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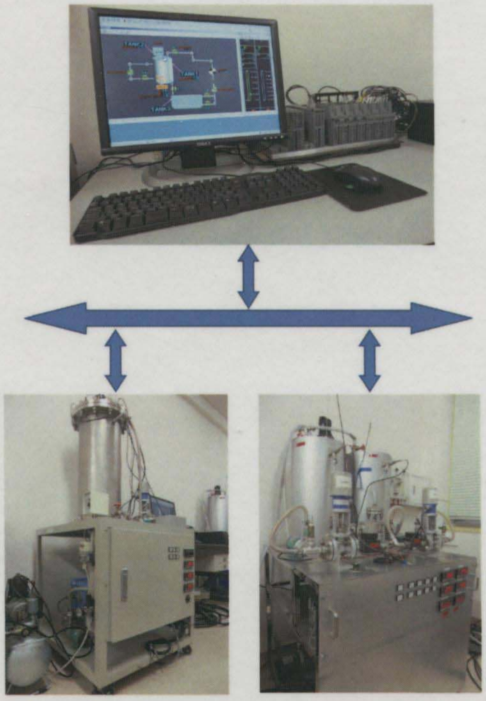
Operator-Based Nonlinear Control Systems

Design and Applications



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OPERATOR-BASED NONLINEAR CONTROL SYSTEMS

CONTENTS

CHAPTER 1

Introduction

1.1 DEFINITION OF NONLINEAR SYSTEMS

In general, a nonlinear system is one that does not satisfy the superposition principle or whose output is not directly proportional to its input. That is, a nonlinear system is any problem where the variable(s) to be solved for cannot be written as a linear combination of independent components. Since most economic, social, and many industrial systems are inherently nonlinear in nature, where mathematical analysis is unable to provide general solutions, nonlinear system problems, especially nonlinear systems dynamics analysis and control problems for industrial systems, are of interest to mathematicians, physicists, and engineers.

1.2 NONLINEAR SYSTEM DYNAMICS ANALYSIS AND CONTROL

Nonlinear systems control is the discipline that applies control theory to design systems with desired behaviors. It can be broadly defined or classified as nonlinear control theory and application. It seeks to understand nonlinear systems dynamics, using mathematical modeling, in terms of inputs, outputs, and various components with different behaviors and to use nonlinear control systems design schemes to develop controllers for those systems in one or many time, frequency, and complex domains, depending on the nature of the design problem. As a result, control of nonlinear systems is a multidisciplinary research field involving the synergistic integration of mechanical and electrical engineering, computer science, and even biological engineering. Control of nonlinear systems will become mainstream consumer products within the next decade, providing a significant growth opportunity for the above-mentioned engineering systems. So far, there are several significant techniques for analyzing nonlinear systems, for example, describing the function method, the phase plane method, Lyapunov-based analysis, the singular perturbation method, the Popov criterion, the center manifold theorem, the small-gain theorem, and passivity analysis. Based on the above techniques, significant results were introduced extending to nonlinear feedback systems design and control. Some cornerstone control

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methods, for example, Lyapunov function method, sliding-mode control method, and nonlinear damping method, are proposed. In view of the input–output nature of the nonlinear system concept itself, it seems useful to establish computer-oriented approaches to nonlinear control systems analysis and design. Addressing this problem, the robust right coprime factorization technique of nonlinear operators, in addition to the above significant techniques, which is based on real and complex variable theory, has been a promising technique, where the operators can be either linear or nonlinear, continuous time or discrete time, finite dimensional or infinite dimensional, and in the frequency domain or time domain [1].

1.3 WHY OPERATOR-BASED NONLINEAR CONTROL SYSTEM?

As a basis for the possible next generation of control of nonlinear systems, the theoretical concept of operator-based nonlinear control has been introduced in recent years. In the operator-based nonlinear control system research approach, since the 1990s, some researchers started with the operator-theoretic nonlinear control approach, and mathematical background was provided. As for the development of the design principle, it is forecasted that the operator-theoretic nonlinear control approach will be applied significantly. As a result, research on operator-based nonlinear system control has great potential to the application for industry and daily life. However, the nonlinear control system analysis design might be difficult and impossible because of the complex uncertain nonlinearities. There was lack of a quantitative stability result, which may guarantee stability and performance of the control system with the uncertain nonlinearities. Addressing the above problem, this book aims to develop a systematic methodology using operator-based design of nonlinear control systems.

1.4 OVERVIEW OF THE BOOK

This book concerns uncertain nonlinearity in this important research field. Starting with major goals and reviews, the book gives a perspective as to how plants can be modeled as operator-based plants. The primary objectives of this book are to guide modeled plants to obtain robust right coprime factorization, provide state-of-the art research on robust stability conditions, and discuss system output tracking and fault detection issues for researchers working in this field. Considering the broad set of the readers whom I would like to reach, I some applications are included for a good understanding. The intent is to help beginning graduate students learn several developments of operator-based nonlinear control system design. This book also summarizes our understanding of the current trend and the likely future of the operator-theoretic approach reported in latest research results on several frontier problems. Motivated by the above consideration, a detailed analysis of nonlinear feedback control systems based on an operator-theoretic approach is considered in this book. Based on the operator theory, nonlinear feedback control systems can be designed and applied, that is, operator-based nonlinear feedback control using robust right coprime factorization [1–2, 8]. For instance, application of the proposed designs to networked control processes is considered and vibration control using piezoelectric actuators, ionic polymer metal composite actuators, and shape memory alloy actuators has been successfully conducted. Meanwhile, a fault detection technique based on an operator-theoretic approach is also developed. In describing these aspects of the operator-based nonlinear control system, it is assumed that the reader is familiar with Banach spaces, linear operator theory, and right coprime factorization and has some elementary knowledge of nonlinear control, found in the excellent text by de Figueiredo and Chen [1]. Some of the work described in this book is based upon a series of recent publications by the author.

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CHAPTER 2

Robust Right Coprime Factorization for Nonlinear Plants with Uncertainties

2.1 PRELIMINARIES

This chapter gives some basic definitions and notation needed throughout this book and some important remarks necessary in describing the problems to be investigated later.

2.1.1 Definition of Spaces

In mathematics, a space is a set with some added structures. There are two basic spaces: linear spaces (also called vector spaces) and topological spaces, where linear spaces are of algebraic nature and topological spaces are of analytic nature. There are three types of linear spaces; real linear spaces (over the field of real numbers), complex linear spaces (over the field of complex numbers), and more generally linear spaces over any field. The discussion in this book is based on linear spaces.

2.1.1.1 Normed Linear Space A space X of time functions is said to be a vector space if it is closed under addition and scalar multiplication. The space X is said to be *normed* if each element x in X is endowed with norm $\|\cdot\|_X$, which can be defined in any way so long as the following three properties are fulfilled:

1. $\|x\|$ is a real, positive number and is different from zero unless x is identically zero,
2. $\|ax\| = |a|\|x\|$, and
3. $\|x_1 + x_2\| \leq \|x_1\| + \|x_2\|$.

It should be mentioned that every normed space is a linear topological space.

2.1.1.2 Banach Space A Banach space is defined as a complete normed space. This means that a Banach space is a vector space X over the real or complex numbers

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with a norm $\|\cdot\|$ such that every Cauchy sequence (with respect to the metric $d(x, y) = \|x - y\|$) in X has a limit in X. Many spaces of sequences or functions are infinite-dimensional Banach spaces.

2.1.1.3 Extended Linear Space Let Z be the family of real-valued measurable functions defined on $[0, \infty)$, which is a linear space. For each constant $T \in [0, \infty)$, let P_T be the projection operator mapping from Z to another linear space, Z_T, of measurable functions such that

$$f_T(t) := P_T(f)(t) = \begin{cases} f(t) & t \leq T \\ 0 & t > T \end{cases} \tag{2.1}$$

where $f_T(t) \in Z_T$ is called the truncation of $f(t)$ with respect to T. Then, for any given Banach space X of measurable functions, set

$$X^e = \{f \in Z : \|f_T\|_X < \infty \text{ for all } T < \infty\} \tag{2.2}$$

Obviously, X^e is a linear subspace of Z. The space so defined is called the extended linear space associated with the Banach space X.

It should be noted that the extended linear space is not complete in the norm in general and hence not a Banach space, but it is determined by a relative Banach space. The reason to use the extended linear space is that all the control signals have finite time duration in practice, and many useful techniques and results can be carried over from the standard Banach space X to the extended space X^e if the norm is suitably defined.

2.1.2 Definition of Operators

Let X and Y be linear spaces over the field of real numbers, and let X_s and Y_s be normed linear subspaces, called the stable subspaces of X and Y, respectively, defined suitably by two normed linear spaces under a certain norm $X_s = \{x \in X : \|x\| < \infty\}$ and $Y_s = \{y \in Y : \|y\| < \infty\}$.

2.1.2.1 Linear and Nonlinear Operator Let $Q : X \to Y$ be an operator mapping from X to Y, and denote by $\mathcal{D}(Q)$ and $\mathcal{R}(Q)$, respectively, the domain and range of Q. If the operator $Q : \mathcal{D}(Q) \to Y$ satisfies the addition rule and multiplication rule

$$Q : ax_1 + bx_2 \to aQ(x_1) + bQ(x_2)$$

for all $x_1, x_2 \in \mathcal{D}(Q)$ and all $a, b \in \mathcal{C}$, then Q is said to be linear. Otherwise, it is said to be nonlinear. Since linearity is a special case of nonlinearity, in what follows "nonlinear" will always mean "not necessarily linear" unless otherwise indicated.

2.1.2.2 Bounded Input–Bounded Output (BIBO) Stability Let Q be a nonlinear operator with its domain $\mathcal{D}(Q) \subseteq X^e$ and range $\mathcal{R}(Q) \subseteq Y^e$. If $Q(X) \subseteq Y$, Q is said to be input–output stable. If Q maps all input functions from X_s into the output space Y_s, that is, $Q(X_s) \subseteq Y_s$, then operator Q is said to be BIBO stable or simply stable. Otherwise, if Q maps some inputs from X_s to the set $Y^e \backslash Y_s$ (if not empty), then Q is said to be unstable. Any stable operators defined here and later in this book are BIBO stable.

2.1.2.3 Invertible An operator Q is said to be invertible if there exists an operator P such that

$$Q \circ P = P \circ Q = I \tag{2.3}$$

where P is the inverse of Q and is denoted by Q^{-1}, I is the identity operator, and $Q \circ P$ [or simply $Q(P(\cdot))$ or QP] is an operation satisfying

$$\mathcal{D}(Q \circ P) = P^{-1}(\mathcal{R}(P) \cap \mathcal{D}(Q)) \tag{2.4}$$

2.1.2.4 Unimodular Operator Let $\mathcal{S}(X, Y)$ be the set of stable operators mapping from X to Y. Then, $\mathcal{S}(X, Y)$ contains a subset defined by

$$\mathcal{U}(X, Y) = \{M : M \in \mathcal{S}(X, Y),\ M \text{ is invertible with } M^{-1} \in \mathcal{S}(Y, X)\} \tag{2.5}$$

Elements of $\mathcal{U}(X, Y)$ are called unimodular operators.

2.1.2.5 Lipschitz Operator For any subset $D \subseteq X$, let $\mathcal{F}(D, Y)$ be the family of nonlinear operators Q such that $\mathcal{D}(Q) = D$ and $\mathcal{R}(Q) \subseteq Y$. Introduce a (semi)-norm into (a subset of) $\mathcal{F}(D, Y)$ by

$$\|Q\| := \sup_{\substack{x,\tilde{x} \in D \\ x \neq \tilde{x}}} \frac{\|Q(x) - Q(\tilde{x})\|_Y}{\|x - \tilde{x}\|_X}$$

if it is finite. In general, it is a seminorm in the sense that $\|Q\| = 0$ does not necessarily imply $Q = 0$. In fact, it can be easily seen that $\|Q\| = 0$ if Q is a constant operator (need not to be zero) that maps all elements from D to the same element in Y.

Let Lip(D, Y) be the subset of $\mathcal{F}(D, Y)$ with its all elements Q satisfying $\|Q\| < \infty$. Each $Q \in \text{Lip}(D, Y)$ is called a Lipschitz operator mapping from D to Y, and the number $\|Q\|$ is called the Lipschitz seminorm of the operator Q on D.

It is evident that a Lipschitz operator is both bounded and continuous on its domain. Next, a generalized Lipschitz operator is introduced which is defined on an extended linear space.

2.1.2.6 Generalized Lipschitz Operator Let X^e and Y^e be extended linear spaces associated respectively with two Banach spaces X and Y of measurable functions defined on the time domain $[0, \infty)$, and let D be a subset of X^e. A nonlinear operator $Q : D \to Y^e$ is called a generalized Lipschitz operator on D if there exists a constant L such that

$$\left\| [Q(x)]_T - [Q(\tilde{x})]_T \right\|_Y \leq L \| x_T - \tilde{x}_T \|_X \tag{2.6}$$

for all $x, \tilde{x} \in D$ and for all $T \in [0, \infty)$. Note that the least such constant L is given by the norm of Q with

$$\begin{aligned} \|Q\|_{\text{Lip}} &:= \|Q(x_0)\|_Y + \|Q\| \\ &= \|Q(x_0)\|_Y \\ &\quad + \sup_{T \in [0,\infty)} \sup_{\substack{x,\tilde{x} \in D \\ x_T \neq \tilde{x}_T}} \frac{\left\| [Q(x)]_T - [Q(\tilde{x})]_T \right\|_Y}{\| x_T - \tilde{x}_T \|_X} \end{aligned} \tag{2.7}$$

for any fixed $x_0 \in D$.

Based on (2.7), it follows immediately that for any $T \in [0, \infty)$

$$\begin{aligned} \left\| [Q(x)]_T - [Q(\tilde{x})]_T \right\|_Y &\leq \|Q\| \| x_T - \tilde{x}_T \|_X \\ &\leq \|Q\|_{\text{Lip}} \| x_T - \tilde{x}_T \|_X \end{aligned} \tag{2.8}$$

Lemma 2.1 [1] Let X^e and Y^e be extended linear spaces associated respectively with two Banach spaces X and Y, and let D be a subset of X^e. The following family of Lipschitz operators is a Banach space:

$$\text{Lip}(D, Y^e) = \left\{ Q : D \to Y^e \middle| \|Q\|_{\text{Lip}} < \infty \text{ on } D \right\} \tag{2.9}$$

Proof First, it is clear that $\text{Lip}(D, Y^e)$ is a normed linear space. Hence, it is sufficient to verify its completeness.

Let Q_n be a Cauchy sequence in $\text{Lip}(D, Y^e)$ such that $\|Q_m - Q_n\| \to 0$ as $m, n \to \infty$. We need to show that $\|Q_n - Q\| \to 0$ for some $Q \in \text{Lip}(D, Y^e)$ as $n \to \infty$.

Let $T \in [0, \infty)$ be fixed. For any $\tilde{x} \in D$, by definition of the Lipschitz norm with an $x_0 \in D$, we have

$$\begin{aligned} &\left\| [(Q_m - Q_n)(\tilde{x})]_T - [(Q_m - Q_n)(x_0)]_T \right\|_Y \\ &\leq \left\| Q_m - Q_n \right\|_{\text{Lip}} \left\| \tilde{x}_T - [x_0]_T \right\|_X \end{aligned} \tag{2.10}$$