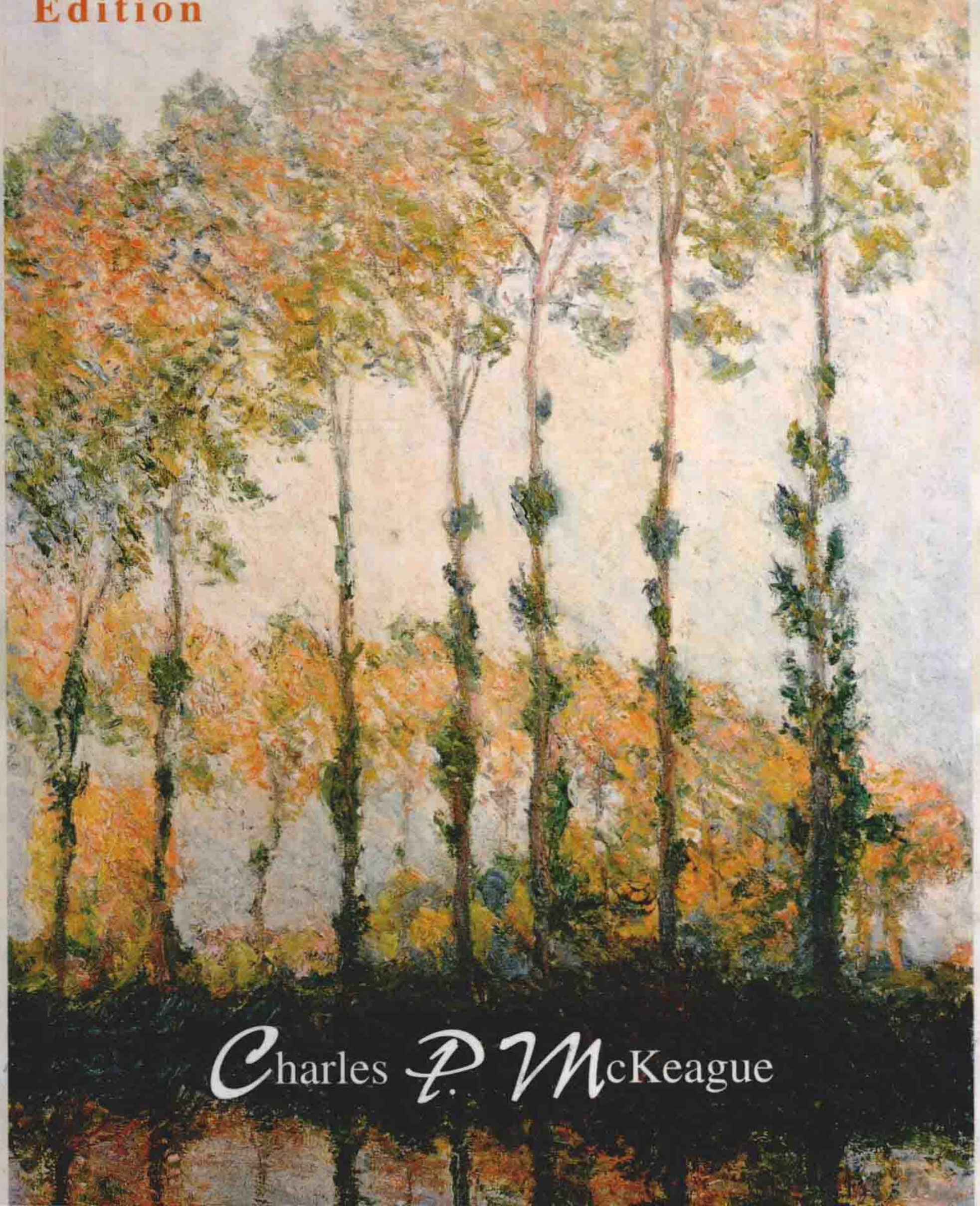
Algebra

for College Students

Third Edition



Charles P. McKeague

THIRD EDITION

Algebra for College Students

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PREFACE TO THE STUDENT

Many of my algebra students are apprehensive at first because they are worried that they will not understand the topics we cover. When I present a new topic that they do not grasp completely, they think something is wrong with them for not understanding it.

On the other hand, some students are excited about the course from the beginning. They are not worried about understanding algebra and, in fact, *expect* to find some topics difficult.

What is the difference between these two types of students?

Those who are excited about the course know from experience (as you do) that a certain amount of confusion is associated with most new topics in mathematics. They don't worry about it because they also know that the confusion gives way to understanding in the process of reading the textbook, working the problems, and getting questions answered. If they find a topic they are having difficulty with, they work as many problems as necessary to grasp the subject. They don't wait for the understanding to come to them; they go out and get it by working lots of problems. In contrast, the students who lack confidence tend to give up when they become confused. Instead of working more problems, they sometimes stop working problems altogether—and that, of course, guarantees that they will remain confused.

If you are worried about this course because you lack confidence in your ability to understand algebra, and you want to change the way you feel about mathematics, then look forward to the first topic that causes you some confusion. As soon as that topic comes along, make it your goal to master it, in spite of your apprehension. You will see that each and every topic covered in this course is one you can eventually master, even if your initial introduction to it is accompanied by some confusion. As long as you have passed a college-level beginning algebra course (or its equivalent), you are ready to take this course.

If you have decided to do well in algebra, the following list will be important to you:

How To Be Successful in Algebra

1. **Attend all class sessions on time** You cannot know exactly what goes on in class unless you are there. Missing class and then expecting to find out what went on from someone else is not the same as being there yourself.

2. **Read the book** It is best to read the section that will be covered in class beforehand. Reading in advance, even if you do not understand everything you read, is still better than going to class with no idea of what will be discussed.
3. **Work problems every day and check your answers** The key to success in mathematics is working problems. The more problems you work, the better you will become at working them. The answers to the odd-numbered problems are given in the back of the book. When you have finished an assignment, be sure to compare your answers with those in the book. If you have made a mistake, find out what it is, and correct it.
4. **Do it on your own** Don't be misled into thinking someone else's work is your own. Having someone else show you how to work a problem is not the same as working the same problem yourself. It is okay to get help when you are stuck. As a matter of fact, it is a good idea. Just be sure you do the work yourself.
5. **Review every day** After you have finished the problems your instructor has assigned, take another fifteen minutes and review a section you have already completed. The more you review, the longer you will retain the material you have learned.
6. **Don't expect to understand every new topic the first time you see it** Sometimes you will understand everything you are doing, and sometimes you won't. That's just the way things are in mathematics. Expecting to understand each new topic the first time you see it can lead to disappointment and frustration. The process of understanding algebra takes time. It requires that you read the book, work problems, and get your questions answered.
7. **Spend as much time as it takes for you to master the material** No set formula exists for the exact amount of time you need to spend on algebra to master it. You will find out as you go along what is or isn't enough time for you. If you end up spending two or more hours on each section in order to master the material there, then that's how much time it takes; trying to get by with less will not work.
8. **Relax** It's probably not as difficult as you think.

CONTENTS

4 Rational Expressions 215

5 Rational Exponents and Roots 283

6 Quadratic Equations 341

7 Systems of Equations 405

C H A P T E R 1

Basic Definitions and Properties

Contents

Introduction

If you major in business or economics, you will see equations and charts like the ones below many times during your college career:

$$P = R - C$$

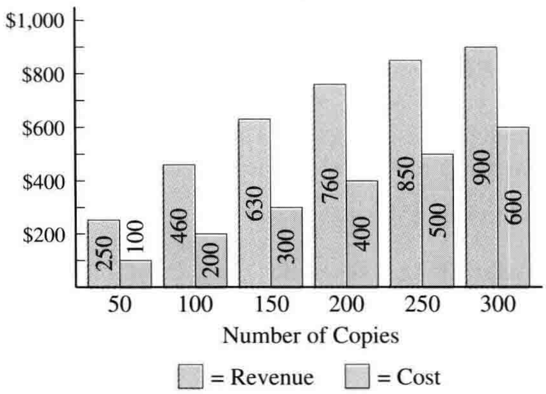
where

P = Profit; the difference between revenue and cost

R = Revenue; the amount of money a company brings in by selling its product

C = Cost; the amount of money a company pays out to produce its product

Revenue and Cost to Duplicate a 30 Minute Video



The formula for profit tells us that we can find the profits in the chart by finding the difference between the heights of each pair of connected columns.

The material we cover in this chapter will help you understand equations and charts like the ones above, and get you started on your way to working with some of the more detailed applications in business and economics.

Overview

The material in Chapter 1 is some of the most important material in the book. Be sure that you master it. Your success in the following chapters is directly related to how well you understand this chapter.

Here is a list of the essential concepts from Chapter 1 that you will need to be successful in the succeeding chapters:

1. You must know how to add, subtract, multiply, and divide both positive and negative numbers.
2. You must understand and recognize the commutative, associative, and distributive properties—the three most important properties of real numbers.
3. You must know the difference between whole numbers, integers, rational numbers, and real numbers.
4. You must be able to use the properties of exponents to simplify expressions involving exponents.
5. You must have a working knowledge of scientific notation.
6. You must be able to add, subtract, multiply, and factor polynomials.

Study Skills

At the beginning of each of the first few chapters of this book you will find a section like this in which we list the skills that are necessary for success in algebra. If you have just completed an introductory algebra class successfully, you have acquired most of these skills. If it has been some time since you have taken a math class, you must pay attention to the sections on study skills.

Here is a list of things you can do to begin to develop effective study skills.

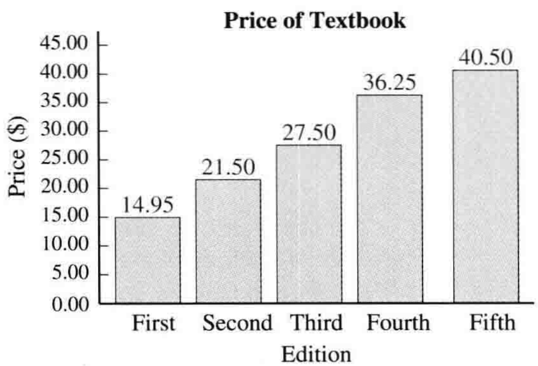
1. **Put yourself on a schedule** The general rule is that you spend 2 hours on homework for every hour you are in class. Make a schedule for yourself, setting aside 2 hours each day to work on algebra. Once you make the schedule, stick to it. Don't just complete your assignments and stop. Use all the time you have set aside. If you complete an assignment and have time left over, read the next section in the book, and work more problems. As the course progresses you may find that 2 hours a day is not enough time for you to master the material in this course. If it takes you longer to reach your goals for this course, then that's how much time you need to spend. Trying to get by with less will not work.
2. **Find your mistakes and correct them** There is more to studying algebra than just working problems. You must always check your answers with the answers in the back of the book. When you have made a mistake, find out what it is, and correct it. Making mistakes is part of the process of learning mathematics. The key to discovering what you do not understand can be found by correcting your mistakes.

3. **Imitate success** Your work should look like the work you see in this book and the work your instructor shows. The steps shown in solving problems in this book were written by someone who has been successful in mathematics. The same is true of your instructor. Your work should imitate the work of people who have been successful in mathematics.
4. **Don't let your intuition fool you** As you become more experienced and more successful in mathematics you will be able to trust your mathematical intuition. For now, though, it can get in the way of your success. For example, if students are asked to "subtract 3 from -5," many will answer -2 or 2. Both answers are incorrect, even though they may seem intuitively true. Likewise, some students will expand $(a + b)^2$ and arrive at $a^2 + b^2$, which is incorrect. In both cases, intuition leads directly to the wrong answer.

SECTION 1.1

Fundamental Definitions and Notation

The diagram below is called a *bar chart*. This one shows the net price of a popular intermediate algebra textbook. (The net price is the price the bookstore pays for the book.)



From the chart, we can find many relationships between numbers. We may notice that the price of the third edition was less than the price of the fourth edition. In mathematics we use symbols to represent relationships between quantities. If we let P represent the price of the book, then the relationship just mentioned, between the price of the third edition and the price of the fourth edition, can be written this way:

$$P(3) < P(4)$$

You will be on your way to understanding expressions like the one above by studying the material in this section.

This section is, for the most part, simply a list of many of the basic symbols and definitions we will be using throughout the book.

Comparison Symbols

IN SYMBOLS	IN WORDS
$a = b$	*a* is equal to *b*
$a \neq b$	*a* is not equal to *b*
$a < b$	*a* is less than *b*
$a \leq b$	*a* is less than or equal to *b*
$a \nless b$	*a* is not less than *b*
$a > b$	*a* is greater than *b*
$a \geq b$	*a* is greater than or equal to *b*
$a \ngtr b$	*a* is not greater than *b*
$a \Leftrightarrow b$	*a* is equivalent to *b*

Operation Symbols

OPERATION	IN SYMBOLS	IN WORDS
Addition	$a + b$	The sum of *a* and *b*
Subtraction	$a - b$	The difference of *a* and *b*
Multiplication	ab, $a \cdot b$, $a(b)$, $(a)b$, or $(a)(b)$	The product of *a* and *b*
Division	$a \div b$, a/b, or $\frac{a}{b}$	The quotient of *a* and *b*

The key words are **sum, difference, product,** and **quotient.** They are used frequently in mathematics. For instance, we may say "the product of 3 and 4 is 12." We mean that both the statements "$3 \cdot 4$" and "12" are called the product of 3 and 4. The important idea here is that the word *product* implies multiplication, regardless of whether it is written $3 \cdot 4$, 12, 3(4), or (3)4.

The following example shows how we translate sentences written in English into expressions written in symbols.

EXAMPLE 1

IN ENGLISH	IN SYMBOLS
The sum of *x* and 5 is less than 2.	$x + 5 < 2$
The product of 3 and *x* is 21.	$3x = 21$
The quotient of *y* and 6 is 4.	$\frac{y}{6} = 4$
Twice the difference of *b* and 7 is greater than 5.	$2(b - 7) > 5$
The difference of twice *b* and 7 is greater than 5.	$2b - 7 > 5$

Exponents

Consider the expression 3^4. The 3 is called the **base** and the 4 is called the **exponent.** The exponent 4 tells us the number of times the base appears in the product. That is,

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

The expression 3^4 is said to be in *exponential form,* while $3 \cdot 3 \cdot 3 \cdot 3$ is said to be in *expanded form.*

EXAMPLES Expand and multiply.

2. $5^2 = 5 \cdot 5 = 25$ Base 5, exponent 2

3. $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ Base 2, exponent 5

4. $4^3 = 4 \cdot 4 \cdot 4 = 64$ Base 4, exponent 3

Order of Operations

It is important when evaluating arithmetic expressions in mathematics that each expression have only one answer in reduced form. Consider the expression

$$3 \cdot 7 + 2$$

If we find the product of 3 and 7 first, then add 2, the answer is 23. On the other hand, if we first combine the 7 and 2, then multiply by 3, we have 27. The problem seems to have two distinct answers depending on whether we multiply first or add first. To avoid this situation, we follow the rule that multiplication in a situation like this will always be done before addition. In this case, only the first answer, 23, is correct.

Here is the complete set of rules for evaluating expressions.

RULE: ORDER OF OPERATIONS

When evaluating a mathematical expression, we will perform the operations in the following order, beginning with the expression in the innermost parentheses or brackets and working our way out.

Step 1 Simplify all numbers with exponents, working from left to right if more than one of these expressions is present.
Step 2 Then, do all multiplications and divisions left to right.
Step 3 Perform all additions and subtractions left to right.

Here are some examples that illustrate the use of this rule.

EXAMPLES Simplify each expression using the rule for order of operations.

5. $5 + 3(2 + 4) = 5 + 3(6)$ Simplify inside parentheses.
$= 5 + 18$ Then, multiply.
$= 23$ Add.

6. $5 \cdot 2^3 - 4 \cdot 3^2 = 5 \cdot 8 - 4 \cdot 9$ Simplify exponentials left to right.
$= 40 - 36$ Multiply left to right.
$= 4$ Subtract.

7. $20 - (2 \cdot 5^2 - 30) = 20 - (2 \cdot 25 - 30)$
$= 20 - (50 - 30)$
$= 20 - (20)$
$= 0$

Simplify inside parentheses, evaluating exponents first, then multiplying, and finally subtracting.

Sets

DEFINITION

A **set** is a collection of objects or things. The objects in the set are called **elements** or **members** of the set.

Sets are usually denoted by capital letters and elements of sets by lowercase letters. We use braces, { }, to enclose the elements of a set.

To show that an element is contained in a set we use the symbol $\in$. That is,

$x \in A$ is read "x is an element (member) of set A"

For example, if A is the set $\{1, 2, 3\}$, then $2 \in A$. On the other hand, $5 \notin A$, means 5 is not an element of set A.

DEFINITION

Set A is a **subset** of set B, written $A \subset B$, if every element in A is also an element of B. That is,

$A \subset B$ if and only if A is contained in B

EXAMPLES

8. The set of numbers used to count things is $\{1, 2, 3, \ldots\}$. The dots mean the set continues indefinitely in the same manner. This is an example of an **infinite** set.

9. The set of all numbers represented by the dots on the faces of a regular die is $\{1, 2, 3, 4, 5, 6\}$. This set is a subset of the set in Example 8. It is an example of a **finite** set, since it has a limited number of elements.

DEFINITION

The set with no members is called the **empty** or **null set.** It is denoted by the symbol $\varnothing$. The empty set is considered a subset of every set.

Operations with Sets

These diagrams are called *Venn diagrams* after John Venn (1834–1923). They can be used to visualize operations with sets. The region inside the circle labeled A is set A, while the region inside the circle labeled B is set B. The shaded region in Figure 1 is the union of A and B. The shaded region in Figure 2 is the intersection of A and B.

Two basic operations are used to combine sets: union and intersection.

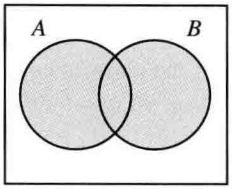


FIGURE 1
The *union* of two sets

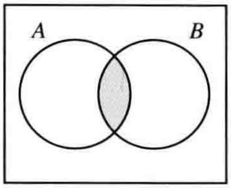


FIGURE 2
The *intersection* of two sets

DEFINITION

The **union** of two sets A and B, written $A \cup B$, is the set of all elements that are either in A or in B, or in both A and B. The key word here is *or*. For an element to be in $A \cup B$ it must be in A or B. In symbols, the definition looks like this:

$$x \in A \cup B \quad \text{if and only if} \quad x \in A \text{ or } x \in B$$

DEFINITION

The **intersection** of two sets A and B, written $A \cap B$, is the set of elements in both A and B. The key word in this definition is the word *and*. For an element to be in $A \cap B$ it must be in both A and B. In symbols,

$$x \in A \cap B \quad \text{if and only if} \quad x \in A \text{ and } x \in B$$

EXAMPLES Let $A = \{1, 3, 5\}$, $B = \{0, 2, 4\}$, and $C = \{1, 2, 3, \ldots\}$. Then:

10. $A \cup B = \{0, 1, 2, 3, 4, 5\}$

11. $A \cap B = \varnothing$ A and B have no elements in common.

12. $A \cap C = \{1, 3, 5\} = A$

13. $B \cup C = \{0, 1, 2, 3, \ldots\}$

Up to this point we have described the sets encountered by listing all the elements and then enclosing them with braces { }. There is another notation we can use to describe sets. It is called **set-builder notation.** Here is how we would write our definition for the union of two sets A and B using set-builder notation:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The right side of this statement is read "the set of all x such that x is a member of A or x is a member of B." As you can see, the vertical line after the first x is read "such that."

EXAMPLE 14 If $A = \{1, 2, 3, 4, 5, 6\}$, find $C = \{x | x \in A \text{ and } x \geq 4\}$.

SOLUTION We are looking for all the elements of A that are also greater than or equal to 4. They are 4, 5, and 6. Using set notation, we have

$$C = \{4, 5, 6\}$$

The set we will work with most often in this book is the set of real numbers. To develop the real numbers, we start with the real number line.

The Real Numbers

The **real number line** is constructed by drawing a straight line and labeling a convenient point with the number 0. Positive numbers are in increasing order to the right of 0; negative numbers are in decreasing order to the left of 0. The point on the line corresponding to 0 is called the **origin.**

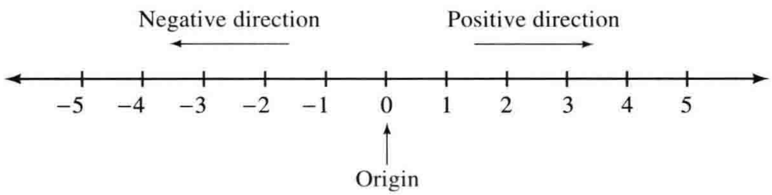


FIGURE 3
Constructing a number line

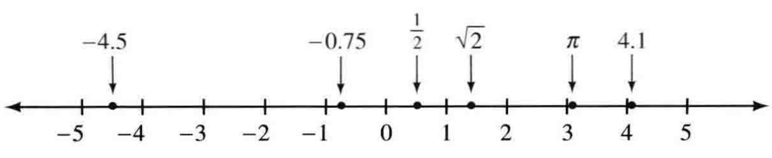
The numbers on the number line increase in size as we move to the right. When we compare the size of two numbers on the number line, the number on the left is always the smaller number.

The numbers associated with the points on the line are called **coordinates** of those points. Every point on the line has a number associated with it. The set of all these numbers makes up the set of real numbers.

DEFINITION

A **real number** is any number that is the coordinate of a point on the real number line.

EXAMPLE 15 Locate the numbers -4.5, -0.75, $\frac{1}{2}$, $\sqrt{2}$, π, and 4.1 on the real number line.



Note: In this book we will refer to real numbers as being on the real number line. Actually, real numbers are *not* on the line; only the points they represent are on the line. We can save some writing, however, if we simply refer to real numbers as being on the number line.

Subsets of the Real Numbers

Next, we consider some of the more important subsets of the real numbers. Each set listed here is a subset of the real numbers:

Counting (or **Natural**) **numbers** $= \{1, 2, 3, \ldots\}$

Whole numbers $= \{0, 1, 2, 3, \ldots\}$

Integers $= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational numbers $= \left\{\frac{a}{b} \,\middle|\, a \text{ and } b \text{ are integers, } b \neq 0\right\}$

Any number that can be written in the form

$$\frac{\text{Integer}}{\text{Integer}}$$

is a rational number. Rational numbers are numbers that can be written as the ratio of two integers. Each of the following is a rational number:

$\frac{3}{4}$	Because it is the ratio of the integers 3 and 4
-8	Because it can be written as the ratio of -8 to 1: $\frac{-8}{1}$
0.75	Because it is the ratio of 75 to 100 (or 3 to 4 if you reduce to lowest terms)
0.333. . .	Because it can be written as the ratio of 1 to 3

There are still other numbers on the number line that are not members of the subsets we have listed so far. They are real numbers, but they cannot be written as the ratio of two integers. That is, they are not rational numbers. For that reason, we call them irrational numbers.

Irrational numbers $= \{x \mid x \text{ is real, but not rational}\}$

The following are irrational numbers:

$$\sqrt{2} \qquad -\sqrt{3} \qquad 4 + 2\sqrt{3} \qquad \pi \qquad \pi + 5\sqrt{6}$$

EXAMPLE 16 For the set $\{-5, -3.5, 0, \frac{3}{4}, \sqrt{3}, \sqrt{5}, 9\}$, list the numbers that are:
(a) Whole numbers (b) Integers (c) Rational numbers
(d) Irrational numbers (e) Real numbers

SOLUTION

(a) Whole numbers $= \{0, 9\}$
(b) Integers $= \{-5, 0, 9\}$ (c) Rational numbers $= \{-5, -3.5, 0, \frac{3}{4}, 9\}$
(d) Irrational numbers $= \{\sqrt{3}, \sqrt{5}\}$ (e) They are all real numbers.