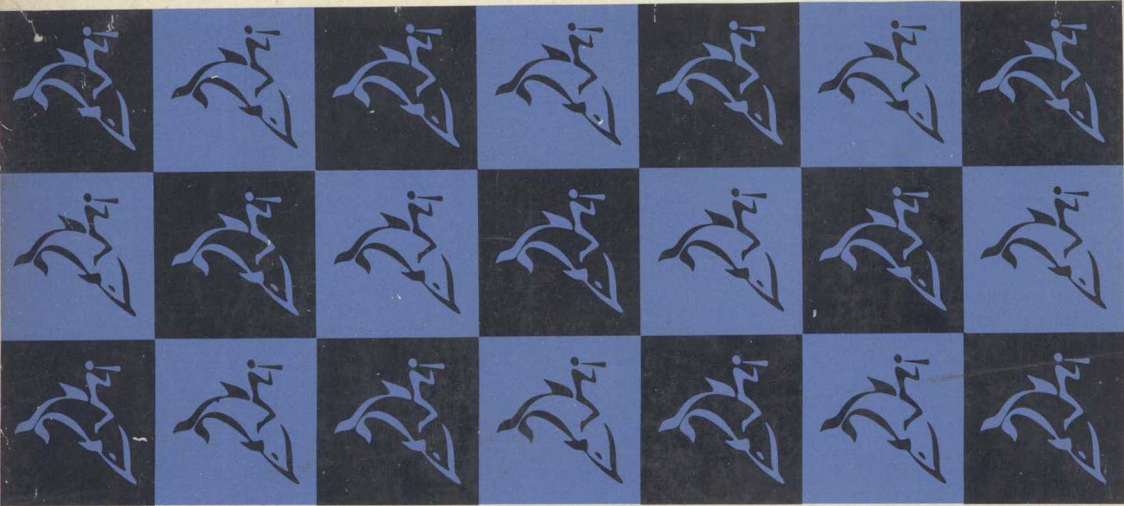
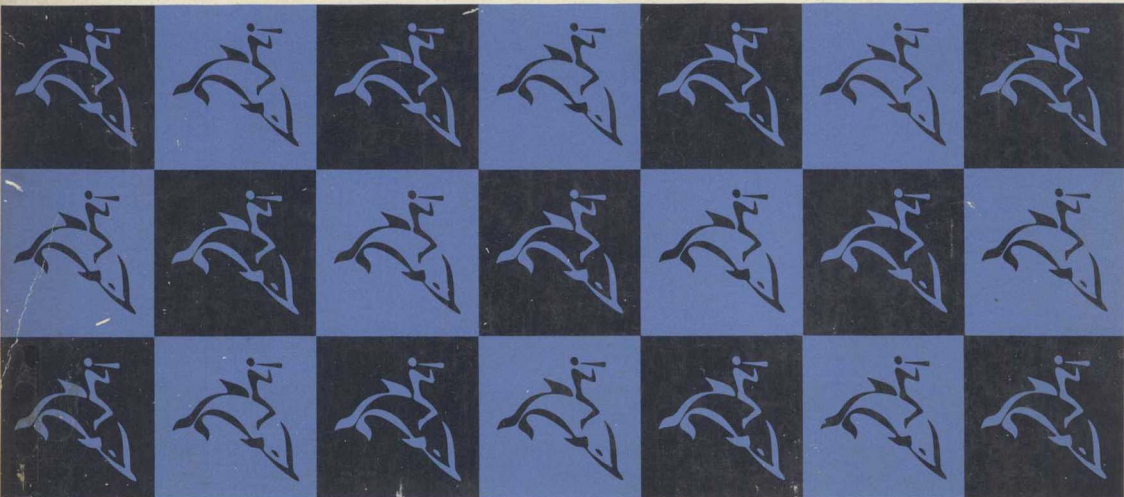


# Business Statistics

Daniel/Terrell



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# **BUSINESS STATISTICS**

**Basic Concepts and Methodology**

**Wayne W. Daniel**

**James C. Terrell**

*Georgia State University*

**Houghton Mifflin Company Boston**

*Atlanta Dallas Geneva, Illinois*

*Hopewell, New Jersey Palo Alto London*

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## PREFACE

*Business Statistics: Basic Concepts and Methodology* has been written as a text for use either in a single statistics course or in a two-course sequence. When used in a single course, the first seven chapters should be covered. If time permits, additional chapters or sections of chapters may be covered on a selective basis at the instructor's discretion. When used in a two-course sequence, the entire text can be covered. However, certain instructors may want to skip or delete certain chapters or sections in order to place more emphasis on others, and thus tailor the material covered to meet the needs of his students. The book was written to facilitate this approach as much as possible.

After a general discussion on the nature and usefulness of statistics, the reader is introduced to descriptive statistics in which we present the concepts of central tendency and dispersion and methods for the summarization and presentation of large collections of data.

The chapters on probability and probability distributions, which follow next, have been kept as concise as possible in the belief that, while these concepts are essential to an understanding of statistical inference, they should not be overemphasized. To make this material as relevant as possible, we have avoided the coin-tossing, card-drawing, and dice-rolling clichés in favor of real-world illustrations from the field of business.

We have gone to great pains to make the next chapter, which deals with sampling distributions, clear and easy to understand, since a grasp of the concepts of sampling distributions provides the key to an understanding of statistical inference. The basic

concepts of statistical inference are presented in Chapters 6 and 7, which deal, respectively, with estimation and hypothesis testing.

The remaining chapters are devoted to more advanced topics in statistics, in the sense that they depend on the ideas presented in the first seven chapters. The topics covered are analysis of variance, simple linear regression, multiple regression, chi-square analysis, nonparametric methods, time series analysis and index numbers, sample survey methods, statistical decision theory, and quality control. These topics provide the models, techniques, and concepts that the business student will need in his other courses and, later, in his everyday work experience.

We feel that a statistics book, in order to meet our stated objectives, should be data oriented and illustrated with examples and exercises that are as realistic as possible. Consequently, we have tried to avoid sacrificing realism for oversimplicity. Our examples and exercises, though fictitious, have an air of authenticity about them that should capture and maintain the interest and enthusiasm of the reader. We have been generous with these in order to give the reader sufficient practice in applying the concepts and techniques that have been presented. We have also used illustrations from a variety of business situations to demonstrate the wide applicability of statistical methods. With this goal in mind, we have included numerous worked-out examples and exercises.

In those chapters where we felt it would serve a useful pedagogical purpose, we have provided exercises both at the end of individual sections and at the end of the chapter. The instructor may wish to assign some of the exercises from the end of the chapter, along with the exercises at the end of a section, to provide maximum reinforcement of the material that has just been covered; or he may wish to save the end-of-chapter exercises until all of the material in the chapter has been covered. This latter procedure will give the student practice in selecting the appropriate technique to solve a particular problem, since this is not as obvious for problems at the end of the chapter as it is for those immediately following the section in which the technique is discussed. This procedure will therefore pose a greater challenge to the student.

As a further aid and challenge to the reader, we have included at the end of each chapter a number of review questions. These questions have been designed to measure the reader's grasp of the major concepts presented in the chapter. We have also included a glossary of important symbols and abbreviations at the end of each chapter. This feature will enable the reader to locate a needed symbol or abbreviation quickly without having to refer to previous chapters. Finally, we have provided the answers to the odd-numbered exercises.

Although the ability to work out numerical examples is useful in developing an understanding of statistical concepts, we have tried to avoid the "cook-book" approach and the necessity of engaging in mathematical gymnastics merely for the sake of mathematics. In each example and exercise our goal has been to help the student attain a piece of information that will serve as an aid in solving the problem. We have tried at all times to be practical rather than theoretical, and to emphasize understanding by intuition rather than mathematical proof. We have, therefore, planned the book so that the level of mathematics required is only that of college algebra.

Throughout the book the reader will encounter references to other books and journal articles. These references, basically, are of three types: (1) those that refer

the reader to a real example of an *application* of a particular technique, (2) those that provide for a *more complete coverage* of a particular topic, and (3) those that provide for a *more advanced treatment* of a particular topic. The references may be used in one of two ways: (1) they may be assigned by an instructor as outside reading, or (2) they may be consulted by the reader who is by nature inquisitive and wants more information in regard to a specific problem or concept.

We would like to express our appreciation to Mrs. Mary Daniel who typed the various drafts of the manuscript, to Professor Pickett Riggs who prepared the solutions manual, and to Professor Geoffrey Churchill who wrote the computer program to calculate the Poisson probabilities appearing in Appendix Table C.

An early draft of the manuscript was reviewed by Professors R. F. Byrne, Myron K. Cox, Thomas R. Hawk, Leon Jay Gleser, L. T. Shiflett, and Kenneth Dunning who made valuable comments and suggestions. Professor Marvin J. Karson, Professor Charles F. Mott, and Professor Fred Steir read the manuscript in detail and offered many specific suggestions for improvement.

Wayne W. Daniel

James C. Terrell

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# THE ROLE OF STATISTICS IN THE DECISION-MAKING PROCESS

## 1.1 Introduction

As man moves further and further into the scientific age, his world is becoming increasingly complex, and both his needs for information and the quantity of information available continue to expand rapidly. One of the major tasks facing the manager or researcher in almost every field of endeavor is to plan carefully, so that the quantity and quality of information obtained will be adequate to meet his needs. The manager will find the techniques and concepts of a *management information system* appropriate for this purpose. The appropriate approach for the researcher is the utilization of the *scientific method*.

A well-planned management information system enables a business firm to determine and examine its informational needs in perspective. That is, the importance of each need is evaluated relative to the overall operation of the firm. To be effective, a management information system requires operational personnel who are skilled in a wide array of quantitative techniques. Even more important to the success of a management information system is a quantitatively oriented management.

The scientific method is characterized by objectivity, inductive reasoning, and a systematic examination and measurement of facts. The accumulation of facts is followed by the formulation of a theory, or hypothesis, which may be modified later as additional facts are collected.

The ultimate objective of the manager and the researcher is to assemble information of sufficient quantity and quality to provide a basis for making sound decisions.

In the utilization of both a management information system and the scientific method, *the person trained in statistics can make an important contribution.*

## 1.2 The Role of Statistics in Decision Making

Statistics may be described as the technology of the scientific method, consisting of a set of tools that are used to facilitate the making of decisions whenever conditions of uncertainty prevail. These tools have great utility in numerous fields of endeavor other than business, for example, biology, medicine, agriculture, psychology, and education. While certain fields require special techniques, the same basic principles and concepts apply to all fields of application. It must be emphasized that statistics is a set of tools whose proper use will *aid* in decision making. Only rarely should these tools be used as the sole basis for making decisions. Statistics presents the decision-maker with relevant facts and, in many cases, provides an estimate of the probability of making a wrong decision. In the modern business world, the concepts, techniques, and results of statistics are indispensable components of the decision-making process.

The ease and rapidity of using statistical methodology have been immeasurably improved by the electronic computer. Because it accurately makes large numbers of complex calculations in seconds, it has made commonplace the use of statistical methodology that was previously impractical.

## 1.3 Basic Principles and Concepts of Special Studies

Much of the information that serves as the basis for decision making within a firm is generated routinely in everyday operations. On occasion, however, routinely available data do not provide an adequate foundation on which to base an important decision. In such circumstances it is necessary to obtain the needed information in a nonroutine manner. The situation may require the collection of additional data or the implementation of a special research project. In the discussion that follows, both nonroutine data-gathering projects and research projects will be referred to as *special studies* or, simply, *studies*. A business firm context will be assumed.

In the execution of special studies, the statistician cannot merely apply statistical techniques; he also must be concerned with the appropriateness and quality of the data involved. The objectives of each study will determine the data needed, the quality requirements of the data, and the particular technique or combination of techniques that will be used in analyzing the data. Since the reason for conducting a study is to fulfill certain objectives, the study should be designed and carried out to meet those objectives as efficiently and effectively as possible. In view of the self-evident truth of this statement, one may wonder why so many studies fail to achieve their objectives. One reason is that most studies are more complex than they appear. Studies require several phases of planning as well as several phases of execution. Each phase must be handled thoroughly and in proper sequence if the study is to be effective. All phases are interrelated, and problems encountered during one phase often require changes in other phases. Thus, even a very carefully designed study may require extensive revisions when unforeseen problems arise. In fact, some studies must be abandoned because of difficulties that either are recognized in the planning phases or arise during the execution phases. It is preferable to recognize

potential problems during the planning phases so that they can be handled by the study design or, if necessary, so the study can be dropped.

When any study is proposed, two questions must be answered: (1) "Can the study be of real value?" and (2) "Is the study feasible?" If the answers are both affirmative, then it must be decided if the study is more desirable than alternative studies that may be equally appropriate. In determining the potential value of a proposed study, the criterion that should be used is the extent of the contribution that the study can make in supplying data needed to meet the firm's goals. Too many studies have been conducted where it should have been apparent that the most thoroughly planned and executed study would be of little value.

In practice, how does one determine the value and feasibility of a study? The first step is to obtain a clear and unambiguous statement of the study objective or objectives. The statement of objectives should be supported by documentation showing why the study is needed and how the results will be used. At this point, the question of the potential value of the study has been answered. If the need for the study and its potential value are accepted, the feasibility and practicality of the proposed study must be determined. The following questions must be answered: (1) Is it logically possible to conduct the study so as to achieve its objective(s)? (2) Are required data available, or can they be obtained with reasonable effort? (3) Will the needed resources—personnel, equipment, and money—be available to the study? (4) Will the study be of sufficient value to the firm to warrant the utilization of these resources?

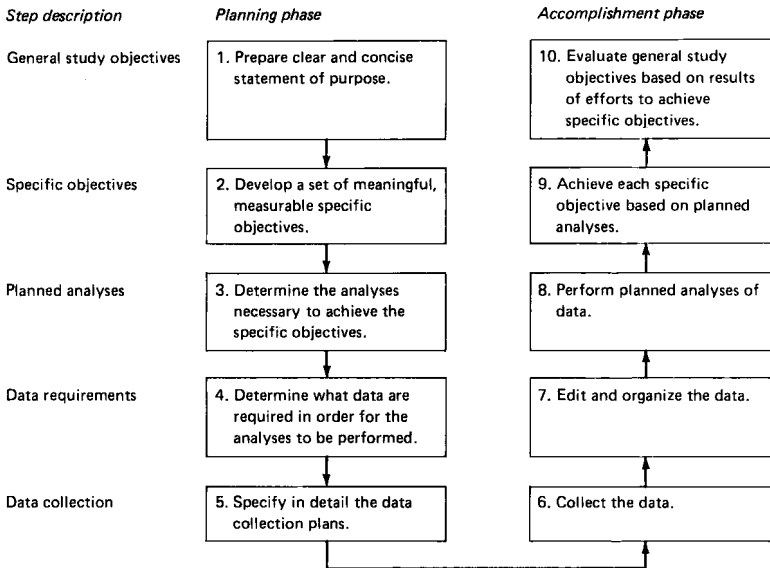
A thorough examination of proposed studies will insure that sound studies are undertaken and that poor studies are discarded before the firm has invested significant resources in them. It must be realized that not all proposed studies are worthwhile; the sooner poor study proposals are recognized, the better. Moreover, once a study is undertaken it should not necessarily be followed through to completion. In spite of the most careful planning, problems may arise during the progress of the study that will prevent the study objectives from being achieved fully. The effect of such problems must be carefully evaluated, and attempts must be made to salvage these studies. Often, the original objectives or a meaningful set of revised objectives may be achieved. However, the discontinuation of a study may be warranted if it is determined that the results that can be achieved will not be of sufficient value to the firm.

## 1.4 Steps Involved in Planning and Conducting Special Studies

This section will present a sequence of steps intended to transform a proposal into a well-designed and well-executed study. The steps include a systematic application of the principles discussed in the preceding section. In fact, the steps that comprise the planning phase may prove very useful in determining the feasibility of a proposed study. These steps should be viewed as a set of recommended procedures, not as inflexible rules. They are intended to provide a logical, systematic approach to meeting the need for "planning before acting." They represent a way of insuring that objectives will be accomplished with a minimum of effort without prohibiting the investigation of leads and hunches that might result in altering the dimensions of the study.

The planning and execution of a study may be viewed as consisting of 10 steps as shown in Figure 1.4.1. The steps may be divided into a planning phase and an

FIGURE 1.4.1 Flow Chart for Planning and Executing a Study



accomplishment phase, each consisting of 5 steps. Alternatively, they may be viewed as 5 steps, each consisting of a planning phase and an accomplishment phase.

Ideally, the planning phase should be completed for all 5 steps before the accomplishment phase begins. Planning is conducted in a step-by-step sequence from the statement of purpose to the specification of plans for data collection. The accomplishment phase is conducted in the reverse order, beginning with the collection of data. In each phase, each step is determined by the steps that precede it. Each step also helps determine the steps that follow.

The ability to revise is an essential part of the planning phase. It is an essential but, hopefully, infrequently used part of the accomplishment phase. For example, during the planning phase, it may become apparent that the data needs of the study cannot be met and, therefore, the planned analyses cannot be accomplished. This means that the planned analyses must be revised to utilize obtainable data which still provide information for achieving the specific objectives. If the necessary revision cannot be made, either the specific objectives must be revised, some method must be devised to obtain the required data, or the study must be dropped.

Studies that cannot be successfully carried out should be identified and dropped during the planning phase. It is a shame not to do studies that need to be done and can be done with a reasonable effort. However, it is perhaps even worse to conduct a study that is unsuccessful, simply because of poor planning.

The chapters that follow will introduce a number of basic statistical techniques and concepts. It is hoped that the reader will gain sufficient mastery of the material so that when confronted with a decision-making situation requiring a knowledge of statistics, he will be able to make a positive contribution. If this contribution consists of no

more than recognizing that the problem requires a higher level of statistical expertise than he possesses, the time spent studying the material of this text will not have been wasted.

## Summary

This chapter was concerned with the business firm's increased need for quality information. Three sources of information were identified: (1) the routine operation of the firm, (2) special data-gathering projects, and (3) special research projects. Sources (2) and (3) were referred to as special studies and were discussed in considerable detail. A step-by-step procedure for conducting a special study was suggested. The importance of statistics to the manager or researcher seeking to meet a firm's informational needs was emphasized.

There is available a wealth of material that delves deeper into the general considerations and more philosophical aspects of special studies. The books mentioned below are a sampling of those with which the authors of this text are most familiar.

A general treatment of the scientific method is given by Walker (1963), Ackoff (1962), and Kerlinger (1964). Business research in general is the subject of the books by Ferber and Verdoorn (1962), Nemmers and Myers (1966), Roberts (1964), Murdick (1969), and Rummel and Ballaine (1963). For a discussion of research in the field of marketing, see Green and Tull (1970). For coverage of research in the area of production management, see the books by Gedye (1965), Johnson et al. (1972), Starr (1971), Levin et al. (1972), and Buffa (1969).

The chapters that follow are concerned primarily with two areas: (1) the analysis of data resulting from special studies and routine operations and (2) the concepts on which these analyses are based.



