MATHEMATICS for MANAGEMENT

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$$\therefore Q^* = \sqrt{\frac{2SD}{I}}$$

Mathematics for Management

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Preface

The last few years have witnessed both a rapid growth in the number of courses offered by universities and polytechnics in the fields of management, economics, and business studies, and an increasing emphasis within these areas on quantitative analysis and the development of numerate skills. Even so, most such degree courses accept students with very diverse backgrounds in mathematics, and first-year classes may well include several students with a recent good 'A' level pass, and others with an average 'O' level taken several years previously. Introductory courses in mathematics thus need to bring the less well qualified student up to the standard of the more advanced students in selected topics, as well as develop the latter's capabilities more in the direction of mathematics applied to subjects less technical than the physical sciences. This text is based on the first-year course in mathematics offered to undergraduate students at the University of Bradford Management Centre, and was developed in the first instance specifically for this course. Despite this particular origin, however, the level of the text is appropriate for recommendation to students with at least mathematics at 'O' level standard, following courses in Business Studies, Economics, Finance, and related social sciences with some quantitative input. Material is also included which should prove useful for understanding the mathematics involved in introductory courses in statistics in the same subject areas (although the book does not attempt to provide a full course in basic statistics, which is better left to a separate specialist volume). Likewise at the postgraduate level, the text is appropriate for those students following MBA, MSc, or research programmes in management whose academic backgrounds have been in non-quantitative subjects.

In line with the usual structure of many degree courses, this text aims to help the student acquire and develop one of the basic analytical skills (mathematics) early in the course, which would then be exercised in relation to appropriate classes of problems (such as operational research or economics) in a subsequent stage of the degree programme. The emphasis is therefore on helping the social science type student become a better mathematician, the techniques covered being selected with a view to those types of application likely to be faced subsequently. This emphasis on developing mathematical skills is in contrast to many texts where the mathematical ability of the reader is assumed, and is exercised to greater or lesser degree on classes of problems, rather than being explicitly developed as skills.

The applied nature of the subject is emphasized in this text through the choice of worked and set examples which run in parallel with the main mathematical techniques discussed. After the introductory chapter, which reviews basic mathematical notation, the text proceeds through five main classes of techniques:

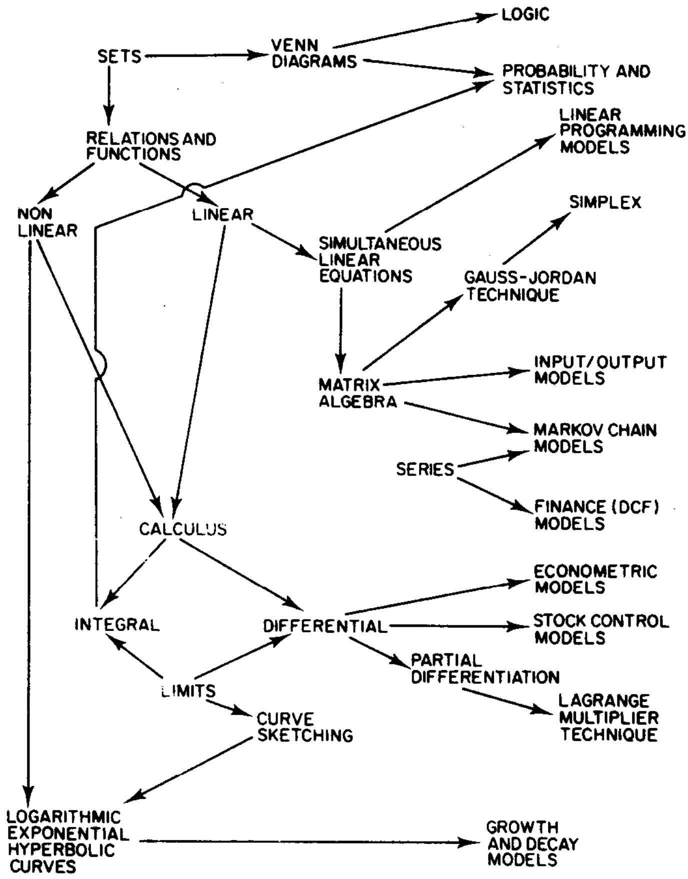
1. Basic set theory, leading into relations, functions, and graphs.
2. Linear relationships.
3. Curvilinear relationships, leading to the development of the differential and integral calculus.
4. Matrix algebra, which is developed from a second look at simultaneous linear equations.
5. Mathematical series.

This material is developed through examples of some applications, so for example:

1. An introduction to linear programming, using graphical representation, is presented after the section on linear relationships.
2. Several aspects of mathematical models are discussed in conjunction with the section on nonlinear relationships.
3. Various discounting techniques in finance are explained and developed from first principles in the section on mathematical series.
4. The theme of optimization under constraints, common to several applications, is emphasized in order to demonstrate their element of similarity.

The chart illustrates the many connections within the material covered. The various 'pure' mathematical inputs toward the left-hand side of the diagram are employed in the applications listed towards the right-hand side.

Besides worked examples within the text, three further sets of examples are included. Simpler examples for the student are included at the end of each chapter (with answers provided at the end of the book); intermediate problems, suitable for tutorial work, also follow each chapter (without answers); and more advanced problems are included at the end of each of the six main sections of the book, the solutions to this latter group being explained more fully, and providing in effect an extension to the material of the main text.


LOGIC
SETS
VENN DIAGRAMS
PROBABILITY AND STATISTICS
RELATIONS AND FUNCTIONS
LINEAR PROGRAMMING MODELS
NON LINEAR
LINEAR
SIMPLEX
SIMULTANEOUS LINEAR EQUATIONS
GAUSS-JORDAN TECHNIQUE
MATRIX ALGEBRA
INPUT/OUTPUT MODELS
MARKOV CHAIN MODELS
SERIES
FINANCE (DCF) MODELS
CALCULUS
ECONOMETRIC MODELS
INTEGRAL
DIFFERENTIAL
STOCK CONTROL MODELS
PARTIAL DIFFERENTIATION
LIMITS
CURVE SKETCHING
LAGRANGE MULTIPLIER TECHNIQUE
LOGARITHMIC EXPONENTIAL HYPERBOLIC CURVES
GROWTH AND DECAY MODELS

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PART ONE

Introduction

This introductory chapter begins by summarizing the basic algebraic rules and notation which will be used and assumed throughout the remainder of the text. Although it should be adequate as a basis for revision, it is deliberately very concise, and any reader who finds himself having great difficulties with this section is advised to refer in the first instance to an introductory text on algebra and arithmetic.

Subsequent sections address themselves to the task of developing the readers' commonsense in spotting errors and avoiding illegal mathematical operations. Aspects of approximation are covered, as well as intuitive or geometric reasoning to assist in the manipulation of algebraic formulae.

CHAPTER 1

Basic Notation

The phrase 'the language of mathematics' suggests that a mathematical expression can, in principle, be translated into an English language form. Or, putting it another way, the underlying logic of a mathematical operation can be expressed either by means of symbols (i.e. usual mathematical notation) or by a statement in English (or Chinese, or French, etc.). However, mathematical notation is

(a) Brief. Complicated operations in terms of logic can be expressed very succinctly.
(b) Universal. A unique language, with one set of rules or grammar which is universally accepted.
(c) Rigorous. English can sometimes be vague, or ambiguous. A mathematical expression should only have one possible interpretation.

For example, the following would be extremely difficult to write in English, and would certainly not be brief

$$3ab\sqrt{xy}/[(\log x + 6 + e^2)^{-2}]^{1/3} + \sum_{i=4}^{7} i^2 \geqslant 6 \times 10^{-8} \sum_{i=2}^{5} x_i.$$

Firstly, therefore, we must understand the conventions of the notation, and how an expression like that above should be interpreted.

1.1 ARITHMETIC OPERATIONS

The four basic operations in arithmetic are addition, subtraction, multiplication, and division, which are written in symbol form as +, −, ×, and /. When combining several operations, the multiplication and division should always be carried out before addition and subtraction, i.e. × and / before + and −. Otherwise, operations are performed in sequence from left to right; for example,

$$5 \times 12 + 9 - 3 \times 2/6 + 7 = 60 + 9 - 6/6 + 7 = 60 + 9 - 1 + 7 = 75.$$

1.2 BRACKETS

When brackets, () or [ ] etc., appear in a statement, all operations inside the bracket are performed *before* those outside the bracket; for example,

$$5 \times (6 - 8) + 4/2 = 5 \times (-2) + 4/2 = -5 \times 2 + 4/2 = -10 + 2 = -8.$$

When brackets are *nested*, the innermost set of operations is carried out first, for example,

$$(3 \times (5 - 2) + 7 \times (8 - 4))/2 = (3 \times 3 + 7 \times 4)/2 = (9 + 28)/2$$
$$= 37/2 = 18{\cdot}5.$$

1.3 MODULUS SIGN

The modulus sign is used to mean *the positive value of*, and is used in the following way (note the similar precedence to brackets):

$$|-6| = 6$$
$$|-3(4) + 1| = |-12 + 1| = |-11| = 11.$$

1.4 ALTERNATIVE NOTATIONS

The multiplications sign ($\times$) is not always used. It can be replaced by a full stop type dot (.), thus:

$$3 \times 4 = 3.4 = 12.$$

When brackets are employed, or alphabetic letters are used, then it may be omitted altogether.

$$3 \times 4 = 3.4 = 3(4) = 12 \quad \text{(but } \textit{not} \text{ 34)}$$

However,

$$y \times z = y.z = yz = y(z)$$

or

$$2 \times y = 2.y = 2y = 2(y).$$

The division sign is often written as $\div$, rather than /.

1.5 EQUALITIES AND INEQUALITIES

If we want to put a statement such as *the weight of an apple is 110 g* into mathematical notation then we might first define a quantity x in the following way:

Let x = weight of an apple in grams, then $x = 110$.

This is an example of an *equality*, and if we operate on both sides of the