

MODERN PHYSICS MONOGRAPH SERIES

# NONRELATIVISTIC MECHANICS

Robert J. Finkelstein

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# Nonrelativistic Mechanics

Robert J. Finkelstein

*University of California, Los Angeles*



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# Nonrelativistic Mechanics

of the book

*Modern Physics Monograph Series*

FELIX M. H. VILLARS, EDITOR

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ROBERT J. FINKELSTEIN

*Nonrelativistic Mechanics, 1973*

ROBERT T. SCHUMACHER

*Introduction to Magnetic Resonance: Principles and Applications, 1970*

*In Preparation:*

HELLMUT J. JURETSCHKE

*Crystal Physics*

## EDITOR'S FOREWORD

Education in physics is going through a phase of rapid evolution. On the frontier of the field new information is literally pouring in, new perspectives are opening up, and new concepts are emerging. For the student, the distance to be covered from freshman year to graduate research work appears to be ever expanding.

Professional education in physics therefore must deal with the very real problem of the need for thoughtful condensation of the material presented, and with the question of what may and should reasonably be achieved in the years between the introductory and the research level.

It is generally agreed, on the one hand, that a thorough presentation of the fundamentals of both classical and quantum physics is essential. On the other hand, there is the understandable desire to let the student participate in the excitement offered by the many interesting new developments in all fields of physics. The discussion of such new topics gives the student an opportunity to see the actual growth process of science: new experiments, new techniques, and the attempts to relate new results to existing or emerging theoretical views. The study of the well established, traditional subjects of physics appears to lack these exciting aspects, and to offer little room for the display of creativity, except as historical fact.

It has at last been recognized that this need not entirely be so; that, in fact, the close ties between the traditional and the modern can be exploited to establish contacts between the classical subjects and current endeavors: classical mechanics and space navigation, wave optics and radar interferometry, or holography, astrophysical applications of classical electromagnetism and hydrodynamics, statistical mechanics as applied to biopolymers, or phase transitions, and so forth. To develop such links wherever they exist, and to put the essential parts of the traditional subjects into a modern perspective is an urgent and rewarding task.

On the undergraduate level, the recent burgeoning of such introductory texts as the Feynman lectures, the Berkeley physics course, and the Massachusetts Institute of Technology introductory physics texts bear witness to the interest that has been aroused by the problem of bringing the fundamentals of physics to the undergraduate in a novel way. This new series of MODERN PHYSICS MONOGRAPHS intends to continue this process at the more advanced level. It will present material for the upper level undergraduate and introductory graduate courses. At this level, there will be, on the one hand, courses of a specialized nature, with the purpose of giving the student an introduction to the great diversity of topics of physical science, from particle physics to nuclear, atomic, solid state, plasma and astrophysics, while, on the other hand, the traditional topics of the undergraduate sequences will be deepened and extended, and their interrelations more clearly established. We hope that the MODERN PHYSICS MONOGRAPH series will help to give the lecturer in the field additional flexibility in choosing his course material and, if he is inclined to experiment, allow him to introduce into his course topics not generally covered in standard textbooks. In addition, the student will have access to a variety of collateral reading material.

For these very reasons, the books of this series are not intended to be textbooks, but rather monographs; that is, works that cover a more restricted area in a space of approximately 200 to 400 pages. They contain problems with and without answers, and could either supplement existing texts or be used in groups as a replacement for a single text.

The present volume, a compact and lucid exposition of the basis structure of both classical and quantum mechanics, in their nonrelativistic version, is a case in point. The

intriguing and profound parallels between classical and quantum-mechanical concepts (often given short shrift in "regular" texts), are here brought to the surface. The Feynman path integral, the Schwinger action principle, and their relation to Schroedinger's formulation are illuminated. Any student of classical and quantum mechanics will find this volume a storehouse of new insights and information.

Felix M. H. Villars

Cambridge, Massachusetts  
January 1973



## PREFACE

This book is based on a course that was designed to integrate the traditional graduate presentation of classical mechanics with an introductory treatment of quantum mechanics. For a number of years this course has been successfully given, in one quarter, to first year graduate students at UCLA. As prerequisites the student was expected to have taken undergraduate courses in both classical and quantum mechanics. Therefore, although the present book is introductory in nature, it is not intended as a first introduction to either classical or quantum mechanics.

The plan of the book is as follows. The first three chapters are devoted to general theory and the last two are concerned with applications. The first two chapters attempt to bring together the assumed prerequisite courses in classical and quantum mechanics in the framework of the underlying invariance group and its associated algebra. The development of the general theory that is begun in the first two chapters is worked out and concluded in the third. Chapter 4 deals with the motion of rigid bodies and Chapter 5 with the orbits of point particles. These last two chapters are tied together in terms of the general theory by making some use of the representation theory of the rotation group. The book is meant to be largely self-contained; however, the reader, depending on his preparation, may have additional work to do.

In combining the classical and quantum formalisms, greater weight must necessarily be given to quantum theory since that is the fundamental discipline. Classical theory unfortunately suffers in such an integration. In addition, although

the relativistic generalization of classical mechanics would not be difficult and would be desirable at this point in the curriculum, the corresponding extension of quantum theory would present much more difficult questions, and we have accordingly decided to limit this book to nonrelativistic theory. Therefore there is less classical theory than one would perhaps like, but the clear advantages of an integrated course seem to us to be decisive. Finally a course in classical mechanics dealing with nonlinear mechanics and other modern topics would complete this program in a logical way.

I should like to thank my colleagues, Ernest Abers, Herbert Fried, David Saxon, and Roscoe White, as well as many students, particularly Joel Kvitky, John Mouton, Darwin Swanson, and Jan Smit, for helpful comments on the manuscript.

I should also like to thank Mr. Ronald Bohn for his skillful preparation of the typescript.

Robert J. Finkelstein

February, 1973  
Los Angeles, California

# Nonrelativistic Mechanics

## **MODERN PHYSICS MONOGRAPH SERIES**

**FELIX M. H. VILLARS, Editor**

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### **Introduction to Magnetic Resonance**

**Principles and Applications**

This book, the first to discuss magnetic resonance at the undergraduate level, was developed as a supplement for courses in atomic, nuclear, or solid state physics, and as a reference for laboratory courses. It emphasizes semiclassical explanations of the characteristic phenomena of magnetic resonance, and illustrates the phenomena with specific examples taken from atomic, nuclear, and solid state physics, and chemistry. The juxtaposition of the characteristic features of magnetic resonance and their diverse applications relates theory to practice. Some of the problems following each chapter emphasize quantitative calculations, while others extend the theory to more advanced levels.

The wide range of subjects treated and the references to further work in each area make the book suitable to a broad audience. "Introduction to Magnetic Resonance: Principles and Applications" also provides an excellent introduction to standard graduate level texts.

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## CHAPTER 1

### HAMILTONIAN FORMULATIONS (CLASSICAL THEORY)



#### 1.1 INTRODUCTION

Most phenomena encountered in our normal experience appear to obey the laws of classical physics, but only because quantum effects are relatively unimportant in these situations. The exact description is always quantal, as far as we now know; and classical physics is only an approximate formalism which provides a good description in limiting situations according to the Bohr correspondence principle.

For example, it is believed that the motion of the earth about the sun is governed by quantum mechanics just as completely as the motion of an electron in the hydrogen atom.



In the planetary case, however, quantum corrections to the classical formulas are negligible while in the atomic case they are dominant. On the other hand one should not give the impression that quantum mechanics is important only in atomic and subatomic systems. There are well-known examples of macroscopic quantum systems: for example, superfluids, which are very cold, and white dwarf and neutron stars which are very dense.<sup>1</sup> As rough tests one may estimate  $h/S$ , the ratio of Planck's constant to an action which characterizes the phenomenon under consideration, or  $\lambda/R$  where  $\lambda$  is a quantum mechanical wave length and  $R$  is a characteristic linear dimension.

Quantum effects are important when these ratios are not small. For example

$$\frac{\lambda}{R} = \frac{h}{MVR}$$

is negligible when  $M$  and  $V$  refer to the mass and velocity of the earth, and  $R$  to its distance from the sun; but the same ratio becomes large in atoms. It also becomes large in macroscopic systems at low temperatures ( $V$  is small) at high densities ( $R$  is small) or even at normal densities and temperatures when  $M$  is small (electron gas in metals).

In this book an arbitrary physical system is described