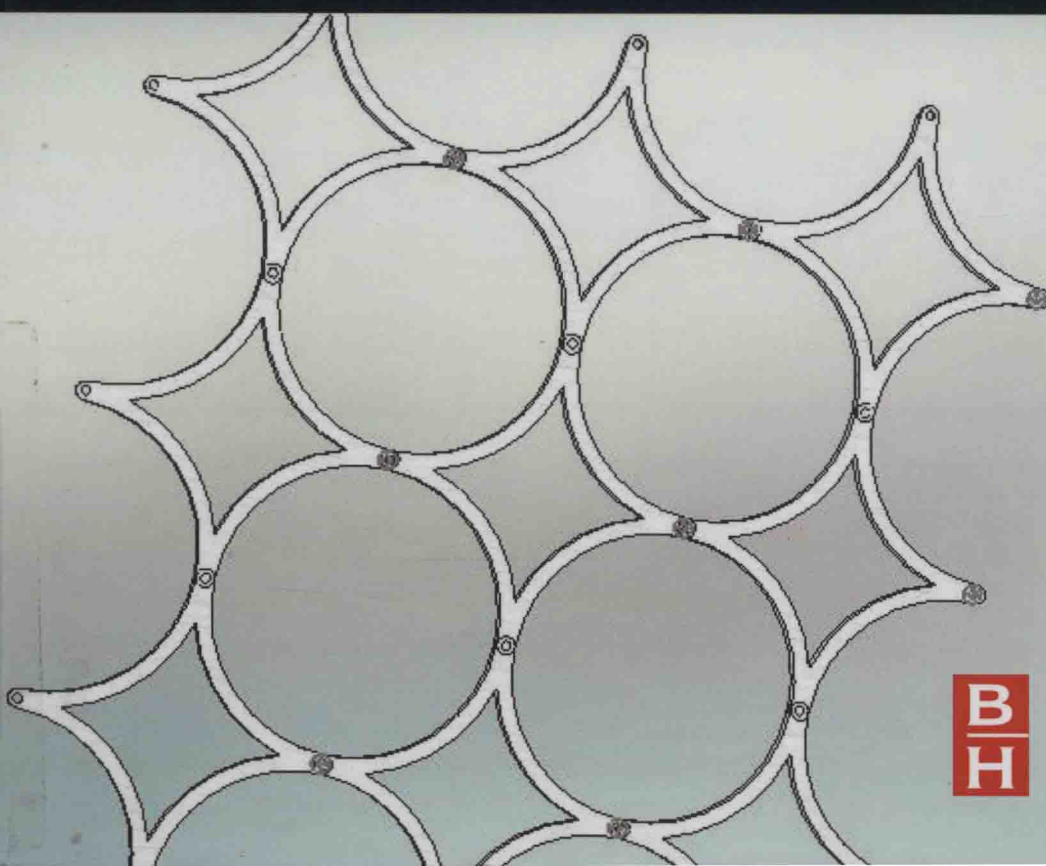
Theory and Experimental Studies



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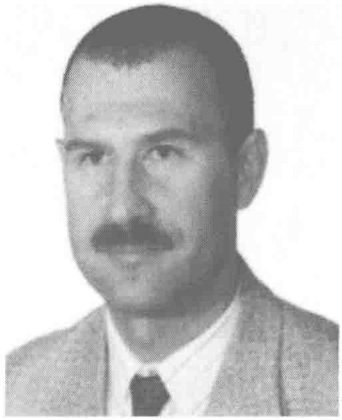
About the author



Andrzej Ziółkowski studied engineering sciences at Warsaw Technical University, Warsaw, Poland. In 1983, he obtained a MSc degree in applied mechanics in the Department of Aviation and Power Systems, Warsaw Technical University, and in 1987 he obtained a second MSc degree in power systems and equipment in the same department at Warsaw Technical University upon directing his interests to nuclear power systems engineering. In 1988, he joined the Institute of Fundamental Technological Research, Polish Academy of Sciences (IPPT PAN) for PhD studies, where he has been employed since 1992. In 1996, he obtained a PhD degree for a dissertation devoted to constitutive and numerical modeling of shape memory alloys' behavior, which has been distinguished for excellence by the Scientific Board of IPPT PAN. In 2007, he was awarded a habilitation degree for a dissertation devoted to experimental studies and theoretical description of pseudoelasticity of shape memory alloys.

Dr. Ziółkowski's research interests are in mechanics and thermodynamics of continuous media, structural mechanics, experimental mechanics, thermodynamics of solid-solid phase transitions, and finite element code simulations, but his primary field of specialty is shape memory alloys. He has published a number of papers devoted to experimental investigation, and constitutive and numerical modeling of SMA materials (NiTi mainly). Dr. Ziółkowski was co-organizer of the Shape Memory Alloys Workshop and a member of the scientific committees of the Shape Memory Materials Symposiums of International Conferences. Much of his research work has been conducted within a framework of international collaboration that was facilitated by his short-stay visits in leading research centers such as the University of Grenoble, the University of Besancon, the Mathematical Institute in Bucharest, Metropolitan University of Tokyo, Kyoto University, Catholic University of Leuven, Cambridge University, and Metz University.

Preface

The quest for new materials with properties superior to those already attainable, more useful for a specific purpose, or exhibiting a combination of required qualities is a continuous subject of interest to researchers and engineers all over the world. A very interesting group of metallic materials make up so-called *shape memory alloys* (SMA), revealing a number of extremely useful practical applications, combinations of functional properties, and utility features not observed together in other metallic materials. For that reason, these materials frequently earn such attributes as *smart*, *intelligent*, or they are sometimes called *living composites*. Intelligence and smartness are inherent attributes of living creatures, and they can be inherited by SMA materials probably to the extent that actual users of them exhibit these virtues, but nobody should object when SMA materials are called *adaptive materials*. The nice, from an engineering point of view, features of SMA materials do not come at any cost. Their behavior is *very complex*, and their efficient use requires comprehensive experimental knowledge about their performance when submitted to various thermomechanical loadings, as well as possessing a credible theoretical model that enables SMA materials to have consistent characterization and the quantitative prediction of appearing stresses, deformations, and/or thermal effects.

The aim of this book is to provide experimental evidence of SMA materials' pseudoelastic behavior and a theoretical framework for the efficient and accurate constitutive modeling of SMA materials' pseudoelasticity. The term *pseudoelasticity* is understood here as the loading-unloading cycle involving *hysteresis*, with the underlying assumption that the state of the material remains the same/does not change after termination of such a loading cycle (no change of material properties occurs). In the materials science literature, this range of SMA materials' behavior is frequently called *superelasticity*, the term stemming from the SMA property that elastic strains involved are close to two orders of magnitude larger than in the case of classical engineering materials (e.g., structural steel). The terms *superelasticity* and *pseudoelasticity*, in the sense delineated above, can be treated as equivalent.

The above definition of SMA pseudoelasticity presumes no change of properties of the material due to the loading-unloading cycle; thus, the considerations present in this book do not embrace in a direct way the Mullins effect, which can be treated as a manifestation of pseudoelastic behavior of rubber-like materials and that is caused by progressive damage to the material due to the sequence of loadings. The general philosophy adopted in this book is that we already have an SMA material with stable properties, by whatever means it is achieved. By stable we mean stable in time, in the intended operational regime, which in one case will mean stable for one cycle of work and in another stable during 100-1000 cycles or even 1 million cycles of operation. Whatever the values of modeling parameters and functions characterizing SMA material, they are assumed to be invariable. Thus, the book does not deliberately discuss strategies on how to obtain SMA material with specific features, which the author believes is the primary task of materials science specialists. The procedures of thermomechanical treatments, training procedures of SMA, and evolution of SMA materials properties are not discussed, nor are material functional or utility properties degradation due to cyclic loadings.

The book focusses on delivering experimental evidence about the essential features of macroscopic, thermomechanical behavior of SMA materials. Upon careful analyses and discussion of the experimental results presented, the concepts and ideas are introduced useful in description of macroscopic, pseudoelastic behavior of SMA materials, within the context of *continuum mechanics* and *nonequilibrium thermodynamics*. The worked-out idealizations and simplifications serve as an introduction to and development of a family of so-called R_L models of pseudoelasticity, the first member of which was originally elaborated as a result of Polish-French-Japanese collaboration by Raniecki, Lexcellent, and Tanaka at the beginning of 1990s.

Shape memory effects (SME) of metallic materials, pseudoelasticity among them, originate on physical grounds from thermoelastic, martensitic phase transformation (p.t.). The latent heat of this phase transition is quite considerable in the case of some SMA materials, which results in strong coupling of two physical fields: mechanical and thermal. For example, in the case of the current commercially dominant SMA material (i.e., NiTi alloy), the thermomechanical coupling originating from latent heat of martensitic p.t. may lead, in adiabatic conditions, to an increase of temperature on the order of 50 °C; an effect that can hardly be disregarded.

The formalism of continuum, macroscopic, nonequilibrium thermodynamics with internal state parameters is employed in this book for phenomenological modeling of SMA materials behavior and characterization. Nevertheless, information from all levels of micro-, meso-, and macroscopic observation has been used in building an effective macroscopic theory of pseudoelasticity. Crystallographic and/or compound martensitic variants, objects that can be distinguished at a microscopic scale of observation, are grouped together in mesoscale to distinguish larger generic objects: *self-accommodating* martensite and *oriented* martensite. Information on specific properties and behavior of microlevel and mesolevel objects in their respective scale of observation enters the macroscopic, constitutive model only in indirect, tacit way.

For example, *mesomechanical* studies are executed to obtain a physical interpretation of terms present in a proposed heuristically form of SMA materials *macroscopic Gibbs free energy function*. The so-called representative volume element (RVE) of the SMA material, a material point with homogeneous properties in a macroscopic model, is treated as a two-phase (multiphase) *elastic medium with eigenstrains*, its *microstructure evolving* due to applied thermomechanical loadings. The assumption on continuity of the displacement field in the RVE done in these studies originates from *microscopic* observations of two-phase austenitic, martensitic microstructures. The incompatibility of phase eigenstrains and constraint of continuity of the displacement field required on interphase boundaries are the key sources for *coherency energy* stored in SMA material RVE. In order to determine the RVE macroscopic effective properties, it is actually treated as a composite with frozen—at some arbitrary stage of loading—and very complex mesostructure to which homogenization methodology is employed.

An effective description of any physical phenomenon requires reaching an amicable compromise between model intricacies and the accuracy required from it for specific purposes. It seems that such an amicable compromise has been reached in the family of R_L models of pseudoelasticity thanks to the introduction of only two internal state parameters. The scalar parameter is the *volume fraction of martensite* (oriented one), and the tensorial parameter is the so-called *ultimate phase eigenstrain*. The product of volume fraction of martensite and ultimate phase eigenstrain gives *macroscopic phase strain*. Evolution of these two parameters with external loading of macroscopic stress and temperature allows predicting the value of macroscopic phase strain describing deformation effects of SMA materials connected with martensitic phase formation, its presence and its mesostructural evolution—reorientation of martensitic phase.

Extensive study and analysis of experimental data on the *macroscopic* behavior of SMA materials in the pseudoelastic range of their behavior, and the belief that matter organization is subject to some higher ordering principles, led to the formulation of rules governing the evolution of internal state parameters.

The so-called rule of *optimum mesostructure rearrangement* of SMA material RVE has been noticed and worked out. It is presumed valid in this entire book. Exploitation of this law leads to the property that ultimate phase eigenstrain tensor always follows/has the direction of macroscopic stress tensor. As a consequence, R_L model theory predicts that macroscopic phase strain tensor always has the direction of macroscopic stress tensor in the pseudoelastic range of SMA materials behavior.

A primary question arises regarding what is actually the key reason for the appearance of pseudoelastic behavior—hysteresis loop in SMA materials behavior, taking into account the axiom posed earlier that upon the loading-unloading cycle the properties and the state of SMA material remains unchanged. In that respect, the fundamental conjecture is adopted that it is *phase instability* of RVE's mesostructural arrangement that causes the appearance of pseudoelastic behavior. Unstable phase equilibrium states lead to the abrupt initiation of qualitative reorganization of SMA material internal mesostructure. The conjecture is used to formulate the criteria of initiation of forward and reverse phase transitions directly, consistently, from the adopted form of Gibbs potential. No ad hoc threshold values are proposed for phase transition initiation analogous to the theory of plasticity, as is frequently done in the literature on modeling SMA materials.

Subsequent progress of forward or reverse martensitic phase transformations evolution of volume fraction of martensitic phase is governed by the kinetics rule of phase transitions. This law has been elaborated, taking advantage of the information that thermoelastic martensitic phase transition is athermal (i.e., it does not depend on time rate), and upon the analysis of experimental data. A specific mathematical form of this rule is adopted, which is responsible for the shape and size of the appearing hysteresis loops. The p.t. kinetics rule embraces a description of internal hysteresis loops. As mentioned earlier, strain effects resulting from reorientation of the already born martensitic phase are governed by optimum microstructure rearrangement principle.

In the broader perspective presented in this book, studies on the pseudoelastic behavior of SMA materials can be perceived as a theory of hysteresis loop; though, it is of a different character than the one exhibited by classical magnetic materials. The description of the pseudoelasticity of SMA materials proposed here is very attractive and opens up very promising new research directions and areas. Relatively uncomplicated adaptations allow reusing considerable parts of already developed theoretical apparatus to describe macroscopic pseudoelastic behavior encountered in magnetic SMA, light-induced shape memory materials, shape memory polymers, biological tissues, sponge and foam materials, and rubber-like materials in general materials exhibiting hysteresis loop under a mechanical load-unload cycle. Specific physical mechanisms underlying pseudoelastic behavior can be completely different in each specific case.

The adaptation process can be straightforward, thanks to the versatility of formalism of nonequilibrium thermodynamics with internal state parameters, which is presented in detail in this book. Adaptation of the present theory for the description of behavior of polymeric SMA, in addition to the modification of kinetics relations describing the progress of glass transition fixing/releasing process, will also require introduction of an additional internal state variable to relevantly describe the evolution of viscoelastic deformation effects. Adaptation of the theory to describe the Mullins effect—pseudoelasticity of rubber-like materials—will require at least replacement of the volume fraction of martensitic phase with some other internal state parameter correctly defined to reflect damage of the internal structure of the rubber material, as well as the discovery and development of its evolution rule.

This monographic book on the pseudoelasticity of SMA can serve well those who are novices in the field, as it provides extensive background information up to the present state of the art in SMA materials. Beginning with a bird's-eye view history of SMA materials development, it explains and/or precisely defines SME in thermomechanical terms, and discusses where, how, and why special behavior of SMA found numerous applications in various engineering areas. The book is valuable for students and faculty, as it explains SME at a fundamental level. It contains a survey of modeling approaches targeted at the reliable prediction of SMA materials behavior on different scales of observation: atomistic, microscopic, mesoscopic, and macroscopic.

Doctoral students and young researchers can find very detailed information on modern methodology used in the process of building constitutive models of advanced materials exhibiting complex behavior; embracing several physical fields—here thermal and mechanical—with special attention on grasping the relationships between the internal organization of materials and their thermomechanical properties. Advanced senior researchers and practicing engineers can find information on the philosophical approaches and many details used to pose theoretical modeling assumptions, leading to considerable simplifications in material model mathematical formulas and, thus, more effective computational models.

Engineering practitioners and designers may find the book to be a helpful, practical tool in carrying out estimates on the applicability of projected engineering solutions grounded on taking advantage the special features of SMA materials using such software as Mathcad and constitutive relations of the R_L models family. Complex, challenging, highly responsible devices made of or including components made of SMA materials usually require finite element code studies. In the web page accompanying the book (http://booksite.elsevier.com/9780128016978/) for demonstration purposes, detailed information is delivered on the process of implementation of R_L model relations into commercial finite element code in the case of axis symmetric problems. The code allows for running fully coupled thermomechanical analyses in which mechanical and thermal effects connected with SMA materials behavior can be reliably evaluated.

Successful exploitation of SMA materials requires extensive interdisciplinary collaboration. It is the hope of the author that this book can fill the gap between mechanicians, physicists, chemists, materials science specialists, and other experts interested in SMA materials, including medical doctors making use of NiTi implants and devices—probably the major and most important current commercial application area of SMA materials (NiTi-based alloys). This is achieved by creating a base of well-defined set of common concepts, objects, and ideas.

This book has evolved from the present author's over 20 years of research work on SMA materials, documented in scientific papers, and his doctoral and habilitation thesis, run in the Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw, Poland.

As with any project of this size there are a number of people to thank. The first person is my mentor and afterward colleague professor Bogdan Raniecki, who introduced and inspired me into the topic of SMA. Discussions and disputes with him, looking from perspective, delivered me a lot of happiness and satisfaction. I had the privilege to experience what I believe had been experienced by Greek peripatetics in the flourishing period of ancient Greece. I wish to thank professor Lech Dietrich for his unconditional support and advice on issues connected with experimental work on SMA alloys. I express my gratitude to all colleagues who entered with me into lengthy, detailed, and sometimes energetic discussions, which I believe make the essence of academic life. Such discussions create a kind of mutual mirror for working out viable ideas, because before you can convince somebody else you must work out arguments to convince yourself.

Many of the results contained in the book have originated from studies performed over the years in cooperation with researchers from France and Japan: Professor Christian Lexcellent, Kiki Tanaka, Hisaaki Tobushi, Suichi Miyazaki. I would like to express my sincere gratitude to all of these individuals for their cooperation and support. I am grateful to Steven Merken and Jeffrey Freeland from Elsevier for their commitment and cooperation to make this project become reality. I also wish to thank Sruthi Satheesh for expertly running production process of the book.

Andrzej Ziółkowski, Warsaw, Poland, February, 2015

List of symbols

$\mathcal{A}$	*austenite* (parent phase). Phase stable at high temperatures with high symmetry crystallographic lattice
A_s^0	austenite start temperature of self-accommodating martensite (Note: superscript "0" standing at phase transition characteristic temperatures denotes that the measurement took place at zero macroscopic stress; these are simply classical DSC-determined temperatures $(A_s^0 = A_s)$.
A_s^{2-1}	austenite start temperature of oriented martensite $(A_s^{2-1} > A_s^0)$
A_f^0	austenite finish temperature of self-accommodating martensite
A_f^{2-1}	austenite finish temperature of oriented martensite $(A_f^{2-1} > A_f^0)$
$\breve{a}(\boldsymbol{x})$	any nonhomogeneous mesoscopic field in the RVE; for example, stress, strain, or elastic properties $\breve{\boldsymbol{\sigma}}(\boldsymbol{x})$, $\breve{\boldsymbol{\sigma}}(\boldsymbol{x})$, $\breve{\mathbf{L}}(\boldsymbol{x})$
$\boldsymbol{\alpha} = \alpha\mathbf{I}$	isotropic thermal expansion tensor
α	thermal expansion coefficient
$\alpha = 1$	numerical denotation of parent phase (austenite)
$\alpha = 2$	numerical denotation of oriented martensitic phase (oriented, detwinned martensitic compound)
$\alpha = 3$	numerical denotation of self-accommodating martensitic phase (self-accommodating, twinned martensitic compound)
${}^0\mathcal{B}$	initial configuration of material body at time instant $t = t_0$
${}^t\mathcal{B}$	configuration of material body at some time instant t
${}^t\mathcal{B}^{in}$	instantaneous natural configuration at time t, that is, configuration at state $\boldsymbol{\sigma} = \mathbf{0}, T = T_0, z = z_t, h_K = h_K(t)$; also called unloaded elastically configuration
$\boldsymbol{\mathcal{B}}_\alpha$	dimensionless stress concentration factor tensors
c_v	heat capacity at constant volume
c_p	heat capacity at constant pressure
D	normalized thermodynamic driving force of phase transition $D \equiv \pi^{1-2}/\pi_N^{1-2}$; π_N^{1-2} (normalizing factor)
$\boldsymbol{d}$	Eulerian strain rate tensor (stretching rate tensor)
$\boldsymbol{d}^e$	elastic stretching rate tensor
$\boldsymbol{d}^{in}$	inelastic stretching rate tensor
$\boldsymbol{E}(n)$	Lagrangean strain measure from Hill's family
$\boldsymbol{E}^e(1)$	elastic Green-Lagrange strain
$\boldsymbol{E}^e(0)$	elastic logarithmic strain
$\boldsymbol{e}(n)$	Eulerian strain measures
$\boldsymbol{e}^e(0)$	elastic Eulerian logarithmic strain
ε_{eq}	equivalent strain $\varepsilon_{eq} = \sqrt{\frac{2}{3}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{ij}}$
$\boldsymbol{\varepsilon}$	small strains tensor
$\boldsymbol{\varepsilon}^e$	macroscopic elastic strain

$\boldsymbol{\varepsilon}^{\mathrm{pt}}$	macroscopic phase transition strain; $\boldsymbol{\varepsilon}^{pt} \equiv z\boldsymbol{\kappa}$
$\boldsymbol{\mathcal{E}}^{\mathrm{de}}$	elastic Zaremba-Jaumann of elastic logarithmic strain and elastic stretching rate connection tensor
$f(y)$	"shape" function of limit surface of pseudoelastic flow
$\boldsymbol{F}$	deformation gradient $\boldsymbol{F} \equiv \mathbf{d}\boldsymbol{x}/\mathbf{d}\boldsymbol{X}$
ϕ	macroscopic specific Helmholtz function; specific Helmholz free energy
ϕ^{tm}	thermoelastic part of free energy
ϕ^{coh}	specific coherency energy
ϕ_{it}	specific interaction energy of austenitic and martensitic phase
ϕ_2^{st}	energy stored in SMA macroelement after full completion of martensitic phase transformation
g	macroscopic, specific Gibbs function; specific Gibbs free energy; specific free enthalpy (J/kg)
g^{ult}	ultimate phase eigenstrain potential
γ	amplitude of pseudoelastic flow in pure shear
$\breve{\boldsymbol{\Gamma}}(\boldsymbol{x})$	local eigenstrain field in RVE, piece-wise continuous; strain field connected with martensitic phase transition
$\breve{\boldsymbol{\Gamma}}_\alpha(\boldsymbol{x})$	eigenstrain field of phase α in RVE; local eigenstrain field of austenitic phase is always zero, $\breve{\boldsymbol{\Gamma}}_1(\boldsymbol{x}) \equiv \mathbf{0}$ for $\boldsymbol{x} \in V_1$
$\boldsymbol{\Gamma}_{\boldsymbol{\alpha}}$	macroscopic value of phase α eigenstrain. Average value over volume of phase α in RVE—$\boldsymbol{\Gamma}_\alpha \equiv \left\langle \breve{\boldsymbol{\Gamma}}(\boldsymbol{x}) \right\rangle_{V_\alpha}$. Macroscopic eigenstrain of austenitic phase and self-accommodating martensitic phase (compound) is always zero $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_3 \equiv \mathbf{0}$
h_K	a set of independent, mesoscopic, internal variables $(K = 1, 2, \ldots, N)$, describing evolving geometry of mesostructure of SMA material RVE
J	determinant of deformation gradient $(J \equiv \det(\boldsymbol{F}) = \rho^0/\rho)$
J^{e}	determinant of elastic deformation gradient $J^{\mathrm{e}} \equiv \det(\boldsymbol{F}^{\mathrm{e}})$
K	bulk modulus of elasticity $K = E/3(1-2v)$
$\boldsymbol{\kappa}$	macroscopic ultimate phase eigenstrain tensor of oriented martensite $(\mathrm{tr}(\boldsymbol{\kappa}) = 0)$.
l_f, $\overset{*}{l}_f$	actual latent heat of complete austenite into oriented martensite p.t., irreversible and reversible thermodynamically, respectively
l_f^{sa}, $l_f^{*\mathrm{sa}}$	actual latent heat of austenite to self-accommodating martensite p.t. irreversible and reversible thermodynamically, respectively
$\mathbf{L}$	tensor of elastic stiffness properties $(\mathbf{L} = \rho\partial^2\phi/\partial\boldsymbol{e}^{\mathrm{e}}(0)\partial\boldsymbol{e}^{\mathrm{e}}(0))$ or, in numerical terms current, instantaneous, elastic tangent $\partial\boldsymbol{\sigma}/\partial\boldsymbol{e}^{\mathrm{e}}(0) = \mathbf{L}$
$\mathbf{L}_\alpha$	tensor of elastic properties of specific phase
$\mathbf{L}^{(\mathrm{Je})}$	current, instantaneous, elastic Zaremba-Jaumann rate tangent $\left(\mathbf{L}^{(\mathrm{Je})} \equiv \mathbf{L}\left[\mathbf{I}^{(4)} + \boldsymbol{\mathcal{E}}^{\mathrm{de}}\right]; J\mathbf{L}^{(\mathrm{Je})} = \partial\left(\overset{\circ}{\boldsymbol{\tau}}{}^{(\mathrm{Je})}\right)/\partial\boldsymbol{d}^{\mathrm{e}}\right)$
$\boldsymbol{L}$	Eulerian velocity gradient tensor
λ_i^{e}	principal elastic stretches
$\{\boldsymbol{m}_i\}$	versors of director triad determining orientation of natural configurations $'\mathcal{B}^{\mathrm{in}}$ in fixed coordinates frame
$\mathcal{M}$	martensite. Phase stable at low temperatures with lower symmetry crystallographic lattice.
$\mathcal{M}_{\mathrm{or}}$	oriented martensite; detwinned martensite $(\mathcal{M}_{\mathrm{or}} \Leftrightarrow \mathcal{M}_2)$
$\mathcal{M}_{\mathrm{sa}}$	self-accommodating martensite; twinned marensite $(\mathcal{M}_{\mathrm{sa}} \Leftrightarrow \mathcal{M}_3)$
M_{d}	limit temperature above which it is impossible to stress-induce martentic transformation

M_s^0	self-accommodating martensite start temperature; throughout the book it is accepted that it is the same as oriented martensite start temperature $\left(M_\mathrm{s}^0 = M_\mathrm{s}^{1-2}\right)$
M_f^0	martensite finish temperature
M_σ	temperature above which first plastic slip deformation is stress-induced and only later martensitic transformation takes place
$\mathbf{M}$	tensor of elastic compliance properties $\left(\mathbf{M} = \mathbf{L}^{-1}\right)$
μ	modulus of shear elasticity $\mu = E/2(1+v)$
$\boldsymbol{N}$	nominal stress tensor
$\boldsymbol{n}_i^\mathrm{e}$	Eulerian (spatial) principal directions of elastic stretch tensors $\boldsymbol{V}^\mathrm{e}$
$\boldsymbol{N}_i^\mathrm{e}$	Lagrangean (material) principal directions of elastic stretch tensors $\boldsymbol{U}^\mathrm{e}$
v	Poisson coefficient
π^{1-2}	thermodynamic driving force of austenite-oriented martensite phase transformation $(\mathcal{A} \rightarrow \mathcal{M}_\mathrm{or})$
π^{1-3}	thermodynamic driving force of austenite-self-accommodating martensite phase transformation $(\mathcal{A} \rightarrow \mathcal{M}_\mathrm{sa})$
π_u^{1-2}	thermodynamic driving force at zero macroscopic stress
π_0^{1-2}	chemical driving force (t.d.f.) of phase transformation $(\mathcal{A} \rightarrow \mathcal{M}_\mathrm{or})$
$\overset{(*)}{\boldsymbol{\Pi}}$	macroscopic nominal stress rate $\overset{(*)}{\boldsymbol{\Pi}} \equiv J^{-1}\boldsymbol{F}\dot{\boldsymbol{N}} = \dot{\boldsymbol{\sigma}} - \boldsymbol{L}\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathrm{tr}(\boldsymbol{d})$
q	specific energy exchanged with environment in the form of heat (q has positive sign, when energy is removed from the macroelement)
$\boldsymbol{R}$	rotation tensor from polar decomposition of deformation gradient
$\boldsymbol{Q}$	proper orthogonal rotation tensor $\left(\boldsymbol{Q}\boldsymbol{Q}^\mathrm{T} = \mathbf{I}, \det(\boldsymbol{Q}) = 1\right)$
ρ	mass density; throughout the book it is accepted that density of austenitic and martensitic phase is the same $\rho_1 = \rho_2 = \rho$
ρ_0	mass density in initial configuration
s	macroscopic specific entropy
s_1^0	entropy of austenite formation at microstress free state (m.r.s.), $\left(\breve{\boldsymbol{\sigma}}(\boldsymbol{x}) = \mathbf{0}, T = T_0\right)$
s_α^0	entropy of martensitic compound formation at microstress free state; $s_2^0 = s_3^0$
s_{12}	entropy material constant
$\Delta s^0 \equiv s_\alpha^0 - s_1^0$	entropy of formation of any object of martensite at microstress free state (m.r.s.), $(\alpha = 2, 3)$
$\boldsymbol{\sigma}^\mathrm{AM}$	critical stress of start of austenite to oriented martensite p.t.
$\boldsymbol{\sigma}^\mathrm{MA}$	critical stress of start of reverese p.t. oriented martensite to austenite
$\boldsymbol{\sigma}$	macroscopic Cauchy stress
$\breve{\boldsymbol{\sigma}}(\boldsymbol{x})$	local mesoscopic Cauchy stress field
$\overset{\circ}{\bar{\boldsymbol{\sigma}}}(0)$	Zaremba-Jaumann rate of Kirchoff stress referred to actual configuration $\left(\overset{\circ}{\bar{\boldsymbol{\sigma}}}^{(0)} = J^{-1}\bar{\boldsymbol{\tau}}\mathrm{o}^{(\mathrm{J})} = \dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\omega} - \boldsymbol{\omega}\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathrm{tr}(\boldsymbol{d})\right)$
σ_ef	effective stress of Cauchy stress tensor $\left(\sigma_\mathrm{ef} \equiv \sqrt{3J_2} = \left[(3/2)\bar{\sigma}_{ij}\bar{\sigma}_{ij}\right]^{1/2}\right)$
$\boldsymbol{t}^{(b)}(\boldsymbol{x})$	boundary surface tractions
T	temperature
T_0	conventional reference state temperature
$T_\mathrm{or}^\mathrm{eq}$	equilibrium temperature of austenite and oriented martensite $\left(T_\mathrm{or}^\mathrm{eq} \equiv \Delta u_2^0/\Delta s_2^0\right)$
$\boldsymbol{T}(n)$	stress measure work conjugate to Lagrangean total strain measure $\boldsymbol{E}(n)$
$\boldsymbol{T}^\mathrm{e}(1)$	elastic II Piola-Kirchoff stress $\left(\boldsymbol{T}^\mathrm{e}(1) \equiv {}^{-1}\boldsymbol{F}^e\boldsymbol{\tau}{}^{-T}\boldsymbol{F}^e\right)$
$\boldsymbol{\tau}$	Kirchoff stress $(\boldsymbol{\tau} \equiv J\boldsymbol{\sigma})$
$\overset{\circ}{\bar{\boldsymbol{\tau}}}^{(\mathrm{J})}$	Zaremba-Jaumann corotational derivative of Kirchoff stress tensor $\left(\overset{\circ}{\bar{\boldsymbol{\tau}}}^{(\mathrm{J})} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau}\boldsymbol{\omega} - \boldsymbol{\omega}\boldsymbol{\tau} = (\rho^0/\rho)\left[\dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\omega} - \boldsymbol{\omega}\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathrm{tr}(\boldsymbol{d})\right]\right)$

$\overset{\circ}{\boldsymbol{\tau}}{}^{(\mathrm{Je})}$	elastic Zaremba-Jaumann corotational derivative of Kirchoff stress tensor $\overset{\circ}{\boldsymbol{\tau}}{}^{(\mathrm{Je})} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau}\boldsymbol{\omega}^e - \boldsymbol{\omega}^e\boldsymbol{\tau}$
u	specific internal energy of macroelement
u_1^0	internal energy of austenite at mesostress free reference state (m.r.s.), $(\breve{\boldsymbol{\sigma}}(\boldsymbol{x}) = 0, T = T_0 \Rightarrow \boldsymbol{\sigma} = \mathbf{0},\ \breve{\boldsymbol{\sigma}}^f(\boldsymbol{x}) = \mathbf{0})$
u_α^0	internal energy of martensite at mesostress free reference state $(\alpha = 2, 3)$; it is adopted in this book that $u_2^0 = u_3^0$
$u_2^0 - \phi_2^0$	internal energy of oriented martensite at t.r.s. $(\boldsymbol{\sigma} = \mathbf{0}, T = T_0)$
$u_3^0 - \phi_3^0$	internal energy of self-accommodated martensite at t.r.s.
$\Delta u^0 \equiv u_\alpha^0 - u_1^0$	internal energy of formation of any object of martensite at microstress free state (m.r.s.), $(\breve{\boldsymbol{\sigma}}(\boldsymbol{x}) = \mathbf{0}, T = T_0)$ $(\alpha = 2, 3)$
$\breve{\boldsymbol{u}}(\boldsymbol{x})$	local displacement field
$\boldsymbol{U}^e$	right elastic stretch tensor
$\boldsymbol{V}^e$	left elastic stretch tensor
V_α	volume fraction of phase (α) in RVE
w	specific external mechanical work (done over macroelement) (J/kg)
w_{Disp}	specific mechanical work dissipation
w_{r}	specific mechanical work done over macroelement in equivalent thermodynamically reversible process
$\boldsymbol{\omega}$	material spin (vorticity) tensor $\left(\boldsymbol{\omega}^Q = \dot{\boldsymbol{Q}}\boldsymbol{Q}^{\mathrm{T}}\right)$
$\boldsymbol{\omega}^{\log}$	logarithmic spin tensor
$\boldsymbol{x}$	coordinates of material point in Eulerian description
$\boldsymbol{X}$	coordinates of material point in Lagrangean description
$Y_{(\alpha)}$	lag threshold value functions of delay in phase transitions start
y	Lode parameter characterizing direction of stress tensor direction $y = \cos(3\theta) = 27J_3/\left(2\sigma_{\mathrm{ef}}^3\right) = 3\sqrt{3}J_3/\left(2J_2^{3/2}\right)$
z_1	mass (volume) fraction of austenitic phase
z_2 or z	mass (volume) fraction of oriented martensite phase
z_3	mass (volume) fraction of self-accommodated martensite phase. (Note: In the pseudoelastic range of SMA behavior no self-accommodating martensite *can ever form*, for this reason throughout this book it is $z_3 = 0$, $z_1 + z_2 \equiv 1$, and subscript "2" is usually omitted (i.e., $z_2 = z$).)