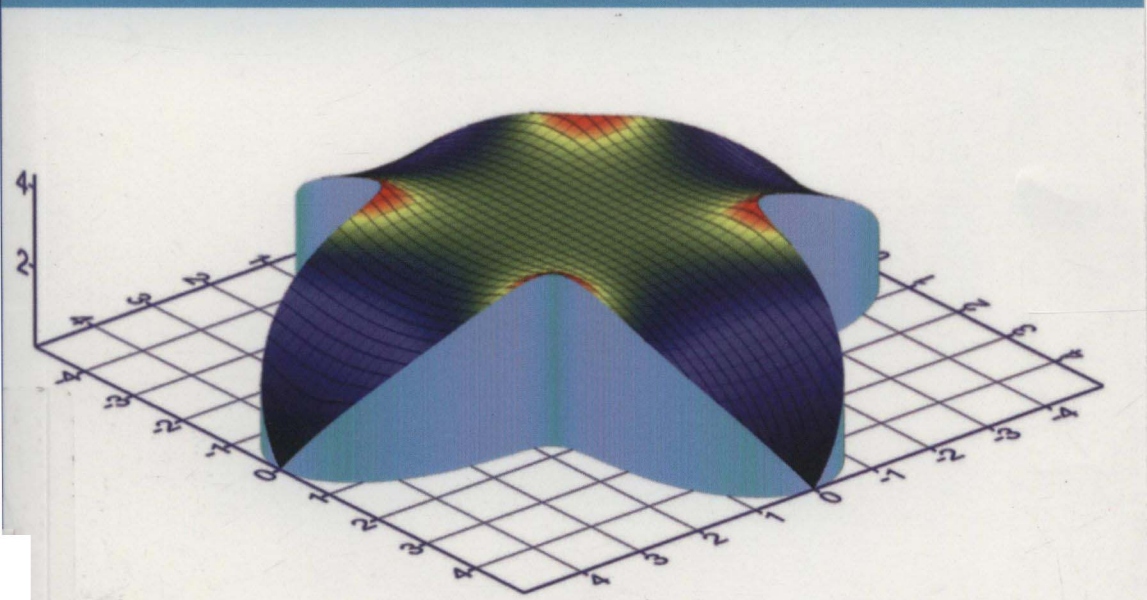
Second Edition

The Boundary Element Method for Engineers and Scientists

Theory and Applications

John T. Katsikadelis



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AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Academic Press is an imprint of Elsevier

Academic Press is an imprint of Elsevier
125 London Wall, London EC2Y 5AS, United Kingdom
525 B Street, Suite 1800, San Diego, CA 92101-4495, United States
50 Hampshire Street, 5th Floor, Cambridge, MA 02139, United States
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, United Kingdom

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British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress.

ISBN: 978-0-12-804493-3

Publisher: Joe Hayton
Acquisition Editor: Brian Guerin
Editorial Project Manager: Edward Payne
Production Project Manager: Nicky Carter
Designer: Mark Rogers

Typeset by MPS Limited, Chennai, India

The Boundary Element Method for Engineers and Scientists

To my wife Efi
for her loving patience and support

Preface to the Second Edition

This second edition with the new title "The Boundary Element Method for Engineers and Scientists: Theory and Applications" reflects new developments that occurred after the first edition was written. Specifically, these are the *dual reciprocity method* (DRM) and the *analog equation method* (AEM), both established methods that have rendered the boundary element method (BEM) an efficient modern computational method for solving all linear and nonlinear problems, static as well as dynamic, in Engineering and Science. These developments are included in three new chapters, namely, in "The BEM for Potential Problems in Inhomogeneous Anisotropic Bodies," "The BEM for Time Dependent Problems," and "The BEM for Nonlinear Problems." Regarding the old chapters, the arrangement of the book remains the same, except for some small additions and emendations.

The first English edition under the title "Boundary Elements. Theory and Applications" appeared in 2002. It was published by Elsevier UK and was widely adopted internationally as a textbook for teaching the BEM as a formal course for boundary elements at the undergraduate or graduate level. This fact is confirmed by the translations of the book into Japanese by Prof. Masa Tanaka of Shinshu University, Nagano (Asakura, Tokyo, 2004), into Russian by Prof. Sergey Aleynikov of Voronezh State Architecture and Civil Engineering University (Издателъство ABC, Publishing House of the Russian Civil Engineering Universities, Moscow, 2007), and into Serbian by Prof. Dragan Spasic of the University of Novi Sad (Gradjevinska Knjiga, Belgrade, 2011) (see Fig. 1). All the three professors, known for their research and writing about the boundary elements and mechanics in general, chose to translate the book as a textbook among several others of the international literature, after having checked its suitability as a textbook with their students.

The success of the first edition encouraged the author to prepare a new revised edition augmented by the three new chapters. They describe the established methods, that is, the DRM and the AEM, which overcome the inherent drawbacks of the BEM, namely, the inability to solve linear problems for which the fundamental solution cannot be established as well as nonlinear and time-dependent problems, using simple known static fundamental solutions. This is illustrated through the application of the DRM and the AEM to problems described by the complete second-order linear or nonlinear equation with variable coefficients (elliptic, parabolic, and hyperbolic).

FIGURE 1 Covers of the English, Japanese, Russian, and Serbian editions.

The material in these new chapters is presented in a systematic and comprehensive manner, as in the old chapters, so that the reader can understand the principles of DRM and AEM as well as their numerical implementation and computer programming. The material related to the AEM is the outcome of the author's long involvement with the method.

In closing, the author wishes to express, from this place too, his sincere thanks to his former student and coworker Dr. A.J. Yiotis for carefully reading the manuscript, his suggestions for constructive emendations, and his

overall contribution to minimize the oversights in the text. Finally, warm thanks belong to Dr. Nikos G. Babouskos, also former student and coworker of the author, not only for his careful reading of the manuscript and his apposite suggestions for improvement of the book, but also for his assistance in developing the computer programs for the new chapters and in producing the numerical results of examples therein.

It is a pleasure to make grateful acknowledgment of many helpful suggestions which have been contributed by readers of the book.

John T. Katsikadelis
Athens
April 2016

Preface to the First Edition

The last three decades have been marked by the evolution of electronic computers and an enormous and widespread availability of computational power. This has boosted the development of computational methods and their application in engineering and in the analysis and design of *structures*, which extend from bridges to aircrafts and from machine elements to tunnels and the human body. New scientific *subfields* were generated in all engineering disciplines being described as "*Computational*," for example, *Computational Mechanics, Computational Fluid Mechanics, Computational Structural Analysis, Computational Structural Dynamics, etc*. The finite element method (FEM) and the boundary element method (BEM) are the most popular of the computational methods. While the FEM has been long established and is most well known in the engineering community, the BEM appeared later offering new computational capabilities with its effectiveness, accuracy, and low computational cost.

Although the BEM is taught as a regular course at an ever increasing number of universities, there is a noticeable lack of a textbook which could help students as well as professional engineers to understand the method, the underlying theory, and its application to engineering problems. An essential reason is that BEM courses are taught mainly as advanced graduate courses, and therefore much of the underlying fundamental knowledge of mathematics and mechanics is not covered in the respective undergraduate courses. Thus, the existing books on the BEM are addressed rather to academia and researchers who, somehow, have already been exposed to the BEM than to students following a BEM course for the first time and engineers who are using boundary element software in industry.

This observation stimulated the author to write the book at hand. His research in the development of the BEM during the last 25 years as well as the experience he acquired by teaching for many years the course of Boundary Elements at the Civil Engineering Department of the National Technical University of Athens, Greece, justify this endeavor. The author's ambition was to make the BEM accessible to the student as well to the professional engineer. For this reason, his main task was to organize and present the material in such a way so that the book becomes "user-friendly" and easy to comprehend, taking into account only the mathematics and mechanics to which students have been exposed during their undergraduate studies. This effort led to an innovative, in many aspects, way of presenting the BEM, including the derivation of fundamental solutions, the integral representation of the solutions and the boundary integral equations for

various governing differential equations in a simple way minimizing a recourse to mathematics with which the student is not familiar. The indicial and tensorial notations, though they facilitate the authors' work and allow to borrow ready to use expressions from the literature, have been avoided in the present book. Nevertheless, all the necessary preliminary mathematical concepts have been included in order to make the book complete and self-sufficient.

In writing the book, topics requiring a detailed study for a deep and thorough understanding of the BEM, have been emphasized. These are:

i. The formulation of the physical problem.
ii. The formulation of the mathematical problem, which is expressed by the governing differential equations and the boundary conditions (boundary value problem).
iii. The conversion of the differential equations to boundary integral equations. This topic familiarizes the reader with special particular solutions, the so-called *fundamental solutions*, shows how they are utilized and helps to comprehend their singular behavior.
iv. The transformation of domain integrals to boundary line integrals or their elimination, in order to obtain pure boundary integral equations.
v. The numerical solution of the boundary integral equations. This topic, which covers a significant part of the book, deals with the numerical implementation of BEM rendering a powerful computational tool for solving realistic engineering problems. It contains the discretization of the boundary into elements, the modeling of its geometry, the approximation of the boundary quantities, as well as the techniques for the evaluation of regular and singular line integrals and in general the procedure for approximating the actual problem by a system of linear algebraic equations.
vi. A detailed description of the FORTRAN programs, which implement the numerical procedure for the various problems. The reader is provided with all the necessary information and the know-how so that he can write his own BEM-based computer programs for problems other than those included in the book.
vii. The use of the aforementioned computer programs for the solution of representative problems and the study of the behavior of the corresponding physical system.

Throughout the book, every concept is followed by example problems, which have been worked out in detail and with all the necessary clarifications. Furthermore, each chapter of the book is enriched with problems-to-solve. These problems serve a threefold purpose. Some of them are simple and aim at applying and better understanding the presented theory, some others are more difficult and aim at extending the theory to special cases requiring a deeper understanding of the concepts, and others are small projects which serve the purpose of familiarizing the student with BEM programming and the programs contained in the CD-ROM.

The latter class of problems is very important as it helps students to comprehend the usefulness and effectiveness of the method by solving real-life engineering problems. Through these problems students realize that the BEM is a powerful computational tool and not an alternative theoretical approach for dealing with physical problems. My experience in teaching the BEM shows that this is the students' most favorite type of problems. They are delighted to solve them, since they integrate their knowledge and make them feel confident in mastering the BEM.

The CD-ROM which accompanies the book contains the source codes of all the computer programs developed in the book, so that the student or the engineer can use them for the solution of a broad class of problems. Among them are general potential problems, problems of torsion, thermal conductivity, deflection of membranes and plates, flow of incompressible fluids, flow through porous media, in isotropic or anisotropic, homogeneous or composite bodies, as well as plane elastostatic problems in simply or multiply connected domains. As one can readily find out from the variety of the applications, the book is useful for engineers of all disciplines. The author is hopeful that the present book will introduce the reader to the BEM in an easy, smooth, and pleasant way and also contribute to its dissemination as a modern robust computational tool for solving engineering problems.

In closing, the author would like to express his sincere thanks to his former student and Visiting Assistant Professor at Texas A&M University Dr Filis Kokkinos for his carefully reading the manuscript and his suggestions for constructive changes. His critic and comments are greatly appreciated. Thanks also belong to my doctoral student, Mr G.C. Tsiatas, MSc, for checking the numerical results and the derivation of several expressions.

John T. Katsikadelis
Athens
January, 2002

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