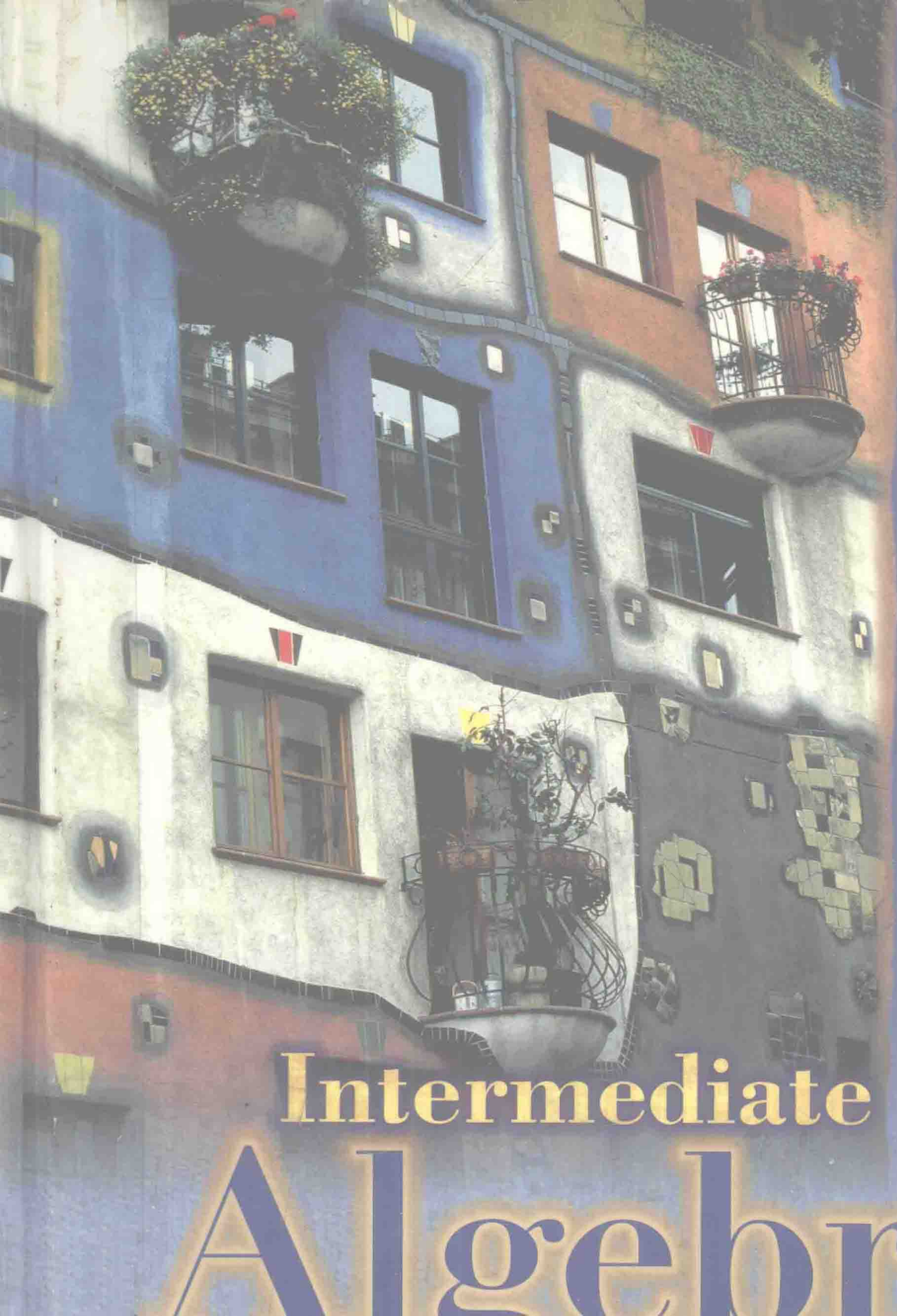

MCKEAGUE

sixth edition

Intermediate Algebra

Intermediate Algebra

SIXTH EDITION

Charles P. McKeague

Cuesta College

SAUNDERS COLLEGE PUBLISHING

A DIVISION OF HARCOURT COLLEGE PUBLISHERS

FORT WORTH PHILADELPHIA SAN DIEGO NEW YORK AUSTIN ORLANDO SAN ANTONIO

TORONTO MONTREAL LONDON SYDNEY TOKYO

Publisher: Emily Barrosse
Executive Editor: Angus McDonald
Product Manager: Julia Downs
Developmental Editor: Carol Loyd
Project Editor: Robin C. Bonner
Production Manager: Alicia Jackson
Art Director: Lisa Adamitis
Text Designer: Lisa Adamitis

Cover Credit: Vienna, Austria (©Murat Ayranci/SuperStock)

INTERMEDIATE ALGEBRA 6/e

ISBN: 0-03-026286-0
Library of Congress Catalog Card Number: 99-067247

Address for domestic orders:
Saunders College Publishing, 6277 Sea Harbor Drive, Orlando, FL 32887-6777
1-800-782-4479
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Address for international orders:
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Address for editorial correspondence:
Saunders College Publishing, Public Ledger Building, Suite 1250,
150 S. Independence Mall West, Philadelphia, PA 19106-3412

Web Site Address
http://www.harcourtcollege.com

Printed in the United States of America

9012345678 032 10987654321

PREFACE TO THE INSTRUCTOR

This sixth edition of Intermediate Algebra retains the same basic format and style as the fifth edition. The book is intended to be used in a lecture format class. Each section of the book can be discussed in a forty-five- to fifty-minute class session.

FEATURES OF THE BOOK

Early Coverage of Graphing The material on graphing equations in two variables has been moved from Chapter 5 to Chapter 3 and combined with an introduction to functions and function notation.

Early Coverage of Functions The material on functions and graphs has been rewritten for this edition. It starts in Section 3.1 with coverage of paired data, bar charts, and line graphs. Functions are introduced in Section 3.5 and function notation in Section 3.6. The concept of a function is then carried through to the remaining chapters in the book.

Using Technology Scattered throughout the book is material that shows how graphing calculators, spreadsheet programs, and computer graphing programs can be used to enhance the topics being covered. This material is easy to find because it appears in boxes under the heading "Using Technology."

Blueprint for Problem Solving The Blueprint for Problem Solving is a detailed outline of the steps needed to successfully attempt application problems. Intended as a guide to problem solving in general, the Blueprint overlays the solution process to all the application problems in the first few chapters of the book. As students become more familiar with problem solving, the steps in the Blueprint are streamlined.

Increased Visualization of Topics This edition contains many more diagrams, charts, and graphs than the previous edition. The purpose is to give the students additional information, in visual form, to help them understand the topics we cover.

Facts from Geometry Many important facts from geometry are listed under this heading. In most cases, an example or two accompanies each of the facts, to give students a chance to see how topics from geometry are related to the algebra they are learning.

Conditional Statements An introductory look at the type of reasoning that is the foundation of mathematics is contained in Section 1.5. This section gives students experience with conditional statements and their associated forms: inverse, converse, and contrapositive.

Number Sequences and Inductive Reasoning An introductory coverage of number sequences and inductive reasoning is contained in Section 1.6. This material is carried into Chapter 3 and other parts of the book. It is expanded upon in Chapter 10. I find that there are many interesting topics I can cover if the students have some experience with number sequences. It is also the easiest way to demonstrate inductive reasoning.

Unit Analysis Chapter 6 contains problems requiring students to convert from one unit of measure to another. The method used to accomplish the conversions is the method they will use if they take a chemistry class. Since this method is the method we use to multiply rational expressions, unit analysis is covered in Section 6.7 as an application of multiplication and division of rational expressions.

Getting Ready for Class Just before each problem set is a list of four problems under the heading *Getting Ready for Class.* These problems require written responses from students and can be answered by reading the preceding section. They are to be done before the students come to class.

Chapter Openings Each chapter opens with an introduction in which a real-world application is used to stimulate interest in the chapter. Most of them are explained using the rule of four: in words, numerically, graphically, and algebraically.

Study Skills Found in the first six chapters, after the opening application, is a list of study skills intended to help students become organized and efficient with their time. The study skills point students in the direction of success.

Organization of the Problem Sets Six main ideas are incorporated into the problem sets.

1. **Drill** There are enough problems in each set to ensure student proficiency in the material.
2. **Progressive Difficulty** The problems increase in difficulty as the problem set progresses.
3. **Odd-Even Similarities** Each pair of consecutive problems is similar. Since the answers to the odd-numbered problems are listed in the back of the book, the similarity of the odd-even pairs allows your students to check their work on an odd-numbered problem and then to try the similar even-numbered problem.
4. **Applying-the-Concepts Problems** Students are always curious about how the algebra they are learning can be applied, but at the same time many of them are apprehensive about attempting application problems. I have found that they are more likely to put some time and effort into trying application problems if they do not have to work an overwhelming number of them at one time and if they work on them every day. For these reasons, I have placed a few application problems in almost every problem set in the book.
5. **Review Problems** Each problem set, beginning with Chapter 2, contains a few review problems. Where appropriate, the review problems cover material that will be needed in the next section. Otherwise, they cover material from the pre-

vious chapter. That is, the review problems in Chapter 5 cover the important material from Chapter 4. Likewise, the review problems in Chapter 6 review the important material from Chapter 5. If you give tests on two chapters at a time, you will find this to be a timesaving feature. Your students will review one chapter as they study the next.

6. **Extending-the-Concepts Problems** Many of the problem sets end with a few problems under this heading. These problems are more challenging than those in the problem sets, or they are problems that extend some of the topics covered in the section.

End-of-Chapter Retrospective Each chapter ends with the following items, which together give a comprehensive reexamination of the chapter and some of the important problems from the previous chapters.

Chapter Summary This lists all main points from the chapter. In the margin, next to each topic being reviewed, is an example that illustrates that type of problem associated with the topics being reviewed.

Chapter Review The Chapter Review is a set of review problems covering the essential ideas of the chapter. Numbers in brackets refer to the section(s) in the text where similar problems can be found.

Chapter Projects The Projects include one group project, for students to work on in class, and one research project, for students to do outside of class.

Chapter Test The Chapter Test is a set of problems representative of all the main points of the chapter.

Cumulative Review Starting in Chapter 2, each chapter ends with a set of problems that reviews material from all preceding chapters. The cumulative review keeps students current with past topics and helps them retain the information they study.

CHANGES IN THE SIXTH EDITION

Chapter 1 Section 1.7 has been deleted and the material there has been distributed among Sections 1.6, 2.4, and 3.1.

Chapter 2 The material in Sections 2.4 and 2.5 is now contained in a single section, Section 2.4. Section 2.8 has been eliminated and the material there integrated into Chapter 3.

Chapter 3 Graphing in two dimensions has been moved from Chapter 5 to Chapter 3 and now includes an introduction to functions and function notation.

Chapter 4 Systems of equations have been moved from Chapter 8 to Chapter 4. The section on matrix solutions to systems of equations has been moved to the appendix.

Chapter 5 There are two fewer sections in this chapter than in the previous edition. The properties of exponents are covered in one section at the beginning of Chapter 5, instead of two sections, and special factoring and the general review of factoring have been combined into a single section.

Chapter 8 The section on solving quadratic inequalities has been moved to the end of the chapter, so that technology can be used in the solution process. The section at the end of the chapter on graphing parabolas that opened left and right has been removed from this edition.

Chapter 9 Exponential functions and inverse functions have been combined with the material on logarithms. Together they make up Chapter 9 of this edition.

Chapter 11 The material on conic sections has been moved to Chapter 11, the last chapter of the book.

Supplements to the Textbook

This sixth edition of *Intermediate Algebra* is accompanied by a number of useful supplements.

For the Instructor

- ***Printed Test Bank and Prepared Tests*** The test bank consists of multiple-choice and short-answer test items organized by chapter, section, and difficulty level. The prepared tests comprise 12 sets of ready-to-copy tests, one set for each chapter and one set for the entire book. Each set comprises 2 multiple-choice and 4 show-your-work tests. Answers for every test item are provided. Also included are 15 sample problems for each section of the text. These problems range in difficulty from easy to hard.
- ***ExaMaster+™ Computerized Test Bank*** A flexible, powerful, computerized testing system, *Examaster+*™ offers teachers a wide range of integrated testing options and features. Available in Macintosh or Windows format, it offers teachers the ability to select, edit, and create not only test items but algorithms for test items as well. Teachers can tailor tests according to a variety of criteria, scramble the order of test items, and administer tests on-line. *Examaster+*™ also includes full-function gradebook and graphing features.

For the Student

- ***Videotape Package*** Free to adopters, the videotape package consists of 11 VHS videotapes, one for each chapter of the book. Each chapter tape is an hour to an hour and a half in length and is divided into lessons that correspond to each section of the chapter. In the videotapes, I work out selected problems from the text.
- ***Core Concepts Video*** This single videotape is over 4 hours in length and contains more than 50 problem-solving sessions, covering most sections of the text. Tailor-made as a take-home tutorial for students with a demanding schedule, this video can be used as a preview of what is to be covered in class, as an aid to completing homework assignments, or as a tool to aid in review for a test.

- ***Student Solutions Manual*** This manual contains complete annotated solutions to every other odd problem in problem sets and to all chapter review and chapter test problems.

- ***MathCue Tutorial*** This computer software package of tutorials has problems that correspond to every section in the text. The software presents problems to solve and tutors students by displaying annotated, step-by-step solutions. Students may view partial solutions to get started on a problem, see continuous record of progress, and back up to review missed problems. Students' scores can also be printed. Available for Windows and Macintosh.

- ***MathCue Solution Finder*** This software allows students to enter their own problems into the computer and obtain annotated, step-by-step solutions in return. This unique program simulates working with a tutor, tracks students' progress, refers students to specific sections in the text when appropriate, and prints students' scores. Available for Windows and Macintosh.

- ***MathCue Practice*** This algorithm-based software allows students to generate large numbers of practice problems keyed to problem types from each section of the book. *Practice* scores students' performance and saves students' scores session to session. Available for Windows and Macintosh.

- ***MathCue F/C Graph*** For use with *Intermediate Algebra* 6th edition, this computer program allows students to graph and analyze any polynomial, logarithmic, exponential, or trigonometric function or conic equation they choose. *F/C Graph* can zoom, trace, display function values or coordinates of selected points, graph up to four functions simultaneously, and save and retrieve set-ups. Students can use *F/C Graph* to relate algebraic and visual forms of functions and conic sections and to explore how changing parameters affect the graph. The set of computer lab exercises accompanying the software directs student investigations of a variety of topics. Available for Windows and Macintosh.

- ***Intermediate Algebra and the Graphing Calculator: A Learning Resource*** This workbook helps both instructors and students understand how to use graphing calculators and presents exercises and exploratory investigations into graphing calculators, linear equations and inequalities, polynomials and factoring, lines and inequalities, quadratic equations, and systems of linear equations.

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ACKNOWLEDGEMENTS

A project of this size cannot be completed without help from many people. Angus McDonald, my editor at Saunders College Publishing, has been very helpful and encouraging in all parts of the revision process. Carol Loyd, my developmental editor, worked especially hard to get the book ready for production. Anne Scanlan-Rohrer and Robin Bonner handled the production part of this revision. They each did an outstanding job, and I am very lucky to have these two on our team. Judy Barclay and Denise Chellsen, my colleagues at Cuesta College, suggested many new exercises for the problem sets. Warren Hawley, the Latin School of Chicago and DePaul University, Mary Lee Seitz, Erie Community College, Mike Rosenborg, and Héctor Muñoz of Santiago, Chile, submitted many new application problems that strengthened this revision. My son, Patrick, assisted me with this revision, and his influence on the use of technology and new application problems has made this a better book. Ann Ostberg and Sandra Beken did an excellent job of accuracy checking and proofreading. I also received a number of excellent suggestions from Mark Turner, Pat Hughes, Steve Herbekian, and Rebecca Otteson, my colleagues at Cuesta College. My thanks to all these people; this book would not have been possible without them.

Thanks also to Diane McKeague and Amy Jacobs for their encouragement with all my writing endeavors.

Finally, I am grateful to the following instructors for their suggestions and comments on this revision. Some reviewed the entire manuscript, while others were asked to evaluate the development of specific topics or the overall sequence of topics. My thanks go to the following:

Susan Akers, Northeast State Technical Community College
Gerald Allen, Angelo State University
Jon Becker, Indiana University Northwest
Sandra Beken, Horry-Georgetown Technical College
Sandra Belcher, Midwestern State University
Karen Sue Cain, Eastern Kentucky University
Pat Roux, Delgado Community College
Margaret Donaldson, East Tennessee State University
Dorothy Gotway, University of Missouri — St. Louis
Cathy Hayes, University of Mobile
Josephine Johansen, Rutgers University
Harriet Kiser, Floyd College
Gene Majors, Fullerton College
Aimee Martin, Amarillo College
Annette Noble, University of Maryland — Eastern Shore
Richard Riggs, New Jersey City University
Margaret Schmid, Black Hawk College
Mark Sigfrids, Kalamazoo Valley Community College
Mark Serebransky, Camden County College
Heidi Staebler, Texas A&M University — Commerce
Katalin Szucs, East Carolina University
Radu Teodorescu, Western Michigan University
Lucy Thrower, Francis Marion University
Pete Witt, Glendale Community College
Alice Wong, Miami-Dade Community College

Charles P. McKeague — October, 1999

PREFACE TO THE STUDENT

Many of my algebra students are apprehensive at first because they are worried they will not understand the topics we cover. When I present a new topic that they do not grasp completely, they think something is wrong with them for not understanding it. On the other hand, some students are excited about the course from the beginning. They are not worried about understanding algebra, and, in fact, expect to find some topics difficult.

What is the difference between these two types of students? Those who are excited about the course know from experience (as you do) that a certain amount of confusion is associated with most new topics in mathematics. They don't worry about it because they also know that the confusion gives way to understanding in the process of reading the textbook, working the problems, and getting questions answered. If they find a topic they are having difficulty with, they work as many problems as necessary to grasp the subject. They don't wait for the understanding to come to them; they go out and get it by working lots of problems. In contrast, the students who lack confidence tend to give up when they become confused. Instead of working more problems, they sometimes stop working problems altogether, and that, of course, guarantees that they will remain confused.

If you are worried about this course because you lack confidence in your ability to understand algebra, and you want to change the way you feel about mathematics, then look forward to the first topic that causes confusion. As soon as that topic comes along, make it your goal to master it, in spite of your apprehension. You will see that each and every topic covered in this course is one you can eventually master, even if your initial introduction to it is accompanied by some confusion. As long as you have passed a college-level beginning algebra course (or its equivalent), you are ready to take this course. If you have decided to do well in algebra, the following list will be important to you:

How To Be Successful in Algebra

1. **Attend all class sessions on time.** You cannot know exactly what goes on in class unless you are there. Missing class and then expecting to find out what went on from someone else is not the same as being there yourself.

2. **Read the book.** It is the best to read the section that will be covered in class beforehand. Reading in advance, even if you do not understand everything you read, is still better than going to class with no idea of what will be discussed.

3. **Work problems every day, and check your answers.** The key to success in mathematics is working problems. The more problems you work, the better you will become at working them. The answers to the odd-numbered problems are given in the back of the book. When you have finished an assignment, be sure to compare your answers with those in the book. If you have made a mistake, find out what it is, and correct it.

4. **Do it on your own.** Don't be misled into thinking someone else's work is your own. Having someone else show you how to work a problem is not the same as working the same problem yourself. It is okay to get help when you are stuck. As a matter of fact, it is a good idea. Just be sure you do the work yourself.

5. **Review every day.** After you have finished the problems your instructor has assigned, take another fifteen minutes and review a section you have already completed. The more you review, the longer you will retain the material you have learned.

6. **Don't expect to understand every topic the first time you see it.** Sometimes you will understand everything you are doing, and sometimes you won't. That's just the way things are in mathematics. Expecting to understand each new topic the first time you see it can lead to disappointment and frustration. The process of understanding algebra takes time. It requires that you read the book, work problems, and get your questions answered.

7. **Spend as much time as it takes for you to master the material.** No set formula exists for the exact amount of time you need to spend on algebra to master it. You will find out as you go along what is or isn't enough time for you. If you end up spending two or more hours on each section in order to master the material there, then that's how much time it takes; trying to get by with less will not work.

8. **Relax.** It's probably not as difficult as you think.

Charles P. McKeague — October, 1999

FEATURES OF THE NEW EDITION

New Design and Art Program

The revamped look of this edition is more open and inviting. Featuring numerous new mathematical and situational figures, the enhanced art program will assist students in visualizing the concepts.

Equations and Inequalities in Two Variables

(© SuperStock)

INTRODUCTION

A student is heating water in a chemistry lab. As the water heats, she records the temperature readings from two thermometers, one giving temperature in degrees Fahrenheit and the other in degrees Celsius. Table 1 shows some of the data she collects. Figure 1 is a scatter diagram that gives a visual representation of the data in Table 1.

TABLE 1 Corresponding Temperatures

In Degrees Fahrenheit	In Degrees Celsius
77	25
95	35
167	75
	100

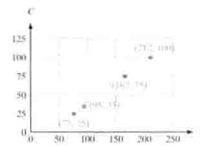

FIGURE 1

...ionship between the Fahrenheit and Celsius temperature scales is ...rmula

$$C = \frac{5}{9}(F - 32)$$

... ways to describe the relationship between the two temperature ... a graph, and an equation. But, most important to us, we don't need ... formula on faith. In Problem Set 3.3, you will derive the formula ... Table 1.

3.4 Linear Inequalities in Two Variables

A small movie theater holds 100 people. The owner charges more for adults than for children, so it is important to know the different combinations of adults and children that can be seated at one time. The shaded region in Figure 1 contains all the seating combinations. The line $x + y = 100$ shows the combinations for a full theater: The y-intercept corresponds to a theater full of adults, and the x-intercept corresponds to a theater full of children. In the shaded region below the line $x + y = 100$ are the combinations that occur if the theater is not full.

Shaded regions like the one shown in Figure 1 are produced by linear inequalities in two variables, which is the topic of this section.

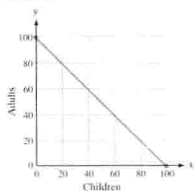

FIGURE 1

A *linear inequality in two variables* is any expression that can be put in the form

$$ax + by < c$$

where a, b, and c are real numbers (a and b not both 0). The inequality symbol can be any one of the following four: $<$, $\leq$, $>$, $\geq$.

Some examples of linear inequalities are

$$2x + 3y < 6 \qquad y \geq 2x + 1 \qquad x - y \leq 0$$

Although not all of these examples have the form $ax + by < c$, each one can be put in that form.

The solution set for a linear inequality is a *section of the coordinate plane*. The *boundary* for the section is found by replacing the inequality symbol with an equal sign and graphing the resulting equation. The boundary is included in the solution set (and is represented with a *solid line*) if the inequality symbol used originally is $\leq$ or $\geq$. The boundary is not included (and is represented with a *broken line*) if the original symbol is $<$ or $>$.

EXAMPLE 1 Graph the solution set for $x + y \leq 4$.

Solution The boundary for the graph is the graph of $x + y = 4$. The boundary is included in the solution set because the inequality symbol is $\leq$.

Figure 2 is the graph of the boundary:

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Chapter Openers Each chapter begins with a real-world application, helping students to make the connection immediately between mathematics and their everyday lives. Similar problems on a variety of subjects also appear in the problem sets and end-of-chapter retrospectives.

Basic Properties and Definitions

(Bruce Ayers/Tony Stone Images)

INTRODUCTION

The following table and diagram show how the concentration of a popular antidepressant changes over time, once the patient stops taking it. In this particular case, the concentration in the patient's system is 80 ng/ml (nanograms per milliliter) when the patient stops taking the antidepressant, and the half-life of the antidepressant is 5 days.

TABLE 1 Concentration of an Antidepressant

Days Since Discontinuing	Concentration (ng/ml)
0	80
5	40
10	20
15	10
20	5

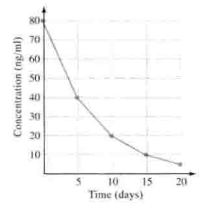


FIGURE 1 Concentration of antidepressant over time

The half-life of a medication tells how quickly the medication is eliminated from a person's system: Medications with a long half-life are eliminated slowly, whereas those with a short half-life are more quickly eliminated. Half-life is the key to constructing the preceding table and graph. When you are finished with Section 1.6 you ... e the half-life of a medication to construct the table and graph.

STUDY SKILLS

At the beginning of each of the first few chapters of this book, you will find a section like this, in which we list the skills necessary for success in algebra. If you have just completed an introductory algebra class successfully, you have acquired most of these skills. If it has been some time since you last took a math class, you must pay attention to the sections on study skills.

Here is a list of things you can do to begin to develop effective study skills.

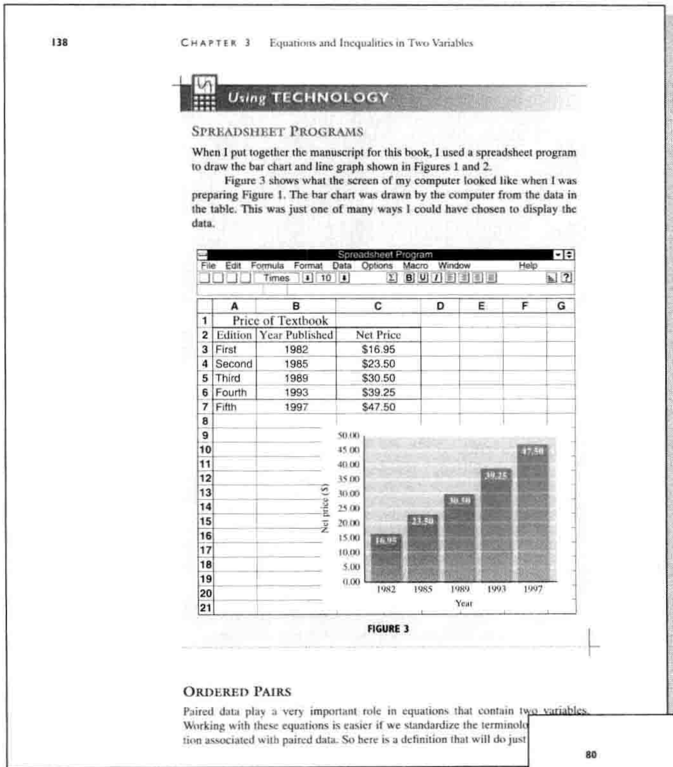
1. **Put Yourself on a Schedule** The general rule is that you spend 2 hours on homework for every hour you are in class. Make a schedule for yourself in which you set aside 2 hours each day to work on algebra. Once you make the schedule, stick to it. Don't just complete your assignments and stop. Use all the time you have set aside. If you complete an assignment and have time left over, read the next section in the book and work more problems. As the course progresses, you may find that 2 hours a day is not enough time for you to master the material in this course. If it takes you longer than 2 hours a day to reach your goals for this course, then that's how much time it takes. Trying to get by with less will not work.
2. **Find Your Mistakes and Correct Them** There is more to studying algebra than just working problems. You must always check your answers with the answers in the back of the book. When you make a mistake, find out what it is and correct it. Making mistakes is part of the process of learning mathematics. One of the number sequences we will study in this chapter is known as the *Fibonacci sequence*, after the Italian mathematician Leonardo Fibonacci (c. 1170–c. 1250). Fibonacci published a book titled *The Book of Squares* in 1225. In the Prologue he has this to say about his writing:

   > I have come to request indulgence if in any place it contains something more or less than right or necessary; for to remember everything and be mistaken in nothing is divine rather than human . . .

   Fibonacci knew, as you know, that human beings make mistakes. You cannot learn algebra without making mistakes. The key to discovering what you do not understand can be found by correcting your mistakes.
3. **Imitate Success** Your work should look like the work you see in this book and the work your instructor shows. The steps shown in solving problems in this book were written by someone who has been successful in mathematics. The same is true of your instructor. Your work should imitate the work of people who have been successful in mathematics.

2

Study Skills List of skills necessary for success in algebra; teaches students to become organized and efficient with their time. The Study Skills appear in Chapters 1 through 6.


138 CHAPTER 3 Equations and Inequalities in Two Variables

Using TECHNOLOGY

SPREADSHEET PROGRAMS

When I put together the manuscript for this book, I used a spreadsheet program to draw the bar chart and line graph shown in Figures 1 and 2.

Figure 3 shows what the screen of my computer looked like when I was preparing Figure 1. The bar chart was drawn by the computer from the data in the table. This was just one of many ways I could have chosen to display the data.

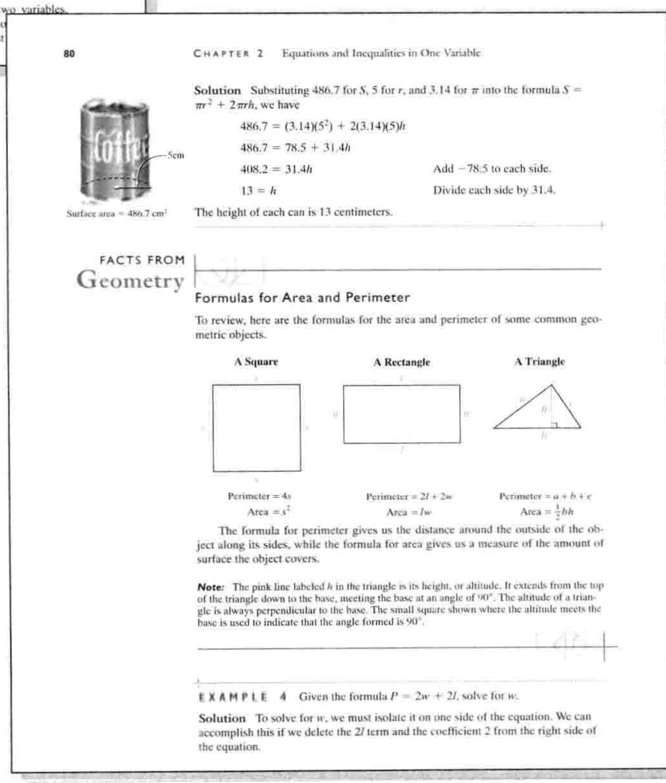
	A	B	C
1	Price of Textbook		
2	Edition	Year Published	Net Price
3	First	1982	$16.95
4	Second	1985	$23.50
5	Third	1989	$30.50
6	Fourth	1993	$39.25
7	Fifth	1997	$47.50

FIGURE 3

ORDERED PAIRS

Paired data play a very important role in equations that contain two variables. Working with these equations is easier if we standardize the terminolo... tion associated with paired data. So here is a definition that will do just...

Using Technology This material illustrates the ways in which graphing calculators, spreadsheet programs, and computer graphing programs can be used to enhance the topics covered. An icon makes the information easy to locate.


80 CHAPTER 2 Equations and Inequalities in One Variable

Solution Substituting 486.7 for S, 5 for r, and 3.14 for π into the formula $S = \pi r^2 + 2\pi rh$, we have

$$486.7 = (3.14)(5^2) + 2(3.14)(5)h$$
$$486.7 = 78.5 + 31.4h$$
$$408.2 = 31.4h \qquad \text{Add } -78.5 \text{ to each side.}$$
$$13 = h \qquad \text{Divide each side by } 31.4.$$

The height of each can is 13 centimeters.

FACTS FROM Geometry

Formulas for Area and Perimeter

To review, here are the formulas for the area and perimeter of some common geometric objects.

A Square	A Rectangle	A Triangle
Perimeter = $4s$	Perimeter = $2l + 2w$	Perimeter = $a + b + c$
Area = s^2	Area = lw	Area = $\frac{1}{2}bh$

The formula for perimeter gives us the distance around the outside of the object along its sides, while the formula for area gives us a measure of the amount of surface the object covers.

Note: The pink line labeled h in the triangle is its height, or altitude. It extends from the top of the triangle down to the base, meeting the base at an angle of 90°. The altitude of a triangle is always perpendicular to the base. The small square shown where the altitude meets the base is used to indicate that the angle formed is 90°.

EXAMPLE 4 Given the formula $P = 2w + 2l$, solve for w.

Solution To solve for w, we must isolate it on one side of the equation. We can accomplish this if we delete the $2l$ term and the coefficient 2 from the right side of the equation.

Facts from Geometry Important facts and associated examples are included under this heading, illustrating to students how geometry relates to the algebra they are learning.

Getting Ready for Class New element designed to reinforce the concepts students have learned by reading the discourse, prior to attending class. Over 250 new writing exercises appear throughout the book under this heading. Students are asked to write on various topics using their own words, thus enhancing their understanding and helping to alleviate math anxiety.

EXAMPLES Let $A = \{1, 3, 5\}$, $B = \{0, 2, 4\}$, and $C = \{1, 2, 3, \ldots\}$. Then

12. $A \cup B = \{0, 1, 2, 3, 4, 5\}$
13. $A \cap B = \varnothing$ (A and B have no elements in common)
14. $A \cap C = \{1, 3, 5\} = A$
15. $B \cup C = \{0, 1, 2, 3, \ldots\}$

Another notation we can use to describe sets is called *set builder* notation. Here is how we write our definition for the union of two sets A and B using set builder notation:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The right side of this statement is read "the set of all x such that x is a member of A or x is a member of B." As you can see, the vertical line after the first x is read "such that."

EXAMPLE 16 If $A = \{1, 2, 3, 4, 5, 6\}$, find $C = \{x \mid x \in A \text{ and } x \geq 4\}$.

Solution We are looking for all the elements of A that are also greater than or equal to 4. They are 4, 5, and 6. Using set notation, we have

$$C = \{4, 5, 6\}$$

Getting Ready for Class

Each section of the book will end with some problems and questions like the ones that follow. They are for you to answer after you have read through the section, but before you go to class. All of them require that you give written responses, in complete sentences. Writing about mathematics is a valuable exercise. If you write with the intention of explaining and communicating what you know to someone else, you will find that you understand the topic you are writing about even better than you did before you started writing. As with all problems in this course, you want to approach these writing exercises with a positive point of view. You will get better at giving written responses to questions as you progress through the course. Even if you never feel comfortable writing about mathematics, just the process of attempting to do so will increase your understanding and ability in mathematics.

After reading through the preceding section, respond in your own words and in complete sentences.

A. What is the meaning of the expression 2^3?
B. Why do we have a rule for the order of operations?
C. What is the intersection of two sets of numbers?
D. Explain the operations that are associated with the words *sum, difference, product,* and *quotient.*

PROBLEM SET 3.7

For the following problems, y varies directly with x.

1. If y is 10 when x is 2, find y when x is 6.
2. If y is 20 when x is 5, find y when x is 3.
3. If y is -32 when x is 4, find x when y is -40.
4. If y is -50 when x is 5, find x when y is -70.

For the following problems, r is inversely proportional to s.

5. If r is -3 when s is 4, find r when s is 2.
6. If r is -10 when s is 6, find r when s is -5.
7. If r is 8 when s is 3, find s when r is 48.
8. If r is 12 when s is 5, find s when r is 30.

For the following problems, d varies directly with the square of r.

9. If $d = 10$ when $r = 5$, find d when $r = 10$.
10. If $d = 12$ when $r = 6$, find d when $r = 9$.
11. If $d = 100$ when $r = 2$, find d when $r = 3$.
12. If $d = 50$ when $r = 5$, find d when $r = 7$.

For the following problems, y varies inversely with the square of x.

13. If $y = 45$ when $x = 3$, find y when x is 5.
14. If $y = 12$ when $x = 2$, find y when x is 6.
15. If $y = 18$ when $x = 3$, find y when x is 2.
16. If $y = 45$ when $x = 4$, find y when x is 5.

For the following problems, z varies jointly with x and the square of y.

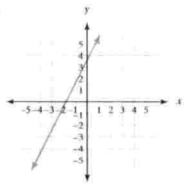
17. If z is 54 when x and y are 3, find z when $x = 2$ and $y = 4$.
18. If z is 80 when x is 5 and y is 2, find z when $x = 2$ and $y = 5$.
19. If z is 64 when $x = 1$ and $y = 4$, find x when $z = 32$ and $y = 1$.
20. If z is 27 when $x = 6$ and $y = 3$, find x when $z = 50$ and $y = 4$.

Applying the Concepts

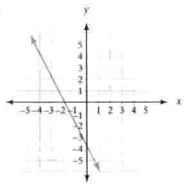
21. **Length of a Spring** The length a spring stretches is directly proportional to the force applied. If a force of 5 pounds stretches a spring 3 inches, how much force is necessary to stretch the same spring 10 inches?
22. **Weight and Surface Area** The weight of a certain material varies directly with the surface area of that material. If 8 square feet weighs half a pound, how much will 10 square feet weigh?
23. **Pressure and Temperature** The temperature of a gas varies directly with its pressure. A temperature of 200 K produces a pressure of 50 pounds per square inch.
    (a) Find the equation that relates pressure and temperature.
    (b) Graph the equation from part (a) in the first quadrant only.
    (c) What pressure will the gas have at 280 K?
24. **Circumference and Diameter** The circumference of a wheel is directly proportional to its diameter. A wheel has a circumference of 8.5 feet and a diameter of 2.7 feet.
    (a) Find the equation that relates circumference and diameter.
    (b) Graph the equation from part (a) in the first quadrant only.
    (c) What is the circumference of a wheel that has a diameter of 11.3 feet?
25. **Volume and Pressure** The volume of a gas is inversely proportional to the pressure. If a pressure of 36 pounds per square inch corresponds to a volume of 25 cubic feet, what pressure is needed to produce a volume of 75 cubic feet?
26. **Wave Frequency** The frequency of an electromagnetic wave varies inversely with the wavelength. If a wavelength of 200 meters has a frequency of 800 kilocycles per second, what frequency will be associated with a wavelength of 500 meters?
27. ***f*-Stop and Aperture Diameter** The relative aperture, or *f*-stop, for a camera lens is inversely proportional to the diameter of the aperture. An *f*-stop of 2 corresponds to an aperture diameter of 40 millimeters for the lens on an automatic camera.
    (a) Find the equation that relates *f*-stop and diameter.
    (b) Graph the equation from part (a) in the first quadrant only.

Problem Sets Just the right number of drill problems is included in each problem set to ensure student proficiency in the material. The problems are progressive in level of difficulty, and each pair of consecutive problems is similar.

33.



34.



35. Give the slope and y-intercept of $y = -2$. Sketch the graph.
36. For the line $x = -3$, sketch the graph, give the slope, and name any intercepts.
37. Find the equation of the line parallel to the graph of $3x - y = 5$ that contains the point $(-1, 4)$.
38. Find the equation of the line parallel to the graph of $2x - 4y = 5$ that contains the point $(0, 3)$.
39. Line l is perpendicular to the graph of the equation $2x - 5y = 10$ and contains the point $(-4, -3)$. Find the equation for l.
40. Line l is perpendicular to the graph of the equation $-3x - 5y = 2$ and contains the point $(2, -6)$. Find the equation for l.
41. Give the equation of the line perpendicular to the graph of $y = -4x + 2$ that has an x-intercept of -1.
42. Write the equation of the line parallel to the graph of $7x - 2y = 14$ that has an x-intercept of 5.
43. Find the equation of the line with x-intercept 3 and y-intercept 2.
44. Find the equation of the line with x-intercept 2 and y-intercept 3.

Applying the Concepts

45. **Deriving the Temperature Equation** The table below resembles the table from the introduction to this section. The rows of the table give us ordered pairs (C, F).

Degrees Celsius C	Degrees Fahrenheit F
0	32
25	77
50	122
75	167
100	212

(a) Use any two of the ordered pairs from the table to derive the equation $F = \frac{9}{5}C + 32$.

(b) Use the equation from part (a) to find the Fahrenheit temperature that corresponds to a Celsius temperature of 30°.

46. **Maximum Heart Rate** The table gives the maximum heart rate for adults 30, 40, 50, and 60 years old. Each row of the table gives us an ordered pair (A, M).

Age (years) A	Maximum Heart Rate (beats per minute) M
30	190
40	180
50	170
60	160

(a) Use any two of the ordered pairs from the table to derive the equation $M = 220 - A$, which gives the maximum heart rate M for an adult whose age is A.

Applying the Concepts These problems are designed to show students how the algebra they are learning can be applied to real-life situations. Over 400 new application problems have been added, and approximately 100 of these emphasize reading and interpreting graphical and tabular data, in keeping with AMATYC standards.

Factor the right side of this equation, and then find h when t is 6 seconds and when t is 3 seconds. [Find $h(6)$ and $h(3)$.]

78. **Height of an Arrow** An arrow is shot into the air with an upward velocity of 16 feet per second from a hill 32 feet high. The equation that gives the height of the arrow at any time t is

$$h(t) = 32 + 16t - 16t^2$$

Factor the right side of this equation, and then find h when t is 2 seconds and when t is 1 second. [Find $h(2)$ and $h(1)$.]

Review Problems

The following problems review material we covered in Section 5.3. Reviewing these problems will help you with the next section.

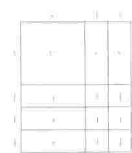
Multiply.

79. $(2x - 3)(2x + 3)$
80. $(4 - 5x)(4 + 5x)$
81. $(2x - 3)^2$
82. $(4 - 5x)^2$
83. $(2x - 3)(4x^2 + 6x + 9)$
84. $(2x + 3)(4x^2 - 6x + 9)$

Extending the Concepts

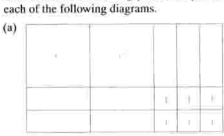
Factor completely.

85. $8x^6 + 26x^3y^2 + 15y^4$
86. $24x^4 + 6x^2y^3 - 45y^6$
87. $3x^2 + 295x - 500$
88. $3x^2 + 594x - 1{,}200$
89. $\frac{1}{8}x^2 + x + 2$
90. $\frac{1}{9}x^2 + x + 2$
91. $2x^2 + 1.5x + 0.25$
92. $6x^2 + 2x + 0.16$
93. **Factoring and Area** The following area model gives us a way to visualize the factorization of the trinomial $x^2 + 5x + 6$. Construct a similar diagram that will allow you to visualize the factorization of $x^2 + 5x + 4$.

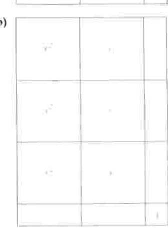


94. **Factoring and Area** Refer to the area model for factoring trinomials mentioned in the preceding exercise. Write the factoring problem represented by each of the following diagrams.

(a)



(b)



Extending the Concepts Many problem sets end with a group of these challenging exercises. Unlike any of the previous problems students have encountered in the chapter, these questions prompt them to synthesize ideas, thus extending their understanding of the topics.