Brownian Ratchets

From Statistical Physics to Bio and Nano-motors

David Cubero and Ferruccio Renzoni

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DAVID CUBERO

University of Seville

FERRUCCIO RENZONI

University College London

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To our families

Preface

This book is about Brownian ratchets, devices that rectify microscopic fluctuations. It was written with two purposes in mind. First, to introduce new readers to the field. To this is devoted the first part of the book, which treats, with as few technicalities as possible, the general ideas as well as set the broad range of physical systems where the concept of ratchets may be of relevance. The second purpose of the book is to address researchers already active in the field, by providing them with a single source for a general and detailed theoretical analysis of ratchets in the different regimes (classical vs. quantum, stochastic vs. deterministic) as well as with a coverage of experimental realizations with different physical systems, each highlighting a specific unique feature of ratchets.

No attempt has been made to provide the reader with an exhaustive list of references. There are excellent review papers, cited in the book, that already serve this purpose. We limit our reference list to the material we used the most, and to some historically important references which, despite their age, are still irreplaceable.

The book is divided into three parts. Part I covers, at an informal level, much of the material that will then be examined in depth in the rest of the book. As such, it can be used by the reader with a broad interest in the topic, but no specific need to go any deeper. This part starts with a historical overview of the Brownian ratchets. Definitions, basic ideas and fundamental concepts are introduced: from the very definition of Brownian ratchet, to the fundamental limitations imposed by the second law of thermodynamics. The historically important Brillouin paradox and Feynman ratchets are reviewed, and used to highlight the main ideas behind the concept of ratchets. This part continues by examining three fundamental models of ratchet devices, used to illustrate the general operation of a Brownian ratchet, and how directed motion is related to two fundamental requirements: out-of-equilibrium settings and symmetry breaking. This part concludes with an overview of the general relevance of the concept of ratchets. It is shown that the concept applies to very diverse fields, from biological systems to game theory.

Part II offers a detailed theoretical anlysis of ratchets. The relationship between symmetry and transport is examined in detail, thus formalizing a necessary condition, symmetry breaking, for the generation of directed motion. The concept of ratchets is also extended beyond the initial formulation in terms of rectification of Brownian motion. Noiseless (deterministic) ratchets are considered, as well as systems where Brownian (white) noise is replaced by Lévy noise. Quantum ratchets, both in the Hamiltonian and in the dissipative regime, are discussed and their unique features highlighted. Finally, a quantitative analysis of the operation of a ratchet is provided in terms of efficiency and coherency.

Part III examines experimental realizations of the ratchet effect in a variety of systems. Ratchets for colloidal particles, cold atom ratchets and solid-state ratchets are discussed. The aim is not to produce a complete list of experiments, already available in review articles. Instead, we tried to report on the different unique features of ratchets, which were highlighted by experiments with different systems. This part is concluded with a discussion of bio-inspired molecular motors. The relationship of ratchets with biological systems is an interesting one. On the one hand, ratchet mechanisms explain some intriguing dynamics at the molecular level. On the other hand, observed biological mechanisms have inspired the design of *artificial* Brownian motors.

In the last few years the authors were fortunate to have the opportunity to speak with many pioneers of the field. We thank all of them for all that we learnt from them, and in particular: Sergej Flach, Peter Hänggi and Fabio Marchesoni.

One author (FR) would like to express a special thank you to his first two PhD students, Michele Schiavoni and Ralf Gommers, with whom he started his long journey in the world of Brownian ratchets. He is also grateful to Sergey Denisov for being always available for many endless discussions on everything to do with ratchets and chaos. The list of people deserving a special thank you would not be complete without Eric Lutz, whose infinite passion for Lévy distributions was a constant source of inspiration.

Another author (DC) would like to thank Jesús Casado Pascual, for introducing him to the fascinating world of ratchets and rectifiers.

Last but not least, a big thank you to our children Marco Cubero, and Caterina Kukua and Ruggero Kwesi Renzoni, who managed to cheer us up every time we realized that everything we wrote had to be rewritten.

Contents

Part I

Historical overview and early developments

1

Limitations imposed by the second law of thermodynamics

Brownian ratchets are devices that rectify microscopic fluctuations, thus producing useful work out of a fluctuating environment. The term "Brownian ratchet" highlights two important elements required for the rectification of fluctuations to take place. First, a fluctuating environment should be present, and indeed the term "Brownian" refers to the archetype of a fluctuating environment: Brownian motion, that is the zig-zag motion of a grain of pollen in a fluid, or more generally the random motion of small particles as the result of the multiple collisions with the molecules surrounding them. The term "ratchet" highlights the second requirement for the rectification of fluctuations: the presence of appropriate asymmetries in the system, so to define a preferential direction of motion. However, there is a third important requirement for the implementation of a Brownian ratchet: the system has to be out of thermal equilibrium. In fact, for systems at equilibrium, the second law of thermodynamics prevents the generation of directed motion out of unbiased fluctuations. This is precisely the topic explored in this chapter.

The basic ideas of a Brownian ratchet goes back to the early twentieth century, when Smoluchowski analyzed a simple mechanical device involving a ratchet and a pawl. This thought experiment, later popularized by Feynman in his book *The Feynman Lectures on Physics, Vol. 1* (1962), was introduced to illustrate the limitations imposed by the second law of thermodynamics. As we shall show later, the implications of the second law in this example seem quite counter-intuitive, at first.

The operating principle of the ratchet machine is the same as that of an electrical rectifier, which was studied by Brillouin in 1950. Both are elementary examples of devices that, if they could perform as intended, harvesting thermal fluctuations from their environment to produce work, would be in violation of the second law of thermodynamics.

1.1 The second law of thermodynamics

Based on empirical evidence, the second law is a postulate of thermodynamics that limits the occurrence of many processes we know from experience do not happen, even though they are allowed by other laws of physics. For example, the water in a glass at room temperature is never seen to cool itself spontaneously to form ice cubes, releasing energy to its environment. Such transformation satisfies the law of conservation of energy, yet it is common sense it never occurs.

Though it may be expressed in several ways, the first formulation of the second law goes back to Sadi Carnot in 1824, who put a limit in the efficiency of any heat engine operating between two given temperatures. The typical efficiencies of Brownian ratchets are discussed in Chapter 6. In this chapter we are specially interested in the equivalent statement of the second law given by Lord Kelvin, which can be formulated as

There is no thermodynamic transformation whose sole effect is to extract heat from a heat reservoir and to convert it entirely into useful work.

A heat reservoir is a system so large that the exchange of heat does not change its temperature. Another common formulation involves the concept of entropy[1]:

The entropy of an isolated system never decreases.

Specializing the isolated system to a sub-system and its surroundings, the previous statement implies that the total entropy, also called the entropy of the universe, never decreases. From this formulation it is clear that, unlike the underlying molecular laws, the second law is not time-symmetric, i.e., is not invariant under a time reversal transformation, and thus displays a preferential direction (or arrow) of time.

1.2 Brillouin paradox

Diodes, the electronic components that allow the electrical current to pass mostly in one direction, were discovered in the second half of the nineteenth century. Their use experienced a boom in the 1950s due to substantial advances in the manufacture of semiconductor diodes, made today the most common type of diode.

The intrinsic asymmetry associated with diodes allows one to examine, with a simple system, important foundational questions regarding the possibility of rectifying fluctuations and the related limitations imposed by the second law of thermodynamics.

In this context, in 1950 the French physicist Léon Brillouin introduced what is now known as *Brillouin paradox*: a simple device that intuitively would lead to the

[1] see Chapter 6 for the definition of entropy in the context of Brownian motion.

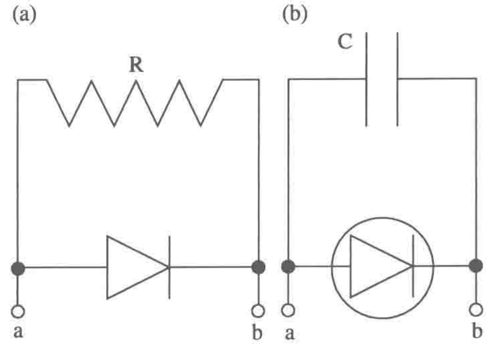

Figure 1.1 (a) The Brillouin paradox, as originally formulated: can a diode rectify the thermal fluctuations in the resistor and produce a direct voltage, like a small battery? (b) The variant of the paradox introduced by Alkemade, with the resistor replaced by a capacitor and a specific type of diode, a vacuum diode, considered.

rectification of fluctuations in a system in equilibrium, in striking violation of the second law of thermodynamics. The device is a simple circuit consisting of a diode and a resistor, as shown in Fig. 1.1(a). At a finite temperature the voltage across the resistor is fluctuating. An intuitive, though wrong, analysis leads immediately to the conclusion that the asymmetry introduced by the diode allows the establishment of a direct current in the circuit, and thus a direct voltage. This voltage would represent a source of electrical, useful energy, obtained from a single heat reservoir (the circuit) at thermal equilibrium, in violation with the second law. A more careful *microscopic* analysis reveals instead how no direct current can be induced in the circuit, thus re-estalishing the agreement with the second law.

Instead of discussing the microscopic analysis of the original Brillouin paradox, we find more illustrative to describe here an evolution of the idea, i.e. a variant of the Brillouin paradox introduced by Alkemade and van Kampen. Let us consider the circuit of Fig. 1.1(b). A capacitor is connected to a *vacuum* diode. In analogy with the original Brillouin paradox, one may wonder whether the rectifying properties of the diode allow the accumulation of a finite charge on the capacitor plates.

An initially uncharged capacitor is considered. At any finite temperature T, the capacitor will hold a small random charge Q due to thermal noise. In the absence of the diode, this charge will fluctuate in time between positive and negative values, so that the (time) average charge is zero. The voltage associated with this charge at a given instant is $V_b - V_a = Q/C$, where C is its capacitance. From the equipartition theorem, the average energy stored in the capacitor will be $\langle Q^2/2C\rangle = k_B T/2$, where $k_B = 1.38065 \times 10^{-23}$J/K is the Boltzmann constant.

However small, it is tempting to try to transform this thermal energy into useful work by connecting a diode to each end of the capacitor. Every fluctuating current with the same direction as the diode that spontaneously appears in the system will

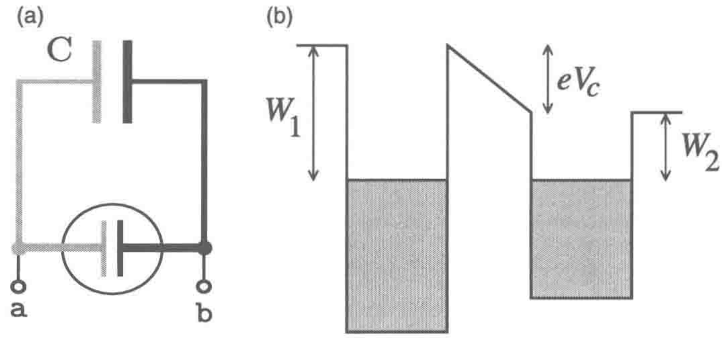

Figure 1.2 Alkemade diode: two metals with different work functions.

be allowed to cross the diode, whereas currents with the opposite directions are impeded. Thus, the asymmetry introduced by the electrical rectifier should result in a voltage offset $V_b - V_a > 0$ that could be used to perform work. This system would be able to extract heat from one heat reservoir and convert it entirely into useful work, in contradiction with the second law of thermodynamics. But then, why is the diode not able to rectify thermal fluctuations? To answer this question, the system has to be examined in more detail, i.e. a *microscopic* analysis of the system is required.

The essential rectifying element of the considered circuit is the vacuum diode, as illustrated in Fig. 1.2. It consists of two parallel electrodes, separated by a short distance, short enough to allow a flux of electrons between the two electrodes. The electrodes are in equilibrium at temperature T, but have different work functions W_1 and W_2. The vacuum diode is connected to an ideal capacitor, of capacitance C. We assume that the capacitance of the diode can be neglected against that of the capacitor.

The natural tendency to equilibrium will produce a net flow of electrons from the electrode with higher Fermi level (i.e., higher chemical potential or lower voltage) to the one with lower Fermi level (lower chemical potential or higher voltage). Thermal equilibrium is obtained when the two Fermi levels are equal, and the net flux of electrons ceases. However, this does not imply that at equilibrium the electric potential in the vacuum between the electrodes is flat. On the contrary, in general a potential difference inside the vacuum between the two plates, and thus also an electric field, is established at equilibrium. This electric field is responsible for the diode's characteristic asymmetry, acting as an effective resistance that slows down electrons in the direction that defines the diode, as we discuss later. It can be traced back to the different work functions of the two metal plates.

The work function W is the minimum energy required to remove an electron from the interior of the metal. Then, the requirement of equilibrium leads to