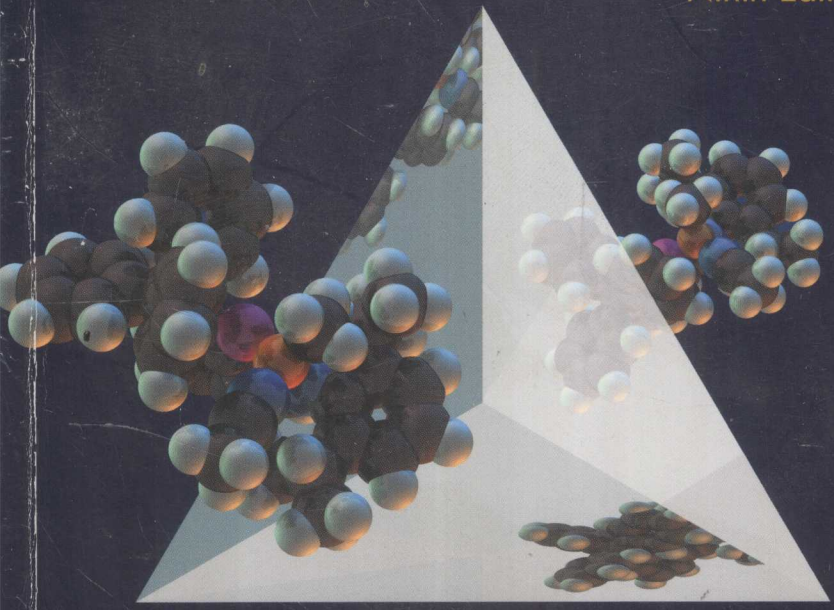
Gary L. Long • Sharon D. Long • Doris I. Lewis

MATH REVIEW TOOLKIT

CHEMISTRY

THE CENTRAL SCIENCE

Ninth Edition



Brown LeMay Bursten

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Preface To The Student

A successful study of general chemistry requires you to use your skills of memorization, basic logic and mathematics. From my years in the classroom I have found that students who are unsure about their math skills generally do not do well on their exams. To put this statement into context, imagine that you holding a chemistry exam containing 20 multiple choice questions, with 15 of them involving the use of mathematical equations. If you have trouble with the first few questions involving math and have only an hour to complete the exam, any weakness in math skills may limit your performance on the exam, regardless of how many facts and figures you have memorized.

The math skills that a student needs to successfully complete general chemistry are basically what you have learned in high school; that is, algebra and trigonometry. Calculus is not required. It is the purpose of this booklet to guide you chapter by chapter in the mathematics that are used in the ninth edition of CHEMISTRY, THE CENTRAL SCIENCE by Brown, LeMay, and Bursten. The math skills for each chapter are outlined and examples are worked in this booklet.

The case of a life sciences student planted the idea for this booklet. This young man was going around for the seventh time in general chemistry. He could never manage to pass the first test so he would drop out and wait until the next semester to try again. Fortunately, he came to me at the beginning of the course. He was desperate; he could not graduate until he passed chemistry. After speaking with him, I found his limitation was not inadequate math training, but a basic fear of math problems. He would stumble over the math problems on the exam. Over the course of the semester I met with the student and explained how to perform these calculations using the methods that are described in this booklet. The student successfully completed the course with a B average.

Although we cannot absolutely guarantee your success in chemistry with the use of this booklet, mastery of the material presented here will enhance your ability to work problems in general chemistry. Take a few moments and complete the "SELF TEST FOR MATH SKILLS"

at the back of this booklet. It will help you assess your skills. Use this guide to help you reinforce your math skills.

Chemistry is an exciting science that touches every aspect of our lives. It is our hope that this booklet may demystify the mathematics used in general chemistry and help you discover this excitement.

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I will never forget my own struggle with chemistry at Wake Forest University. As a freshman, I had not yet developed productive study habits. I faced every new chemistry chapter with a certain dread. There is a tendency to procrastinate taking hold of something we fear or find distasteful, or so it was in my case. However, if you cope with chemistry in this manner, as I did, the stress of catching up and preparing for a test will cost you your well being until the test is over. The two grades of C that I made in freshman chemistry taught me this painful lesson. The point being, even if you dislike chemistry and are just taking it only to satisfy the requirements for your major, make your effort consistent.

Many college students have been forced to change their career dreams because they could not make it through chemistry. This unpleasantness does not give this science a good name nor does it give chemistry professors an easy grace at social functions when asked their occupation. Some of you may need to change your career option to a different discipline for which you are gifted. A very kind English professor pointed this out to me at a time when organic chemistry was causing me great distress. Time is too valuable to spend it in the wrong place. However, for those of you heading in the right direction, we do not want chemistry to be a stumbling block for your dreams.

We hope that your experience with chemistry will be a victorious one and that in some small way this math book will make a difference. Write us and tell us your stories as well as your triumphs.

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Contents

Preface ..... *vii*

1 Your Calculator and Its Functions *1*
- 1.1 Arithmetic Operations *1*
- 1.2 Trigonometric Operations *2*
- 1.3 Logarithmic Functions *2*
- 1.4 Exponential Notation and Your Calculator *4*
- 1.5 Choosing a Calculator *5*

2 Skills for Chapter 1: Introduction *7*
- 2.1 Significant Figures *7*
- 2.2 Exponential Notation *8*
- 2.3 Mean and Standard Deviation *10*

3 Skills for Chapters 3 & 4: Stoichiometry and Aqueous Reactions *12*
- 3.1 Yields *12*
- 3.2 Molarity *13*
- 3.3 Dilution *15*
- 3.4 Titration *16*

4 Skills for Chapter 5: Thermochemistry *19*
- 4.1 Calorimetry *19*
- 4.2 Enthalpies of Reaction *20*

5 Skills for Chapter 6: Electronic Structure of Atoms *22*
- 5.1 Radiant Energy *22*
- 5.2 Rydberg Equation *23*
- 5.3 Dual Nature of the Electron *24*

6 Skills for Chapter 10: Gases *26*
- 6.1 Gas Laws *26*
- 6.2 Partial Pressures *29*
- 6.3 Graham's Law *30*

7 Skills for Chapter 13: Properties of Solutions *33*
7.1 Concentration Units *33*
7.2 Colligative Properties *33*

8 Skills for Chapter 14: Chemical Kinetics *35*
8.1 Reactions Rates *35*
8.2 Concentration and Time *37*
8.3 Activation Energies *41*

9 Skills for Chapter 15: Chemical Equilibrium *42*
9.1 Calculations with Equilibrium Expressions *42*
9.2 Calculations of Equilibrium Concentrations *44*

10 Skills for Chapter 16: Acid-Base Equilibria *50*
10.1 pH Calculations *50*
10.2 Equilibrium Calculations *53*

11 Skills for Chapter 17: Aspects of Equilibria *57*
11.1 Common Ion Effect *57*
11.2 Buffers *60*
11.3 Acid- Base Titrations *64*
11.4 Solubility *65*
11.5 Effect of Common Ions and pH *69*
11.6 Complex Ions *69*

12 Skills for Chapter 19: Thermodynamics *72*
12.1 Enthalpy and Entropy Changes *72*
12.2 Gibb's Free Energy *72*
12.3 Free Energy and Equilibrium *73*

13 Skills for Chapter 20: Electrochemistry *75*
13.1 emf, *K*, and the Nernst Equation *75*
13.2 Electrolysis *79*

14 Skills for Chapter 21: Nuclear Chemistry *80*

15 Self Test for Math Skills *82*

16 Chemistry and Writing *86*
16.1 The Scientific Notebook *86*
16.2 The Scientific Report *88*
16.3 Technical Writing *90*
16.4 A Notebook Example *93*

17 Chemistry and Career Planning *97*
17.1 Materials Science *97*
17.2 Polymer Science *98*
17.3 Environmental Science and Technology *98*
17.4 Biochemistry and Biotechnology *99*
17.5 Medicinal Chemistry and Clinical Chemistry *99*
17.6 Forensic Science *100*
17.7 Radiochemistry *100*
17.8 Patent Attorneys and Patent Agents *101*
17.9 Chemical Education *101*
17.10 Chemical Information Careers *102*
17.11 Other Chemistry Careers *103*

18 Answers for Math Skills Test *104*

Section 1

Your Calculator and Its Functions

A recent Peanuts cartoon displays Snoopy holding up a calculator with the expression, "I don't need to think, I have batteries..." This statement is far from reality in any study that requires the use of mathematics. Calculators are invaluable tools when working with long equations and involved calculations. What you can do in a few seconds on your calculator now would have taken a chemistry student 20 years ago several minutes to do by hand. Even with this greater potential speed in problem solving, you have to set up the equations before you use the calculator. Besides writing the equations down on paper, you may have to rearrange them so that you can use your calculator's functions to solve the problem. You will need to become acquainted with your calculator so you will know how to properly set up the problem. The mastery of the equations and your calculator's operation will allow you to get these "speedy" answers.

In your general chemistry course you will encounter problems using arithmetic operations (addition, subtraction, multiplication, and division), trigonometric operations (cos, sin, $\cos^{-1}$, and $\sin^{-1}$), and logarithmic operations (log, ln, 10^x, e^x, powers and roots). You will also deal with exponential notation in many of these problems. In the following sections, we will discuss these functions and how to use your calculator to solve problems in General Chemistry.

1.1 Arithmetic Operations

These operations are generally straightforward. To add two numbers we enter the first number, press the [+] key, enter the second number and press the [=] key. The sum of the numbers should appear on the display.

Some calculators may not follow this sequence because they are designed to use Reverse Polish Notation, RPN. These types of calculators do not have an [=] key. Calculations are performed by first entering the numbers into memory "stacks" on the calculator and then

pressing the desired key. Although this method seems horribly backward at first, the use of RPN can be quite advantageous in dealing with calculations contained within a large formula.

1.2 Trigonometric Operations

You will encounter several equations in chemistry that will use the trigonometric operations of sin and cos. The process of determining the sin or cos of an angle is similar for conventional and RPN calculators. To determine the sin of 30°, enter 30 into the calculator and depress the [sin] key. The value of 0.50 should appear. *Note: It is most important that your calculator is in the "degree" mode and not in "radian" or "grd" when performing these calculations.*

Other calculations will require you to determine what degree value in sin x will correspond to a certain value, say 0.60 = sin x. You can determine this x by using one of several keys on your calculator. Look for the [$\sin^{-1}$] or [arcsin] key. Enter the value of 0.60 and then depress the [$\sin^{-1}$], or the [arcsin] key. A value of 36.9° should appear on the display. The use of this function may require you to first depress a [2^{nd}], [INV], [f], or [g] key.

1.3 Logarithmic Functions

These functions will include the use of log, 10^x, ln, and e^x. Also to be discussed in this section are roots and powers.

In calculations involving logarithmic relationships, there are two different mathematical bases that are used. On your calculator you will find base 10 logarithmic functions, [10^x] and [log], and base$_e$ functions, [e^x] and [ln]. These logarithmic functions are not the same. Base$_{10}$ math is structured around examining the properties of raising the number 10 to different powers, while base$_e$ math is structured on using 2.71828…. You will find that many calculations, such as thermodynamic relationships, are defined using log, not ln. Part of this reasoning is historical, due to the fact that calculations involving base$_{10}$ math are much easier for people to work with than base$_e$ mathematics. Many of

these relationships were developed and refined by chemists before the advent of the computer and pocket calculator. For this reason the base$_{10}$ approach was favored. As an example, consider the calculation of the log of 1,000. Here, we must figure out to what power 10 must be raised to equal 1,000. We know from our algebra training that if we cube 10, we should obtain 1,000. The calculation of ln (1,000), which involves base$_e$, is another matter. To determine this value we must use the calculator. We first enter 1,000 and then press the [ln] key. The value of 6.908 should appear on the display.

The calculations of 10^x and e^x are sometimes referred to as anti-logs in calculator manuals. Most calculators have a [10^x] and an [e^x] key. Some models may require you to press the [INV] key to use an anti-log function. In this scheme of thinking [e^x] = [INV] [ln]. To determine $e^{1.5}$, enter 1.5 into the calculator and depress the [e^x] or the [INV] [ln] key(s). A value of 4.5 should appear on the display.

Powers and roots are another important use of the calculator in chemistry. All calculators possess square root [$\sqrt{x}$] and square keys [x^2]. The use of these functions is straightforward. However, you may need to determine cubic (and higher) roots in your studies. Your calculator may have a [y^x] key that will enable you to determine these roots as well as to calculate a number raised to a power. To determine the roots with the [y^x] it is necessary to enter the root as the value of $1/x$. To find the cubic root of 77, enter 77 into the calculator (as the y value), enter 1/3 (as the x value), and depress the [y^x] key. You should see the value of 4.3 on the display.

Don't be discouraged if your calculator does not have a [y^x] key. If it has a [log] and a [10^x] or [INV] [ln] key(s), you can determine any root of a number. To calculate the cubic root of 77:

- enter 77 as the y value
- use the [log] button (1.8864 will be displayed)
- divide the result by 3, the x^{th} root, (0.6288 will be displayed)
- press the [10^x] button on the quotient (4.254 will be displayed)
- the result (4.3) is the cube root of 77

This method should allow you to take the n^{th} root of any number that you may encounter in your studies in chemistry. For instance, try

computing the fourth root of 1.9×10^{-11}. The log of this number is -10.72. Dividing this number by 4 and taking the anti-log yields the answer 2.1×10^{-3}. This method will be useful to you in Chapter 17 where you will be determining the solubility of salts.

1.4 Exponential Notation and Your Calculator

Exponential notation is a method used to express very large and very small numbers on your calculator. These numbers are expressed by multiplying the significant portion of the number by a multiplier based on 10^x.

To use the exponential notation feature on your calculator, you will use a [sci], [exp] or [EE] function. Consider the process of entering 1.6×10^{-17}. First enter the significant portion of 1.6 into the display. Next press the [sci] or equivalent key and enter the number 17. You will need to change the sign of the power of 17 (which should appear in the right hand portion of the display), so press the [+/-] key to cause the power to read -17. Depending on the type of calculator you have, you may need to press the [=] key to end the sequence.

You will also be able on your calculator to convert from regular notation to exponential notation. If the number 1,000,000 was in the display of your calculator, you could depress the [sci] then [=] keys to cause the number to appear in exponential notation (1.000×10^6) on the display.

Please be aware that different calculators may require different methods for the entering of numbers in exponential notation or the conversion of numbers to and from exponential notation. As trite as it may seem, you may need to consult the manual that came with your calculator. I have done this on more than one occasion for the various calculators I use.

More information on exponential notation and its use in chemical calculations can be found in Appendix A of your textbook.

1.5 Choosing A Calculator

There are many excellent brands of calculators on the market available to the chemistry student. If you have not yet purchased a calculator, consider the following questions.

- Will you be pursuing a science, business, or liberal arts degree?
- Do you need a programmable or graphing calculator?

The calculator that you need in this course must be able to perform the basic functions in addition to trigonometric and logarithmic functions. It must support exponential notation. Prices of "scientific" calculators usually start at $20.

With regards to the first question, if you are planning to pursue a field of study in the liberal arts, the above mentioned calculator would well suit your needs. Science majors may wish to consider a calculator that contains a statistical analysis package and programmability in addition to the basic functions. Business majors would be well advised to purchase the basic "scientific" calculators. The calculations that you would use in later business courses will require you to use specific formulas and functions that are programmed into "business" calculators.

Programmability on a calculator is a wonderful feature that allows you to quickly solve repetitive calculations that are employed in science and engineering fields. Unfortunately, programmability adds to the cost of a calculator. Models that graph mathematical functions on a small LCD screen are also becoming popular, but at a higher cost. If you are operating on a restricted budget, choose a basic "scientific" model and upgrade later to a programmable or graphing model.

After you have answered these questions, examine the brands that would meet your calculating and financial needs. Examine the calculators for the size of the display and buttons (and their spacing) on the keypad. You want a calculator that has an easy to read display and that is easy to use. In order to put more "full feature functions" some manufacturers have down-sized the keys and put three to four functions on the same key. If the keypad legend is not well designed, these types of calculators can be difficult to use.

A final factor to consider is the power source. Solar powered calculators are great while working in well lighted areas. Unfortunately, these calculators may not work well in some large lecture halls because of insufficient room lighting. However, some calculators have dual power sources: solar and battery. With this type of design, your batteries will power the calculator if the light levels are too weak. (This is especially good to know before you take your first chemistry test.)

Your calculator can be a valuable tool to you in your study of general chemistry. Take the time to acquaint yourself with its capabilities.

Section 2

Skills for Chapter 1: Introduction

2.1 Significant Figures

Every measurement that is made in the laboratory is subject to error. The level of this error (or certainty) depends upon the instruments used in the measurement and the skill of the person performing the operations. Even if we can eliminate the systematic errors (*i.e.* miscalibration of the pan balance) we will still encounter random errors in lab. These random errors will determine the accuracy of our measurement.

Rather than stating the error with each number a chemist may use in a calculation, the practice of using "significant figures" in calculations is employed. The significant figures of a number can be thought of as those digits in the number that do not change when the uncertainty is factored in. (The rules used for determining the number of significant figures in an expressed value are listed in Section 1.5 of your textbook.) As an example of the effect of uncertainty on the number of significant figures, consider the food sample which was found to have 1.33 mg of Ca per serving with an error of 0.1 mg for the determination. Based on the uncertainty we should only say that the sample contains 1.3 mg of Ca. The number has only 2 significant figures. It is of no importance to attempt to convey that the calculation indicated a second 3 (*i.e.* 1.3$\underline{3}$) in the decimal. The error made this digit "insignificant".

You will be asked in this course to use significant figures in the estimation of answers to your calculations. As an example, consider the problem of determining the circumference of a cylinder. With a simple ruler you could determine the diameter to be 2.5 inches. From the relationship of $C = \pi \times d$, you could use your calculator to determine the circumference, C. The display of the calculator would read (7.853981. . .). What value do you report? What is significant?

The basic rule is that you report your answer using the least number of significant figures. In the multiplicative operation above, the

diameter was only reported to two significant figures. The answer should be reported as 7.9 in.

Rules for addition and subtraction are slightly different. Consider adding a 0.001 g weight to an object resting on a pan balance and indicating a value of 23.2 g in mass. The addition of 0.001 to 23.2 would yield a theoretical answer of 23.201 g. However, if the pan balance used had an error of 0.1 g, the addition of 0.001 g to this weight would be insignificant. The uncertainty will limit our ability to express accurately the sum of the combined weights. Sample Exercises 1.5-1.8 are problems with significant figures. Exercises 1.33-1.40 address the use of significant figures in calculations.

2.2 Exponential Notation

The calculations that you will use in this course will involve those numbers that are exceedingly large or small. In order that you may perform the calculations (let alone write them on one sheet of paper) exponential notation is employed. As mentioned in your text, exponential notation is a way to express these numbers by reducing the number to a significant portion (discussed above) and a multiplier based on 10^x. The metric system takes advantage of this system in allowing us to express small masses such as nanograms (1×10^{-9} grams) to larger masses such as kilograms (1×10^3 grams).

The use of exponential notation in equations also allows us to quickly estimate the answer to problems involving multiplication and division. For multiplication we sum the powers of the values multiplied to obtain the power of the final answer. Consider the equation:

$$x = (1.00 \times 10^3) \times (2.00 \times 10^6)$$

The product of the significant portion is 2.00. The sum of 3 and 6 is 9. For this problem

$$x = 2.00 \times 10^9$$

Division is based upon the subtraction of powers. For the problem:

$$x = \frac{6.00 \times 10^5}{2.00 \times 10^9}$$

The quotient of the significant portion is 3.00. The difference in powers is 4. By inspection, the answer to this problem is 3.00×10^{-4}.

This method of inspection can be a powerful tool. Let's say we wish to convert 1000 ft (1.000×10^3 ft) into millimeters. From the conversion table on the inside back cover of your textbook, we can find that 1 inch is equal to 25.4 mm. Since there are twelve inches in one foot, 1 ft equals 12 times this value (304.8 mm/ft or 3.048×10^2 mm/ft). If we set up the problem correctly,

$$x \text{ mm} = (1.000 \times 10^3 \text{ ft}) \times (3.048 \times 10^2 \text{ mm/ft})$$
$$x \text{ mm} = 3.048 \times 10^5 \text{mm}$$

Remember when multiplying we can sum the powers to see that the answer will be in the 10^5 range. The product of the significant portion is 3.048. The answer to this question then is there are 3.048×10^5 mm in 1000 ft or 304,800 mm in 1000 ft.

This solution did not require an electronic calculator. The key to the mental calculation lies in the inspection of the powers and the estimation of the product and the significant portion of the two values. It should be noted that before an answer can be determined, the units in the problem must be inspected. The final answer must appear in mm. The units of ft canceled by appearing both in the numerator and the denominator. If your inspection of the set up problem does not reduce to the desired units, the constants chosen or basic set up is wrong. This topic is further discussed in your textbook in Section 1.6, Dimensional Analysis.

A final example of exponential notation concerns a problem that you will encounter in Chapter 6 of your textbook where you will determine the energy of a photon. If a HeNe laser has a wavelength of 632.8 nm, what is the energy in J of a single photon? The answer is calculated using $E = hc/\lambda$, where h and c are constants and λ is the wavelength in meters. We can quickly estimate the energy by