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Lectures in Econometrics

Lawrence R. Klein

with a chapter on modelling
socialist economy by Wladyslaw Welfe

LECTURES IN ECONOMETRICS

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INTRODUCTION TO THE SERIES

The aim of the series is to cover topics in economics, mathematical economics and econometrics, at a level suitable for graduate students or final year undergraduates specializing in economics. There is at any time much material that has become well established in journal papers and discussion series which still awaits a clear, self-contained treatment that can easily be mastered by students without considerable preparation or extra reading. Leading specialists will be invited to contribute volumes to fill such gaps. Primary emphasis will be placed on clarity, comprehensive coverage of sensibly defined areas, and insight into fundamentals, but original ideas will not be excluded. Certain volumes will therefore add to existing knowledge, while others will serve as a means of communicating both known and new ideas in a way that will inspire and attract students not already familiar with the subject matter concerned.

The Editors

Chapter I

MODELS OF THE ECONOMY AS A WHOLE

What is a Model?

A schematic simplification that strips away the nonessential aspects to reveal the inner workings, shape, or design of a more complicated mechanism constitutes a model. Social systems are enormously complicated, so much so that we can never grasp the complete explanation of all aspects of society at once. We break the problem into parts, but even that is not sufficient for complete human comprehension; therefore, modeling is an important step. A social model consists of simplifying assumptions; approximate but understandable relationships; and some explanation of reality. It, by itself, is not reality, but merely a simplified picture of reality that man can understand.

Models are far reaching in social science analysis. In this volume, I am going to concentrate on economic models because I feel completely at ease in that area, but inevitably it will become necessary to break outside the confines of the purely economic aspects of behavior to more general social models. The necessity will arise because economics alone is not all-embracing or self contained. To explain some basic aspects of economic behavior, it will be necessary to explain related aspects of more general social behavior, encompassing politics, demography, and even social engineering.

At the beginning, the discussion will be entirely concerned with quantitative models that are capable of mathematical expression. Later, however, more general possibilities will be considered — historical, qualitative, non-mathematical models.

A Macroeconomic Model — A New Approach

The theory of employment and output determination developed in the 1930's formed the basis of abstract and, later, statistical model building for the economy as a whole. The real meaning of the *Keynesian Revolution* became clear when model comparisons of alternative systems — the classical, neoclassical, Keynesian, Marxian — were formulated side-by-side in mathematical equation systems. Model building took an unfortunate doctrinal turn at that time. Although I participated as fully as anyone else in that approach to model building, I have lately come to feel that a more rewarding approach that is neutral as far as doctrine is concerned will be through the accounting structure.

Model structure is both art and science. Inspiration, leads, pieces of evidence, and many sources of information are needed to put together a good model. If all the economic accounts associated with a problem are laid

Table 1
The NIA system.

	Households (H)		
(*EH1*)	Consumer expenditures = (*RB1*)	(*RH1*)	Receipts from *B* = (*EB1*)
(*EH2*)	Taxes paid by *H* = (*RG1*)	(*RH2*)	Receipts from *G* = (*EG1*)
(*EH3*)	Personal Saving = (*S1*)		
	Business (B)		
(*EB1*)	Income paid to *H* = (*RH1*)	(*RB1*)	Consumer expenditures = (*EH1*)
(*EB2*)	Taxes paid by *B* = (*RG2*)	(*RB2*)	Sales to *G* = (*EG2*)
(*EB3*)	Imports = (*RF1*)	(*RB3*)	Exports = (*EF1*)
(*EB4*)	Capital consumption = (*S5*)	(*RB4*)	Sales by *B* to *B*, Investment = (*U1*)
(*EB5*)	Business saving = (*S2*)		
	Government (G)		
(*EG1*)	Wages and transfers paid to *H* = (*RH2*)	(*RG1*)	Taxes received from *H* = (*EH2*)
(*EG2*)	Purchases from *B* = (*RB2*)	(*RG2*)	Taxes received from *B* = (*EB2*)
(*EG3*)	Government saving = (*S3*)		
	Foreigners (F)		
(*EF1*)	Exports = (*RB3*)	(*RF1*)	Imports = (*EB3*)
(*EF2*)	Foreign saving = (*S4*)		
	Sources and Uses (S/U)		
(*U1*)	Business investment = (*RB4*)	(*S1*)	Personal saving = (*EH3*)
		(*S2*)	Business saving = (*EB5*)
		(*S3*)	Government saving = (*EG3*)
		(*S4*)	Foreign saving = (*EF2*)
		(*S5*)	Capital consumption = (*EB4*)

out in advance with the necessary accounting balance equations stated, we can see immediately what is needed in order to get a full explanation of the whole system. As an instructive example of a closed accounting system and an associated model, let us consider a simplified national income accounting (NIA) (see table 1) system.[1]

The units of economic action: Households (H)
Businesses (B)
Government (G)
Foreigners (F)

The accounts: Balance systems of receipts and expenditures, on a double entry basis. These are income accounts for each unit.

In order to make the system double entry, there must be provision for separate entries for balancing items in each account — the various savings accounts. These are offset in the final account for Sources and Uses (S/U) by entries for gross investment (final sales by business to other business) and depreciation or capital consumption.

In this system, every entry occurs twice and is labeled on the left by one source. On the right, we label the other place in the accounting system where it occurs. There are 28 entries and, by the dual principle, we need only 14 equations to model the system. Associated with each account, there are balance equations, five in all:

$$(EH1)+(EH2)+(EH3)=(RH1)+(RH2), \tag{1.1}$$

$$(EB1)+(EB2)+(EB3)+(EB4)+(EB5) = (RB1)+(RB2)+(RB3)+(RB4), \tag{1.2}$$

$$(EG1)+(EG2)+(EG3)=(RG1)+(RG2), \tag{1.3}$$

$$(EF1)+(EF2)=(RF1), \tag{1.4}$$

$$(U1)=(S1)+(S2)+(S3)+(S4)+(S5). \tag{1.5}$$

Next, we must formulate 9 more statistical equations for the behavioral, institutional, and technological structure of the economy. Some plausible relationships are:

consumption function

$$(EH1)=\alpha_0+\alpha_1\left[(RH1)+(RH2)-(EH2)\right]+\alpha_2(EH1)_{-1}+e_6, \tag{1.6}$$

[1] There is a good deal of generality in this and succeeding accounting designs in this chapter. The same is true of the associated macromodels, yet this entire exposition is pitched for the structure of the typical OECD (Western industrial market) economy. A different structure would be appropriate for the centrally planned and developing economies.

personal tax function

$$(EH2) = \beta_0 + \beta_1 [(RH1) + (RH2)] + e_7, \tag{1.7}$$

wage equation

$$(EB1) = \gamma_0 + \gamma_1 [(RB1) + (RB2) + (RB3) + (RB4)] + e_8, \tag{1.8}$$

business tax function

$$(EB2) = \delta_0 + \delta_1 [(RB1) + (RB2) + (RB3) + (RB4) - (EB1) - (EB3) - (EB4)] + e_9, \tag{1.9}$$

import function

$$(EB3) = \varepsilon_0 + \varepsilon_1 [(RB1) + (RB2) + (RB3) + (RB4)] + e_{10}, \tag{1.10}$$

depreciation equation

$$(EB4) = \zeta_1 (RB4) + \zeta_2 (EB4)_{-1} + e_{11}, \tag{1.11}$$

investment function

$$(RB4) = \eta_0 + \eta_1 [(RB1) + (RB2) + (RB3) + (RB4) - (EB3)] + \eta_2 (RB4)_{-1} + e_{12}, \tag{1.12}$$

$$(EG1) + (EG2) = \text{exogenous}, \tag{1.13}$$

$$(EF1) = \text{exogenous}. \tag{1.14}$$

The first 5 equations of the model are self evident by virtue of the accounting balances used in data construction and presentation. The last 2 are not really structural equations; they are simply part of the specification of the model, explaining what is to be internally generated and what is to be imposed from the outside. The external or exogenous variables are *driving forces* in the system. Eq. (1.13) states simply that total government expenditures (wage payments plus purchases) are not determined on the basis of rules, either behavioral or statutory; they are set by the legislative and administrative branches of government in a way that serves contemporary political and economic interests. In a more complicated system, parts of the government spending process can be explained endogenously, but that discussion is to be given later.

Eq. (1.14) defines exports as exogenous, and this is a first approximation, assuming that exports depend primarily on external economic activity. As in the case of government expenditure an endogenous treatment of exports is postponed until the development of more elaborate and complicated systems below.

Eqs. (1.6)–(1.12) are simple structural equations of the macro model. They are all expressed in linear form, although that is not necessary. It is done only for convenience, simplicity, and because linearity may provide a fairly good empirical approximation in the present context. They are all *stochastic*, containing additive error terms $e_6, \ldots, e_{12}$. The probability properties of the errors are used in making statistical inferences from sample data about the sizes of the unknown coefficients in the linear expressions. The model is dynamic, by virtue of the time lags introduced in eq. (1.6), (1.11) and (1.12), associated with the processes of consumer spending, capital consumption, and capital formation. The lag variable, carried over from the previous observation period, has the same role as an exogenous variable in the first period of a solution of the model. Lags and exogenous variables are *inputs* to the solution process. As additional solution periods are developed, after the first, exogenous values must be repeatedly obtained as external input, while lagged values can be generated from prior solutions as delayed internal input. In chapter 2 on *Simulation*, dynamic solutions of models with lags are taken up in detail.

The accounting structure tells us what to look for. It gives little or no insight into the particular specification of structural equations. As an exercise, and with that alone in mind, we might set for ourselves the task of devising an interrelated model made up entirely of interrelationships among the entries in the simple NIA accounting system.

Eq. (1.6) is the familiar *propensity to consume*. It expresses personal consumption expenditures as a function of disposable personal income, i.e., *income payments to persons plus transfers from the government less personal taxes paid to government*. Previous consumption is introduced as an added explanatory variable to show either habit persistence or the transformation of a distributed lag of past disposable income values, stretching back in time with geometrically declining weights.

In constructing the concept of disposable income in (1.6), we subtracted personal tax payments from personal income. These taxes are not, however, independent of the size of personal income. By law they vary with income and a great many other special factors according to a whole schedule of income taxes. The tax laws apply, however, to individuals in a complicated nonlinear way. In the aggregate, there is a simple smooth relation between total taxes paid and income received, although the parameters of this function will have to be re-evaluated when tax laws change substantially. Also, large inflationary movements and other causes of income redistribution by tax bracket can affect the coefficients of a simple linear relation; therefore much care must be exercised to allow for parameter shifts in the simple linear form.

Transfer payments from government to persons could be treated like taxes, but with opposite direction of flow. This was not done in the present model in order to condense the accounts as much as possible. Transfer payments are lumped with other payments (wages and interest) from government to persons.

The wage eq. (1.8) shows the distribution of payments in the business account between wages and other factor payments, principally profits. In this system, business retained profits are entered as business saving. Payments to persons are dominated by wage payments but also include interest and dividends. The explanatory variable in this equation is gross business receipts.

Like households, businesses must also pay taxes to government. This tax function uses *net* business income as a tax base, after subtraction of factor and import material costs. The parameters of this aggregate linear tax function are also subject to statutory changes, but in the United States are not scheduled on a steeply graduated basis, as are personal income taxes; therefore, the linear approximation, in the aggregate, is sounder in (1.9) than in (1.7).

Although, as a first approximation, it may seem all right to classify exports in the exogenous category, this would not be suitable treatment for imports. They depend closely on internal economic activity. We have accordingly specified the propensity to import as a simple linear function of gross business receipts.

The depreciation or capital consumption eq. (1.11) is based on the assumption that capital use is proportional to the outstanding stock of capital. A direct statement of this relation must, however, be transformed in order to express it in terms of the variables of the NIA system, as follows:

$$(EB4)_t = \zeta \sum_{i=-\infty}^{t} [(RB4)_i - (EB4)_i] + e_t,$$

$$(EB4)_{t-1} = \zeta \sum_{i=-\infty}^{t-1} [(RB4)_i - (EB4)_i] + e_{t-1},$$

$$(EB4)_t - (EB4)_{t-1} = \zeta(RB4)_t - \zeta(EB4)_t + (e_t - e_{t-1}),$$

$$(EB4)_t = \frac{\zeta}{1+\zeta}(RB4)_t + \frac{1}{1+\zeta}(EB4)_{t-1} + \frac{1}{1+\zeta}(e_t - e_{t-1}).$$

The simple relationship in (1.11) does not recognize the parameter restriction implied in this transformation. There is only one unknown parameter in the derived relationship, but (1.11) suggests that there are two independent parameters. The relationship between the two parameters could be taken into account in the estimation process, but that will be more complicated than the straightforward estimation of two independent parameters. Also, it should be noted, that if e_t and e_{t-1} are uncorrelated, the transformed variable $[1/(1+\zeta)](e_t - e_{t-1})$ will be serially correlated. Conversely, if the transformed error is serially uncorrelated, e_t will be serially correlated. This fact may be utilized in the estimation process.

Depreciation values in NIA systems are statutory values that are associated with business tax policy of government. Estimates of physical depreciation are superior, but their introduction into the system would complicate the accounting balance in terms of published figures; therefore (1.11) is specified in terms of statutory data, and the parameters are subject to change as tax guideline rules for capital write-off change.

Finally, the *propensity to invest* (1.12) is modeled like the propensity to consume, either as habit persistence or a transformation of a distributed lag, with geometrically declining weights, in business receipts (net of imports). A transformation of a distributed lag to derive (1.12) or (1.6) can introduce serial correlation of error in much the same way as illustrated above for the depreciation equation. This is an important consideration to be taken into account in estimation of the equations from data.

Simple and direct consideration of accounting structure, institutional knowledge, and common sense behavior patterns have been used in the specification of this simple macroeconomic model. While there is much to be said in favor of this simple model, there are many deficiencies that come about as a result of restricting the analysis to a very simple framework. It is too restrictive to insist that the model be formulated entirely in terms of variables listed in the NIA system. The accounting structure is logical, neat, and indicative, but it does not contain all the information and all the variables needed for adequate model structure.

What is wrong with this model? It lacks,

(1) significant nonlinearities,
(2) adequate dynamics,
(3) market variables — prices, interest rates, wage rates,
(4) appropriate homogeneity properties.

The first two deficiencies are not particularly related to the restrictions imposed by working exclusively within the NIA framework, but the latter two deficiencies arise because prices do not enter explicitly in the accounting structure; current priced variables are used throughout. Some partial accounts can be constructed in constant prices, but the full balance of the system requires statement of magnitudes in current prices. It does not follow that the mathematical model has to be in current prices alone, but correspondence between the full accounting system and the *values stated therein* for model construction does lead to the specification of a current priced equation system. Some investigators actually prefer current priced relationships in their models, but received doctrine states that rational behavior in equilibrium does not admit "money illusion". Equilibrium relationships need not be strictly imposed on dynamic macroeconomic models, but specifications should provide for no money illusion when the dynamic systems are in equilibrium.

This requires decomposition of current priced value entries in the NIA system into price (index) and quantity (index) components. Thus the propensity to consume should be written as

$$\frac{EH1}{PC} = \alpha_0' + \alpha_1' \frac{[(RH1)+(RH2)-(EH2)]}{PC} + \alpha_2' \frac{(EH1)_{-1}}{(PC)_{-1}} + e_6'. \quad (1.6')$$

This says that *real* consumer spending is a distributed lag function of real disposable income. Similar revisions may be made of other equations, but it should be stressed that (1.7) and (1.9) are based on statutory considerations that specify the relationship to be in current prices and not corrected for price change. Specifications like these make complete linearity impossible, but they are important.

The propensities to consume, import, or invest ought to depend on price relatives — the ratio between present and future prices (interest rate) and between foreign and domestic prices. Also, if domestic prices are explicitly introduced, exports should be made endogenous, depending on the relationship between (domestic) prices of export goods and foreign prices. Wage rates, physical labor productivity and other new considerations can be introduced when the scope of model specification is extended beyond the NIA system. The accounting structure should be built in detail and studied first in model specification; then the whole supply–demand balance of the economy should be introduced with equations of price, interest, and wage rate determination. Accounting structure complemented by market process for rate determination leads us to more complete and more satisfactory models. When the model is built and solved for application purposes, however, it is useful and important to present all the price/quantity values in the detailed form of a complete accounting structure. In large applied models, after the mathematical solution is obtained, it is transformed into standard accounting tables for a readily understandable description of the quantitative economy.

A skeleton model that has the same degree of aggregation as the NIA system but allows for market variables (price, wage rate, interest rate) and uses homogeneity (no money illusion) conditions where necessary is useful as a base from which to begin macro model construction. By disaggregation and elaboration of this skeleton system, useable models resembling those now operating in most of the industrial market economies of the world can be constructed. The skeleton system will be stated in conventional symbolism used in most model presentations and not directly in terms of the simplified accounting system.

Identities

definition of GNP

$$C_t + I_t + G_t + E_t - (IM)_t = X_t, \tag{1.15}$$

reconciliation of GNP and NI

$$p_t X_t - T_{1t} - p_t D_t = Y_t + T_{2t} + T_{3t} - T_{rt}, \tag{1.16}$$

definition of NI

$$w_t L_t + P_t = Y_t + T_{2t} + T_{3t} - T_{rt}, \tag{1.17}$$

definition of capital stock

$$K_t = K_{t-1} + I_t - D_t. \tag{1.18}$$

Behavior and Technological Relations

consumption function

$$C_t = \alpha_0 + \alpha_1 (Y_t / p_t) + \alpha_2 C_{t-1} + u_{1t}, \tag{1.19}$$

investment function

$$I_t = \beta_0 + \beta_1 X_t + \beta_2 r_t + \beta_3 K_{t-1} + u_{2t}, \tag{1.20}$$

export function

$$E_t = \gamma_0 + \gamma_1 (WT)_t + \gamma_2 [(p_w)_t / p_t] + \gamma_3 E_{t-1} + u_{3t}, \tag{1.21}$$

import function

$$(IM)_t = \delta_0 + \delta_1 X_t + \delta_2 (p_t / (p_m)_t) + \delta_3 (IM)_{t-1} + u_{4t}, \tag{1.22}$$

production function

$$\ln L_t = \varepsilon_0 + \varepsilon_1 \ln X_t + \varepsilon_2 \ln K_{t-1} + \varepsilon_3 \ln L_{t-1} + u_{5t}, \tag{1.23}$$

price formation equation

$$p_t = \zeta_0 + \zeta_1 (w_t L_t / X_t) + \zeta_2 (p_m)_t + u_{6t}, \tag{1.24}$$

wage formation equation

$$\Delta \ln w_t = \eta_0 + \eta_1[(LF)_t/((LF)_t - L_t)] + \eta_2 \Delta \ln p_t + u_{7t}, \tag{1.25}$$

labor force participation rate

$$[(LF)_t/N_t] = \theta_0 + \theta_1[((LF)_t - L_t)/(LF)_t] + \theta_2(w_t/p_t) + u_{8t}, \tag{1.26}$$

velocity equation

$$\ln(p_t X_t/M_t) = \iota_0 + \iota_1 r_t + \iota_2 \Delta \ln p_t + u_{9t}, \tag{1.27}$$

depreciation equation

$$D_t = \kappa K_{t-1} + u_{10t}. \tag{1.28}$$

Legal or Institutional Equations

indirect tax equation

$$T_{1t} = \Upsilon_{10} + \Upsilon_{11}(p_t X_t) + u_{11t}, \tag{1.29}$$

personal direct tax equation

$$T_{2t} = \Upsilon_{20} + \Upsilon_{21} Y_t + u_{12t}, \tag{1.30}$$

business direct tax equation

$$T_{3t} = \Upsilon_{30} + \Upsilon_{31} P_t + u_{13t}, \tag{1.31}$$

transfer equation

$$T_{rt} = \Upsilon_{r0} + \Upsilon_{r1}[(LF)_t - L_t] + \Upsilon_{r2} w_t + u_{14t}. \tag{1.32}$$

Endogenous or dependent variables

C_t = real consumer expenditures,
Y_t = nominal personal disposable income,
p_t = general price level,
I_t = real gross capital formation (investment),
X_t = real *GNP*,
r_t = nominal interest rate,
K_t = end-of-period stock of real capital,
E_t = real exports,
$(IM)_t$ = real imports,
L_t = employment,
w_t = wage rate,

$(LF)_t$ = labor force,
D_t = real capital consumption (depreciation),
T_{1t} = nominal indirect taxes,
T_{2t} = nominal personal direct taxes,
T_{3t} = nominal business direct taxes,
T_{rt} = nominal transfer payments to persons,
P_t = nominal nonwage income (profits).

Exogenous or independent variables

G_t = real public expenditures on goods and services,
$(WT)_t$ = real volume of world trade,
$(p_w)_t$ = price of world trade,
$(p_m)_t$ = price of imports,
N_t = population,
M_t = nominal money supply.

An explanation of this model is in order, both in terms of its relation to the NIA accounting system and to economic or statistical analysis. The first and third identities (1.15) and (1.17) are simply definitions of some aggregates. They are not arbitrary in the sense that these are widely used aggregates (gross national product and national income), but they are not restrictive, because they are simply combinations of some of the variables of the system into other aggregative variables. Eq. (1.16) is, however, an accounting restriction that says that *GNP* and NI are equal if some reconciling items are taken into account. These are taxes and capital consumption. We shall now show that the accounting identity (1.5) is essentially the same as (1.16).

By substitution we have

$$w_t L_t + P_t + T_{1t} + p_t D_t = p_t(C_t + I_t + G_t + E_t - (IM)_t).$$

The corresponding concepts from the NIA system are

$$w_t L_t = RH1 + RH2, \qquad p_t C_t = EH1,$$

$$P_t = EB5, \qquad p_t I_t = U1,$$

$$T_{1t} = EB2, \qquad p_t G_t = EG1 + EG2,$$

$$p_t D_t = EB4, \qquad p_t E_t = EF1,$$

$$p_t (IM)_t = RF1.$$

If we replace the skeleton econometric model symbols by the NIA symbols