

Donald J. Albers
Gerald L. Alexanderson
William Dunham
EDITORS

THE G. H. HARDY READER

CAMBRIDGE



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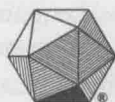
The G. H. Hardy Reader

edited by

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CAMBRIDGE
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Cambridge University Press
University Printing House
Cambridge
CB2 8BS
United Kingdom
www.cambridge.org

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Library of Congress Control Number: 2015948223

Paperback ISBN: 978-1-10759-464-7

Hardcover ISBN: 978-1-10713-555-0

Printed in the United States of America

Current Printing (last digit):

10 9 8 7 6 5 4 3 2 1

The G. H. Hardy Reader

Cambridge University Press

Cambridge

CB2 3RU

United Kingdom

www.cambridge.org

0 521 87622 3

Donald J. Alderson

Cambridge University Press

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0 521 87622 3

The following description of the book is taken from the

introductory note to the edition for the Press of the

Library of Congress Control Number: 2012-48222

ISBN 978-1-107-12522-0

ISBN 978-1-107-12522-0

Printed in the United States of America

Cambridge University Press

0 521 87622 3

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Overview

G. H. Hardy (1877–1947) ranks among the great mathematicians of the twentieth century. He was, as well, a colorful, mildly eccentric individual with expository skills of the first order.

Hardy was a product of the English educational system as it existed in the decades before 1900. He attended the prestigious Winchester College and then “went up” to Trinity College, Cambridge, where he distinguished himself as Fourth Wrangler and as winner of the Smith’s Prize—honors that meant a lot in turn-of-the-century Britain, even if they leave modern readers somewhat puzzled. From there, his career took him to professorships at Cambridge and Oxford and to visiting positions at Princeton and Cal Tech.

Mathematically, Hardy is regarded as one of history’s most accomplished analytic number theorists, although his papers ranged across other topics like divergent series, integration, and the theory of partitions. Throughout his career, he knew a Who’s-Who of scholars, from Norbert Wiener (his student), to George Pólya (his co-author), to Bertrand Russell (his Cambridge colleague), to John Maynard Keynes (his friend). Of course, he is best remembered for his extraordinarily fruitful collaborations with J. E. Littlewood and Srinivasa Ramanujan. Clearly, G. H. Hardy kept good company.

His mathematical papers, in seven volumes, were published by Oxford University Press in the 1960s and 1970s. But there seemed to be the need for a book that gave a sense of Hardy, the man. For years, Don Albers of the Mathematical Association of America campaigned for just such a work. Everyone whom he approached agreed that it would be a worthy undertaking, but no one volunteered to do it.

Consequently, Don modified his vision from a full-blooded biography to a “reader” that would allow Hardy to speak for himself. Hardy was, after all, a gifted writer with a distinctive voice that could be inspirational, funny, or caustic—sometimes all at once. Accompanying his words would be material

written by present-day authors to provide the necessary introductions and transitions. When Albers pitched his idea to the two of us—Alexanderson and Dunham—we thought we'd give it a try. This book is the result.

Our volume is divided into five main parts. We begin with our own Hardy biography, where we quote liberally from the man himself, especially from his classic, *A Mathematician's Apology*. In addition, we were fortunate to get the rights to reprint the Epilogue from Robert Kanigel's biography of Ramanujan, *The Man Who Knew Infinity* (Scribners, 1991).

Next comes a section called, 'Writings by and about G. H. Hardy.' As the title suggests, we have selected polemics, quips, anecdotes, and other passages from Hardy and from those who knew him. In their subject matter, these roam far and wide, but we think they provide a better look at Hardy and his times.

Our third section focuses on Hardy's mathematics. Here we have selected pieces that he wrote for general mathematical audiences rather than for narrow specialists. For instance, we included his survey articles on the theory of numbers, on geometry, and even a tongue-in-cheek item titled, "A Mathematical Theorem on Golf." On a more technical level, we prepared our own accounts of four of Hardy's mathematical gems, ranging from his proof of a peculiar inequality to his introduction of what is now called the Hardy-Weinberg Law of genetics.

The fourth section contains some of Hardy's tributes to other mathematicians. These pieces—written as memorials to those he knew—have a personal dimension not found in the standard obituary.

Hardy was a regular reviewer of mathematics books, and our last section contains some of his reviews. When Hardy was enthusiastic about a book, his comments could soar; when he was not, he could be strikingly harsh. Either way, his reviews make for great reading.

Three words of warning: First, writings collected from so many different sources inevitably introduce repetition. Our authors did not coordinate with one another, so certain stories, phrases, and quotations—especially the famous ones—show up here more than once. We believed, however, that it was important to keep the original writings intact, so we shall risk the repetition.

Second, we are aware that certain statements in the book do not reflect current mathematical knowledge. Again, we chose to retain the original wording rather than try to update all such items (e.g., former conjectures that are now theorems).

Finally, Hardy regularly used what we today regard as sexist language. Many passages, taken literally, suggest that only males studied mathematics or that only males were in the professoriate. Of course, Hardy knew

better; after all, he mentored both Mary Cartwright and Olga Taussky. But such was the custom of the time.

As we assembled this book, the three of us were continually reminded that G. H. Hardy was a brilliant and unfailingly interesting character. This should be evident in the pages that follow.

Donald J. Albers
Gerald L. Alexanderson
William Dunham

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I

Biography

Can you name a twentieth-century English intellectual who has appeared just as clearly in a popular novel as in a serious one? A historical figure, and a modern-day hero? Better yet, can you name one who was also led by a famous author to having written? . . . In his own time, Hardy was one of the most perceptive English intellectuals.

It is wonderful, hardly thinkable, that Hardy was not. He left deep footprints, both as a scholar and as a writer, and he is a worthy example of the English intellectual from a century ago.

Like the way the novel is Edward Lewis's *The Golden Thread*, the play is *The Unpleasantness at the Bellamys*, the film is *The Man Who Knew Too Much*, the history is Kenneth Robinson's *The Case of the Philanthropist*, and the famous author is C. E. Spence, whose education for Hardy's private life knows no bounds.

It is a pity that has been written about Hardy, his own memoirs, a *Memorial*, and a *Biography*, a series of books. It is a pity that only reflecting upon his life as a creative philosopher. It is a pity, a personal document, written at an age when writing was a departed kind of the world. It is not clear what he has to say about his own life. The essay is preceded by a moving introduction from the aforementioned world and the story, C. E. Spence, who knew Hardy well when they were at Cambridge together. Their friendship continued up to the time of Hardy's death in 1942.

Although a *Memorial* for a century has its merits, it is certainly not a work Hardy used to describe it in the opening page. It stands as an unvarnished testament for a century as a creative art. In it Hardy found "pure" mathematics, which was for him, the only kind that required the of nature recognized the personal side of the subject as it applied to our world and this, but even here his power was lost. He searched for a more authentic form of truth, after all, the ordinary person of English culture is in