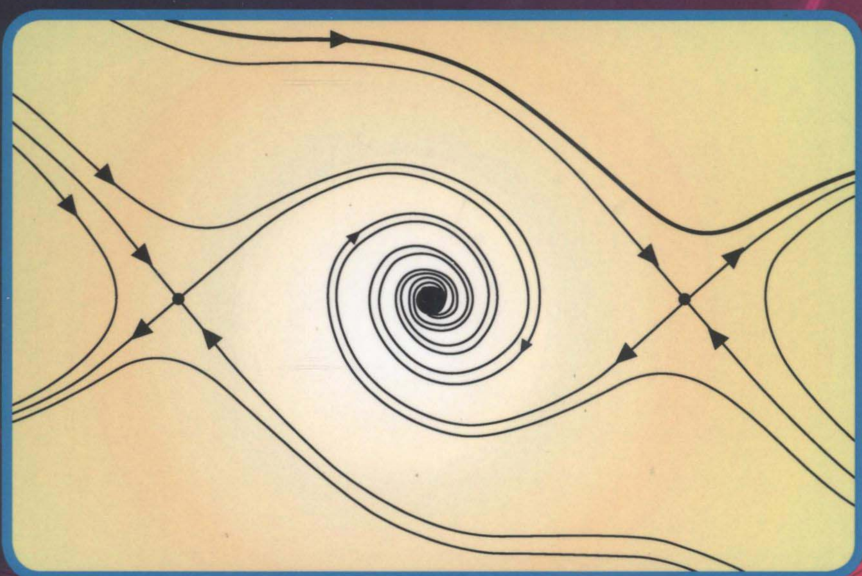
Nonlinear Systems Stability Analysis

Lyapunov-Based Approach



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Preface

The dynamic properties of a physical system can be described in terms of ordinary differential, partial differential, and difference equations, or any combination of these subjects. In addition, the systems can be time-varying, time-invariant and/or time-delayed, and continuous or discrete systems. These equations are often nonlinear in one way or the other and it is rarely possible to find their solutions. Numerical solutions for such nonlinear dynamic systems with an analog or digital computer are impractical. This is due to the fact that a complete solution must be carried out for every possible initial condition in the solution space. Graphical techniques, which can be employed for finding the solutions for special cases of first- and second-order ordinary systems, are not useful tools for other types of systems as well as higher-order ordinary systems. However, there are different theorems and methods concerning existence, uniqueness, stability, and other properties of nonlinear systems and/or their solutions. Among these qualitative properties, the stability of a given system is the most crucial systems issue. Without the guaranteed stability, the system will be of no value.

Many researchers have worked on stability robustness analysis for different systems. For a good list of these studies, one may read chapter five of sensitivity analysis by Eslami (e1). The aim of this book is to introduce some advanced tools for stability analysis of nonlinear systems. Toward this end, first, standard stability techniques are discussed with the shortcomings highlighted; then some recent developments in stability analysis are introduced, which can improve the applicability of standard techniques. Finally, stability analysis of special classes of nonlinear systems, for example, time-delayed systems and fuzzy systems, are proposed.

This book is organized as follows: In the first chapter, the stability of ordinary time-invariant differential equations will be considered. In Chapter 2, Lyapunov stability analysis will be studied. The subject of the third chapter is time-invariant systems. Chapter 4 deals with time-delayed systems. The stability analysis of fuzzy linguistic systems models is considered in Chapter 5.

This book is intended for graduate students of all disciplines who are involved in stability analysis of dynamic systems.

S.K.Y. Nikravesh

September 2010

(50th anniversary of the establishment of Amirkabir University of Technology [AUT])

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Contents

1 Basic Concepts

Introduction: In this chapter, the stability analysis of a system, the dynamics of which are represented in time domain by nonlinear time-invariant ordinary differential equations, is considered. This chapter consists of the following subsections:

1.1 Mathematical model for nonlinear systems.
1.2 Qualitative behavior of second-order linear time-invariant systems (LTI).

1.1 MATHEMATICAL MODEL FOR NONLINEAR SYSTEMS

A nonlinear system may mathematically be represented in the following form:

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t), \\
\dot{x}_2 &= f_2(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t), \\
\vdots \quad & \quad \vdots \\
\dot{x}_n &= f_n(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t),
\end{aligned} \tag{1.1}$$

where $\dot{x}_i,\ i = 1, 2, \ldots, n$ denotes the derivative of x_i (the ith state variable) with respect to the time variable t and u_j, $j = 1, 2, \ldots, m$ denote the input variables. Equation (1.1) could be written in the following state-space form:

$$\dot{x} = f(x, u, t), \tag{1.2}$$

where,

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{pmatrix} \quad \text{and} \quad f(x, u, t) = \begin{pmatrix} f_1(x, u, t) \\ f_2(x, u, t) \\ \vdots \\ f_n(x, u, t) \end{pmatrix}.$$

The measurable outputs (a p-dimensional vector) are functions of the states, the inputs, and the time such that:

$$\begin{aligned}
y_1 &= h_1(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t), \\
y_2 &= h_2(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t), \\
\vdots \quad & \quad \vdots \\
y_p &= h_p(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m, t).
\end{aligned} \tag{1.3}$$

or, in the following general form:

$$y = h(x,u,t). \tag{1.4}$$

Equations (1.2) and (1.4) together are called the *mathematical dynamic equations*, or:

$$\begin{aligned} \dot{x} &= f(x,u,t), \\ y &= h(x,u,t). \end{aligned} \tag{1.5}$$

These equations could be simulated using operational amplifiers (integrators) and function generators as shown in Figure 1.1 (a) and (b).

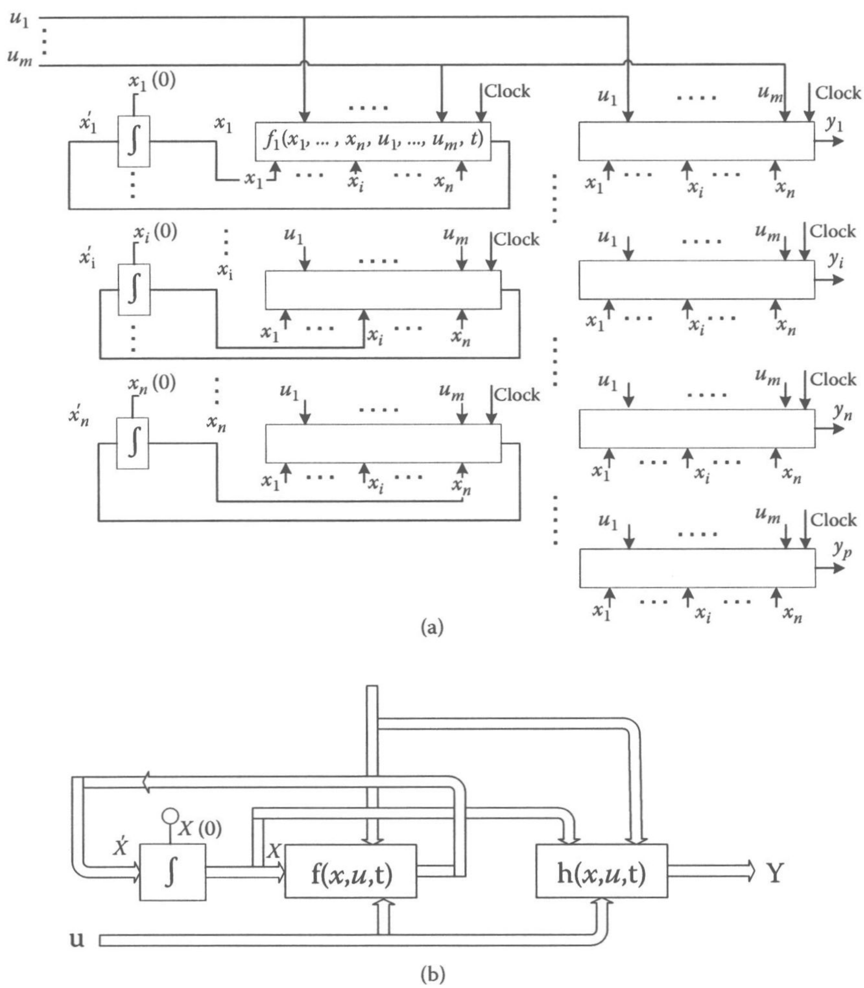

FIGURE 1.1 System dynamic simulation.

It seems the dynamic systems could be simulated to obtain their responses, having signal generators f_i and h_i. However, there is a drawback with this approach, since for each initial condition the simulation must be repeated. To have the actual response of dynamic systems, (1.2) and (1.4), the system must be at least locally Lipschitz in $x, \forall\, x \in D \subset R^n$ and continuous in t, for every t.

Throughout this book, wherever this type of dynamic equation occurs, the satisfaction of these conditions is assumed. The Lipschitz conditions are discussed shortly in this chapter.

Although in theory, the simulation could be proposed as a solution for the stability analysis, it is impractical or impossible, since in nonlinear system studies, every initial condition should be used.

Special Cases: If a system is a feedback system, then the system's inputs would be functions of the states, thus:

$$u \triangleq g(x,t). \tag{1.6}$$

Substituting (1.6) into (1.5) yields the following unforced dynamic equations:

$$\dot{x} = f(x,u,t) = F(x,t) \triangleq f(x,t), \;\; y = h(x,u,t) = H(x,t) \triangleq h(x,t), \tag{1.7a}$$

If the dynamic system (1.7a) is time invariant, then the system is called an *autonomous* (either *forced* or *unforced*) system.

$$\begin{aligned} \dot{x} &= f(x,u), \;\; \text{or} \;\; f(x) \\ y &= h(x,u), \;\; \text{or} \;\; h(x). \end{aligned} \tag{1.7b}$$

If the linearization technique is used in dynamic equations (1.5) or (1.7b), then linear time-varying (1.8) or linear time-invariant (forced or unforced) (1.9) equations yield:

$$\begin{aligned} \dot{x}_n &= \left(\frac{\partial f}{\partial x} \Big|_{\substack{x0 \\ u0}} \right) x_n + \left(\frac{\partial f}{\partial u} \Big|_{\substack{x0 \\ u0}} \right) u_n \triangleq A(t)x_n + B(t)u_n, \\ y_n &= \left(\frac{\partial h}{\partial x} \Big|_{\substack{x0 \\ u0}} \right) x_n + \left(\frac{\partial h}{\partial u} \Big|_{\substack{x0 \\ u0}} \right) u_n \triangleq C(t)x_n + D(t)u_n, \end{aligned} \tag{1.8}$$

or:

$$\begin{aligned} \dot{x}_n &\triangleq Ax_n + Bu_n, \\ y_n &\triangleq Cx_n + Du_n. \end{aligned} \tag{1.9}$$

The index "n" stands for new variable. Note that (1.8) or (1.9) can only predict the local behavior of the nonlinear system of (1.5) or (1.7), respectively.

1.1.1 Existence and Uniqueness of Solutions [k1]

The existence and uniqueness of the solution of (1.6) are given by the following theorem.

Theorem 1.1:

Let *f(x,t)* be a single valued continuous function in a region defined by $|x_i - x_i(o)| < h_i,\ i = 1,2,\dots,n$ and $o \le t - t_1 < T$ in which $|f(x,t)| < M$ for some $o < M < \infty$, and t_1 is the domain of piecewise continuity of *f(x,t)*. If *f(x,t)* satisfies the following Lipschitz condition in *x*:

$$\|f(x_1,t) - f(x_2,t)\| \le L\|x_1 - x_2\| \quad ,o < L < \infty,$$

$$\forall x_1, x_2 \in B = \{x \in R^n \, \|x - x_o\| \le r\}, \forall t \in (t_o, t_1), \quad r > o,$$

then there exists some $\delta > o$ such that the state equation $\dot{x} = f(x,t)$ with $x(t_o) = x_o$ has a unique solution over $[t_o, t_o + \delta]$; $\delta = \min(T, \frac{h_i}{M})$. ■

When n = 1 and *f(x)* is autonomous, then the Lipschitz condition implies,

$$\frac{|f(x_1) - f(x_2)|}{|x_1 - x_2|} \le L,$$

that is, in a plane of *f(x)* versus *x*, a straight line joining any two points of *f(x)* cannot have a slope with absolute value greater than *L*. Therefore, a discontinuous function is not locally Lipschitz at the points of discontinuity.

More generally, if for $t \in I \subset R$ and $x \in D \subset R^n$, *f* (*x*,t) and its partial derivatives $\partial f_i / \partial x_j$ are continuous, then *f* (*x*,t) is locally Lipschitz in *x* on *D*. *f*(*x*,t) is globally Lipschitz in *x* if and only if (iff) $\frac{\partial f_i}{\partial x_j}$ are globally uniformly bounded in *t*.

Example 1.1:

Note that $\dot{x} = f(x) = x^{1/3}$ is not locally Lipschitz, at $x = o$ since:

$$f'(x) = \frac{1}{3}x^{-2/3} \to \infty \quad as\ x \to 0 \quad x(t) = \left(\frac{2t}{3}\right)^{\frac{3}{2}} \quad \text{and } x(t) = 0$$

are the two different solutions for this differential equation, when the initial state is

$$x(0) = 0.$$

■