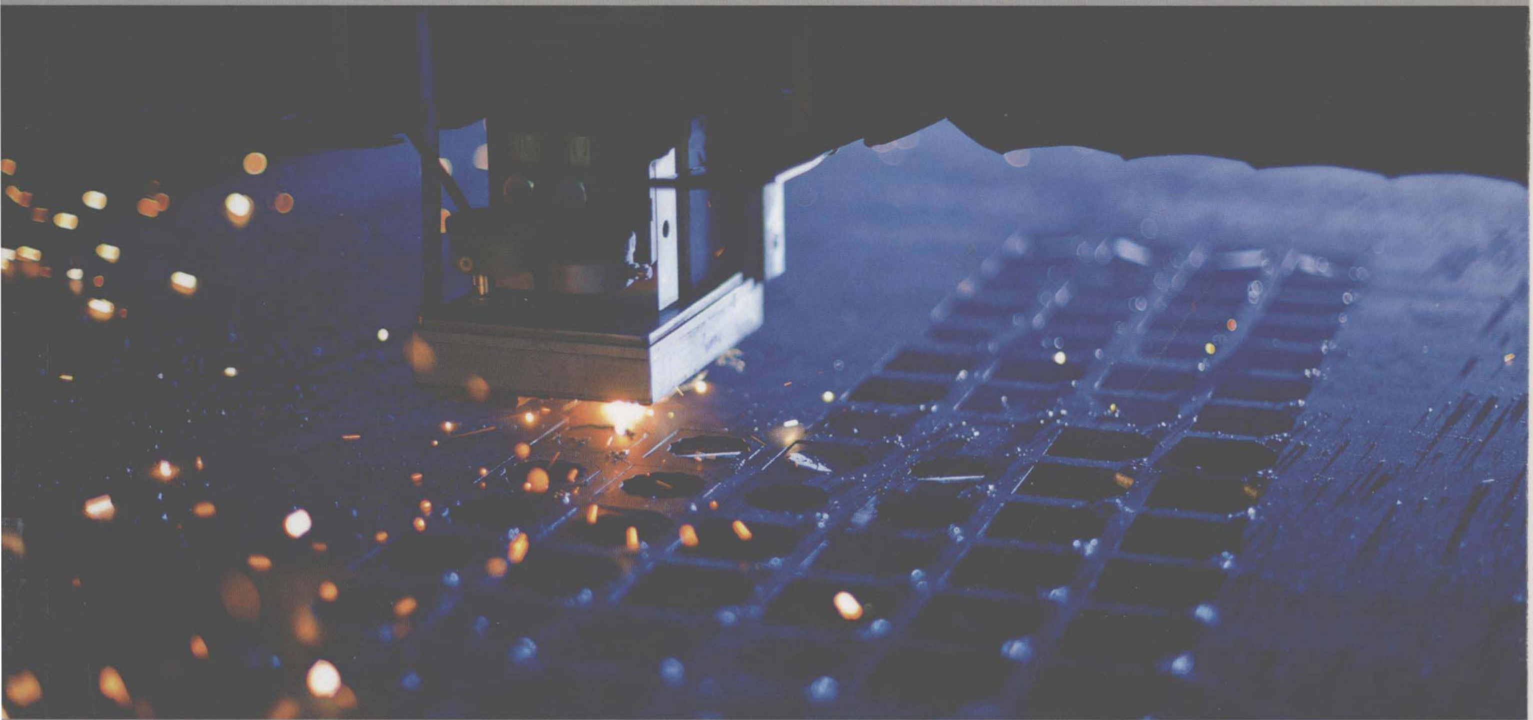
DOUGLAS C. MONTGOMERY | GEORGE C. RUNGER

Applied Statistics and Probability for Engineers

Sixth Edition

EXCLUSIVE TO THIS VERSION

- New exercise sets
- Coverage of SI units



INTERNATIONAL STUDENT VERSION

WILEY

Applied Statistics and Probability for Engineers

International Student Version

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WILEY

To:
Meredith, Neil, Colin, and Cheryl
Rebecca, Elisa, George, and Taylor

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Preface

INTENDED AUDIENCE

This is an introductory textbook for a first course in applied statistics and probability for undergraduate students in engineering and the physical or chemical sciences. These individuals play a significant role in designing and developing new products and manufacturing systems and processes, and they also improve existing systems. Statistical methods are an important tool in these activities because they provide the engineer with both descriptive and analytical methods for dealing with the variability in observed data. Although many of the methods we present are fundamental to statistical analysis in other disciplines, such as business and management, the life sciences, and the social sciences, we have elected to focus on an engineering-oriented audience. We believe that this approach will best serve students in engineering and the chemical/physical sciences and will allow them to concentrate on the many applications of statistics in these disciplines. We have worked hard to ensure that our examples and exercises are engineering- and science-based, and in almost all cases we have used examples of real data—either taken from a published source or based on our consulting experiences.

We believe that engineers in all disciplines should take at least one course in statistics. Unfortunately, because of other requirements, most engineers will only take one statistics course. This book can be used for a single course, although we have provided enough material for two courses in the hope that more students will see the important applications of statistics in their everyday work and elect a second course. We believe that this book will also serve as a useful reference.

We have retained the relatively modest mathematical level of the first five editions. We have found that engineering students who have completed one or two semesters of calculus and have some knowledge of matrix algebra should have no difficulty reading all of the text. It is our intent to give the reader an understanding of the methodology and how to apply it, not the mathematical theory. We have made many enhancements in this edition, including reorganizing and rewriting major portions of the book and adding a number of new exercises.

ORGANIZATION OF THE BOOK

Perhaps the most common criticism of engineering statistics texts is that they are too long. Both instructors and students complain that it is impossible to cover all of the topics in the book in one or even two terms. For authors, this is a serious issue because there is great variety in both the content and level of these courses, and the decisions about what material to delete without limiting the value of the text are not easy. Decisions about which topics to include in this edition were made based on a survey of instructors.

Chapter 1 is an introduction to the field of statistics and how engineers use statistical methodology as part of the engineering problem-solving process. This chapter also introduces the reader to some engineering applications of statistics, including building empirical models, designing engineering experiments, and monitoring manufacturing processes. These topics are discussed in more depth in subsequent chapters.

Chapters 2, 3, 4, and 5 cover the basic concepts of probability, discrete and continuous random variables, probability distributions, expected values, joint probability distributions, and independence. We have given a reasonably complete treatment of these topics but have avoided many of the mathematical or more theoretical details.

Chapter 6 begins the treatment of statistical methods with random sampling; data summary and description techniques, including stem-and-leaf plots, histograms, box plots, and probability plotting; and several types of time series plots. Chapter 7 discusses sampling distributions, the central limit theorem, and point estimation of parameters. This chapter also introduces some of the important properties of estimators, the method of maximum likelihood, the method of moments, and Bayesian estimation.

Chapter 8 discusses interval estimation for a single sample. Topics included are confidence intervals for means, variances or standard deviations, proportions, prediction intervals, and tolerance intervals. Chapter 9 discusses hypothesis tests for a single sample. Chapter 10 presents tests and confidence intervals for two samples. This material has been extensively rewritten and reorganized. There is detailed information and examples of methods for determining appropriate sample sizes. We want the student to become familiar with how these techniques are used to solve real-world engineering problems and to get some understanding of the concepts behind them. We give a logical, heuristic development of the procedures rather than a formal, mathematical one. We have also included some material on nonparametric methods in these chapters.

Chapters 11 and 12 present simple and multiple linear regression including model adequacy checking and regression model diagnostics and an introduction to logistic regression. We use matrix algebra throughout the multiple regression material (Chapter 12) because it is the only easy way to understand the concepts presented. Scalar arithmetic presentations of multiple regression are awkward at best, and we have found that undergraduate engineers are exposed to enough matrix algebra to understand the presentation of this material.

Chapters 13 and 14 deal with single- and multifactor experiments, respectively. The notions of randomization, blocking, factorial designs, interactions, graphical data analysis, and fractional factorials are emphasized. Chapter 15 introduces statistical quality control, emphasizing the control chart and the fundamentals of statistical process control.

WHAT'S NEW IN THIS EDITION

We received much feedback from users of the fifth edition of the book, and in response we have made substantial changes in this new edition.

- Because computer intensive methods are so important in the modern use of statistics, we have added material on the bootstrap and its use in constructing confidence intervals.
- We have increased the emphasis on the use of P-value in hypothesis testing. Many sections of several chapters were rewritten to reflect this.
- Many sections of the book have been edited and rewritten to improve the explanations and try to make the concepts easier to understand.
- The introductory chapter on hypothesis testing now includes coverage of equivalence testing, a technique widely used in the biopharmaceutical industry, but which has widespread applications in other areas.
- Combining P-values when performing multiple tests is included.
- Decision theory is briefly introduced in Chapter 15.
- We have added brief comments at the end of examples to emphasize the practical interpretations of the results.
- Many new examples and homework exercises have been added.

FEATURED IN THIS BOOK

Definitions, Key Concepts, and Equations

Throughout the text, definitions and key concepts and equations are highlighted by a box to emphasize their importance.

4-3 Cumulative Distribution Functions

An alternative method to describe the distribution of a discrete random variable can also be used for continuous random variables.

Cumulative Distribution Function

The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du \quad (4\text{-}3)$$

for $-\infty < x < \infty$.

The cumulative distribution function is defined for all real numbers. The following example illustrates the definition.

Example 4-3 Electric Current For the copper current measurement in Example 4-1, the cumulative distribution function of the random variable X consists of three expressions. If $x < 4.9$, $f(x) = 0$. Therefore,

$$F(x) = 0, \quad \text{for} \quad x < 4.9$$

and

$$F(x) = \int_{4.9}^{x} f(u)\,du = 5x - 24.5, \quad \text{for} \quad 4.9 \le x < 5.1$$

Finally,

$$F(x) = \int_{4.9}^{x} f(u)\,du = 1, \quad \text{for} \quad 5.1 \le x$$

Therefore,

$$F(x) = \begin{cases} 0 & x < 4.9 \\ 5x - 24.5 & 4.9 \le x < 5.1 \\ 1 & 5.1 \le x \end{cases}$$

The plot of $F(x)$ is shown in Fig. 4-6.

Notice that in the definition of $F(x)$, any $<$ can be changed to $\le$ and vice versa. That is, in Example 4-3 $F(x)$ can be defined as either $5x - 24.5$ or 0 at the end-point $x = 4.9$, and $F(x)$ can be defined as either $5x - 24.5$ or 1 at the end-point $x = 5.1$. In other words, $F(x)$ is a continuous function. For a discrete random variable, $F(x)$ is not a continuous function. Sometimes a continuous random variable is defined as one that has a continuous cumulative distribution function.

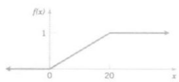


FIGURE 4-6 Cumulative distribution function for Example 4-3.

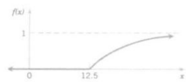


FIGURE 4-7 Cumulative distribution function for Example 4-4.

Learning Objectives

Learning objectives at the start of each chapter guide the students in what they are expected to take away from this chapter and serve as a study reference.

Learning Objectives

After careful study of this chapter, you should be able to do the following:

1. Determine probabilities from probability density functions
2. Determine probabilities from cumulative distribution functions and cumulative distribution functions from probability density functions, and the reverse
3. Calculate means and variances for continuous random variables
4. Understand the assumptions for some common continuous probability distributions
5. Select an appropriate continuous probability distribution to calculate probabilities in specific applications
6. Calculate probabilities, determine means and variances for some common continuous probability distributions
7. Standardize normal random variables
8. Use the table for the cumulative distribution function of a standard normal distribution to calculate probabilities
9. Approximate probabilities for some binomial and Poisson distributions

Seven-Step Procedure for Hypothesis Testing

The text introduces a sequence of seven steps in applying hypothesis-testing methodology and explicitly exhibits this procedure in examples.

9-1.6 GENERAL PROCEDURE FOR HYPOTHESIS TESTS

This chapter develops hypothesis-testing procedures for many practical problems. Use of the following sequence of steps in applying hypothesis-testing methodology is recommended.

1. **Parameter of interest:** From the problem context, identify the parameter of interest.
2. **Null hypothesis, H_0:** State the null hypothesis, H_0.
3. **Alternative hypothesis, H_1:** Specify an appropriate alternative hypothesis, H_1.
4. **Test statistic:** Determine an appropriate test statistic.
5. **Reject H_0 if:** State the rejection criteria for the null hypothesis.
6. **Computations:** Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
7. **Draw conclusions:** Decide whether or not H_0 should be rejected and report that in the problem context.

Figures

Numerous figures throughout the text illustrate statistical concepts in multiple formats.

TABLE • 11-1 Oxygen and Hydrocarbon Levels

Observation Number	Hydrocarbon Level $x(\%)$	Purity $y(\%)$
1	0.99	90.01
2	1.02	89.05
3	1.15	91.43
4	1.29	93.74
5	1.46	96.73
6	1.36	94.45
7	0.87	87.59
8	1.23	91.77
9	1.55	99.42
10	1.40	93.65
11	1.19	93.54
12	1.15	92.52
		90.56
		89.54
		89.85
		90.39
		93.25
		93.41
		94.98
		87.33

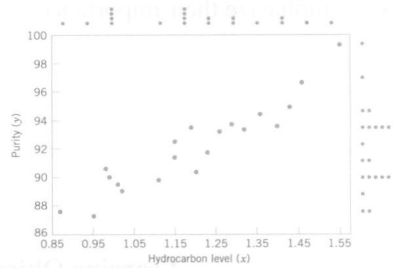


FIGURE 11-1 Scatter diagram of oxygen purity versus hydrocarbon level from Table 11-1.

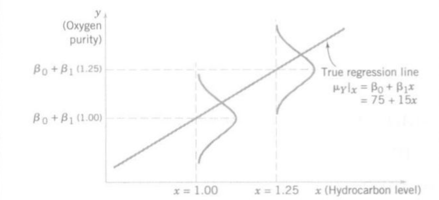


FIGURE 11-2 The distribution of Y for a given value of x for the oxygen purity-hydrocarbon data.

Computer Output

Examples throughout the book use computer output to illustrate the role of modern statistical software.

Stem-and-leaf of Strength

N = 80	Leaf	Unit = 1.0
1	7	6
2	8	7
3	9	7
5	10	1 5
8	11	0 5 8
11	12	0 1 3
17	13	1 3 3 4 5 5
25	14	1 2 3 5 6 8 9 9
37	15	0 0 1 3 4 4 6 7 8 8 8 8
(10)	16	0 0 0 3 3 5 7 7 8 9
33	17	0 1 1 2 4 4 5 6 6 8
23	18	0 0 1 1 3 4 6
16	19	0 3 4 6 9 9
10	20	0 1 7 8
6	21	8
5	22	1 8 9
2	23	7
1	24	5

FIGURE 6-6 A typical computer-generated stem-and-leaf diagram.

Example Problems

A set of example problems provides the student with detailed solutions and comments for interesting, real-world situations. Brief practical interpretations have been added in this edition.

Example 10-1 Paint Drying Time A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using $\alpha = 0.05$?

We apply the seven-step procedure to this problem as follows:

1. **Parameter of interest:** The quantity of interest is the difference in mean drying times, $\mu_1 - \mu_2$, and $\Delta_0 = 0$.
2. **Non hypothesis:** $H_0: \mu_1 - \mu_2 = 0$, or $H_0: \mu_1 = \mu_2$.
3. **Alternative hypothesis:** $H_1: \mu_1 > \mu_2$. We want to reject H_0 if the new ingredient reduces mean drying time.
4. **Test statistic:** The test statistic is

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad \text{where } \sigma_1^2 = \sigma_2^2 = (8)^2 = 64 \text{ and } n_1 = n_2 = 10.$$

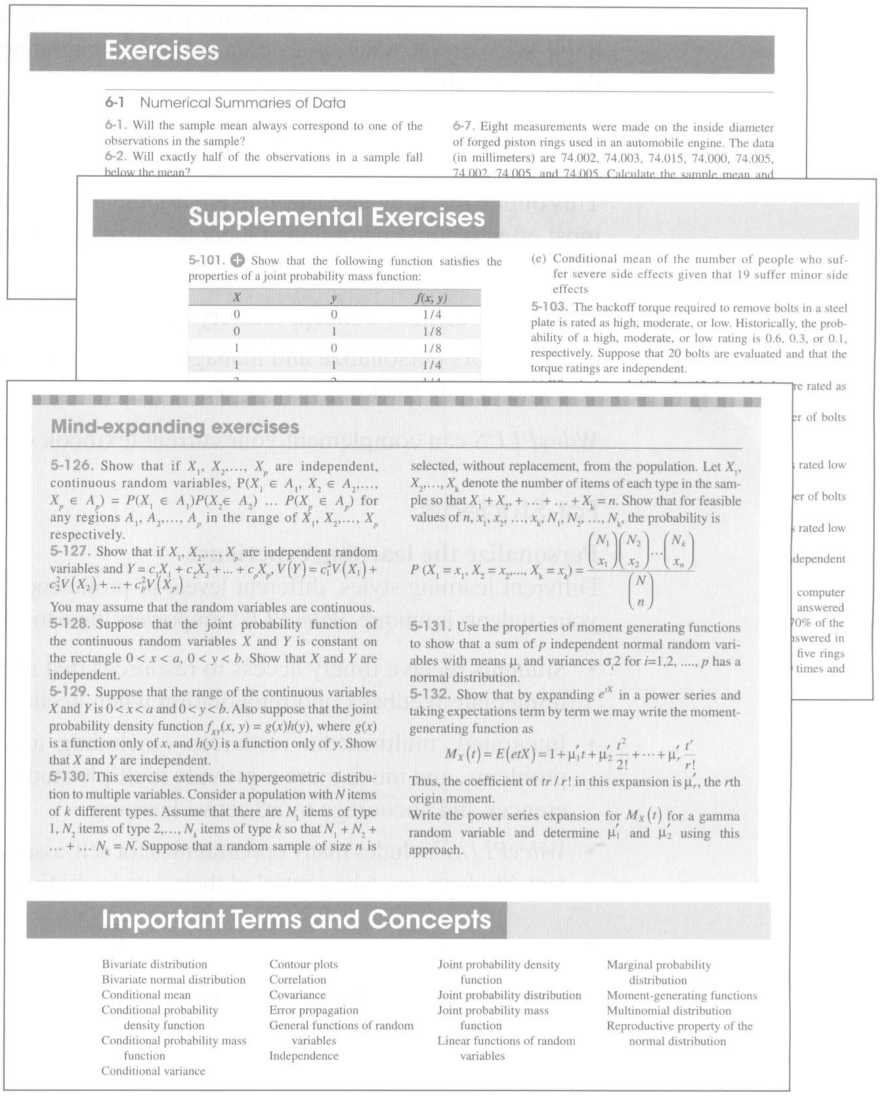
5. **Reject H_0 if:** Reject $H_0: \mu_1 = \mu_2$ if the P-value is less than 0.05.
6. **Computations:** Because $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, the test statistic is

$$z_0 = \frac{121 - 112}{\sqrt{\frac{(8)^2}{10} + \frac{(8)^2}{10}}} = 2.52$$

7. **Conclusion:** Because $z_0 = 2.52$, the P-value is $P = 1 - \Phi(2.52) = 0.0059$, so we reject H_0 at the $\alpha = 0.05$ level.

Practical Interpretation: We conclude that adding the new ingredient to the paint significantly reduces the drying time. This is a strong conclusion.

Exercises
Each chapter has an extensive collection of exercises, including **review exercises** that emphasize the material in each section, **supplemental exercises** that cover the scope of chapter topics and require the student to make a decision about the approach they will use to solve the problem, and **mind-expanding exercises** that often require the student to extend the text material somewhat or to apply it in a novel situation. Answers are provided to most odd-numbered exercises in Appendix C in the text, and the *WileyPLUS* online learning environment includes for students complete detailed solutions to selected exercises.



Exercises

6-1 Numerical Summaries of Data

6-1. Will the sample mean always correspond to one of the observations in the sample?

6-2. Will exactly half of the observations in a sample fall below the mean?

6-7. Eight measurements were made on the inside diameter of forged piston rings used in an automobile engine. The data (in millimeters) are 74.002, 74.003, 74.015, 74.000, 74.005, 74.002, 74.005, and 74.005. Calculate the sample mean and

Supplemental Exercises

5-101. Show that the following function satisfies the properties of a joint probability mass function:

X	y	$f(x, y)$
0	0	1/4
0	1	1/8
1	0	1/8
1	1	1/4

(e) Conditional mean of the number of people who suffer severe side effects given that 19 suffer minor side effects

5-103. The backoff torque required to remove bolts in a steel plate is rated as high, moderate, or low. Historically, the probability of a high, moderate, or low rating is 0.6, 0.3, or 0.1, respectively. Suppose that 20 bolts are evaluated and that the torque ratings are independent.

Mind-expanding exercises

5-126. Show that if $X_1, X_2, \ldots, X_p$ are independent, continuous random variables, $P(X_1 \in A_1, X_2 \in A_2, \ldots, X_p \in A_p) = P(X_1 \in A_1)P(X_2 \in A_2) \ldots P(X_p \in A_p)$ for any regions $A_1, A_2, \ldots, A_p$ in the range of $X_1, X_2, \ldots, X_p$ respectively.

5-127. Show that if $X_1, X_2, \ldots, X_p$ are independent random variables and $Y = c_1X_1 + c_2X_2 + \ldots + c_pX_p$, $V(Y) = c_1^2V(X_1) + c_2^2V(X_2) + \ldots + c_p^2V(X_p)$

You may assume that the random variables are continuous.

5-128. Suppose that the joint probability function of the continuous random variables X and Y is constant on the rectangle $0 < x < a$, $0 < y < b$. Show that X and Y are independent.

5-129. Suppose that the range of the continuous variables X and Y is $0 < x < a$ and $0 < y < b$. Also suppose that the joint probability density function $f_{XY}(x, y) = g(x)h(y)$, where $g(x)$ is a function only of x, and $h(y)$ is a function only of y. Show that X and Y are independent.

5-130. This exercise extends the hypergeometric distribution to multiple variables. Consider a population with N items of k different types. Assume that there are N_1 items of type 1, N_2 items of type 2,…, N_k items of type k so that $N_1 + N_2 + \ldots + \ldots N_k = N$. Suppose that a random sample of size n is selected, without replacement, from the population. Let $X_1, X_2, \ldots, X_k$ denote the number of items of each type in the sample so that $X_1 + X_2 + \ldots + \ldots + X_k = n$. Show that for feasible values of $n, x_1, x_2, \ldots, x_k, N_1, N_2, \ldots, N_k$, the probability is

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_k = x_k) = \frac{\binom{N_1}{x_1}\binom{N_2}{x_2}\cdots\binom{N_k}{x_n}}{\binom{N}{n}}$$

5-131. Use the properties of moment generating functions to show that a sum of p independent normal random variables with means μ_i and variances $\sigma_i 2$ for i=1,2, ….., p has a normal distribution.

5-132. Show that by expanding e^{tX} in a power series and taking expectations term by term we may write the moment-generating function as

$$M_X(t) = E(etX) = 1 + \mu_1' t + \mu_2' \frac{t^2}{2!} + \cdots + \mu_r' \frac{t^r}{r!}$$

Thus, the coefficient of $tr / r!$ in this expansion is μ_r', the rth origin moment.

Write the power series expansion for $M_X(t)$ for a gamma random variable and determine μ_1' and μ_2' using this approach.

Important Terms and Concepts

Bivariate distribution
Bivariate normal distribution
Conditional mean
Conditional probability density function
Conditional probability mass function
Conditional variance
Contour plots
Correlation
Covariance
Error propagation
General functions of random variables
Independence
Joint probability density function
Joint probability distribution
Joint probability mass function
Linear functions of random variables
Marginal probability distribution
Moment-generating functions
Multinomial distribution
Reproductive property of the normal distribution

Important Terms and Concepts
At the end of each chapter is a list of important terms and concepts for an easy self-check and study tool.

STUDENT RESOURCES

- **Data Sets** Data sets for all examples and exercises in the text. Visit the student section of the book Web site at www.wiley.com/college/montgomery to access these materials.

INSTRUCTOR RESOURCES

The following resources are available only to instructors who adopt the text:

- **Solutions Manual** All solutions to the exercises in the text.
- **Data Sets** Data sets for all examples and exercises in the text.
- **Image Gallery of Text Figures**
- **PowerPoint Lecture Slides**
- **Section on Logistic Regression**

These instructor-only resources are password-protected. Visit the instructor section of the book Web site at www.wiley.com/college/montgomery to register for a password to access these materials.

WileyPLUS

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- With *WileyPLUS* you can identify those students who are falling behind and intervene accordingly, without having to wait for them to come to office hours.
- *WileyPLUS* simplifies and automates such tasks as student performance assessment, making assignments, scoring student work, keeping grades, and more.

COURSE SYLLABUS SUGGESTIONS

This is a very flexible textbook because instructors' ideas about what should be in a first course on statistics for engineers vary widely, as do the abilities of different groups of students. Therefore, we hesitate to give too much advice, but will explain how we use the book.

We believe that a first course in statistics for engineers should be primarily an applied statistics course, not a probability course. In our one-semester course we cover all of Chapter 1 (in one or two lectures); overview the material on probability, putting most of the emphasis on the normal distribution (six to eight lectures); discuss most of Chapters 6 through 10 on confidence intervals and tests (twelve to fourteen lectures); introduce regression models in Chapter 11 (four lectures); give an introduction to the design of experiments from Chapters 13 and 14 (six lectures); and present the basic concepts of statistical process control, including the Shewhart control chart from Chapter 15 (four lectures). This leaves about three to four periods for exams and review. Let us emphasize that the purpose of this course is to introduce engineers to how statistics can be used to solve real-world engineering problems, not to weed out the less mathematically gifted students. This course is not the "baby math-stat" course that is all too often given to engineers.

If a second semester is available, it is possible to cover the entire book, including much of the supplemental material, if appropriate for the audience. It would also be possible to assign and work many of the homework problems in class to reinforce the understanding of the concepts. Obviously, multiple regression and more design of experiments would be major topics in a second course.

USING THE COMPUTER

In practice, engineers use computers to apply statistical methods to solve problems. Therefore, we strongly recommend that the computer be integrated into the class. Throughout the book we have presented typical example of the output that can be obtained with modern statistical software. In teaching, we have used a variety of software packages, including Minitab, Statgraphics, JMP, and Statistica. We did not clutter up the book with operational details of these different packages because how the instructor integrates the software into the class is ultimately more important than which package is used. All text data are available in electronic form on the textbook Web site. In some chapters, there are problems that we feel should be worked using computer software. We have marked these problems with a special icon in the margin.

In our own classrooms, we use the computer in almost every lecture and demonstrate how the technique is implemented in software as soon as it is discussed in the lecture. Student versions of many statistical software packages are available, and students can either purchase their own copy or use the products available through the institution. We have found that this greatly improves the pace of the course and student understanding of the material.

Users should be aware that final answers may differ slightly due to different numerical precision and rounding protocols among softwares.

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We would like to express our grateful appreciation to the many organizations and individuals who have contributed to this book. Many instructors who used the previous editions provided excellent suggestions that we have tried to incorporate in this revision.

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Douglas C. Montgomery

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