2008恒隆数学奖
获奖论文集

区国强 吴恭孚 丘成桐 主编

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數學獎
HANG LUNG
MATHEMATICS AWARDS

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Hang Lung Mathematics Awards

Collection of Winning Papers 2008

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The Institute of Mathematical Sciences
香港中文大學數學系
Department of Mathematics,
The Chinese University of Hong Kong
恒隆地產
HANG LUNG PROPERTIES

Preface

Since the inaugural Hang Lung Mathematics Awards (HLMA) held in 2004, we have been extremely pleased by the enthusiasm and interest displayed among students, teachers and secondary schools who have participated. 2008 HLMA marked the third occasion of the competition held in Hong Kong.

Before the launch of the competition in 2004, we were concerned whether students and teachers in Hong Kong would be interested in this type of research-based competition, and whether it would be too demanding of the students. In hindsight, we are pleased to say that those concerns have been unfounded. Students and teachers have capably risen to the challenge, demonstrated creativity in selecting their research topics and displayed remarkable ingenuity in applying mathematics to solve problems or prove a proposition.

This book has gathered the research papers of the winners of the 2008 HLMA which have reached a high level of academic standards in terms of methodology, execution and scholarship. It is exciting to witness students' tackling a wide variety of challenging mathematical problems with great skill and imagination. They seemed to have discovered how enjoyable mathematics can be.

We are also proud to see that HLMA has been winning tremendous and continuous support from not only the local, but also the international academic community since its establishment. It is our hope that HLMA will continue to motivate youngsters to think more critically and induce their interest in mathematics. We also yearn to position HLMA as a pioneering model encouraging the business sector to support other academic pursuits, so that the government, the business sector and the education community can join forces to discover talents for our society.

Professor Shing-Tung Yau	Mr. Ronnie C. Chan
Director, The Institute of Mathematical Sciences The Chinese University of Hong Kong	Chairman Hang Lung Properties Ltd.

October 2012

Acknowledgement

The publishing of this book has been made possible by donations from Hang Lung Properties Ltd.

The contribution to mathematics education from the initiators of Hang Lung Mathematics Awards, Mr. Ronnie C. Chan and Professor Shing-Tung Yau, will be marked in history. Professor Yau provides the academic leadership and vision for the Awards and he is also one of the editors of this volume. Hang Lung Properties Ltd. has been providing the financial support for the Awards and additional funding for the editorial and publishing cost of this volume.

The reputation of an academic award is always established upon its rigorous process of assessment and review. The Scientific Committee is the cornerstone of this intellectual excellence. The Screening Panel would like to thank Wenyi Chen, Feng Hui, Jiaxin Hu, Jianguo Huang, Wen Huang, Qinghui Liu, Jihua Ma, Kung-Fu Ng, Yanhui Qu, Hui Rao, Song Shao, Yuguang Shi, Baorui Song, Bo Tan, Zhenhan Tu, Gengsheng Wang, Jiayan Yao, Changjian Zhang, Guobiao Zhou, Xiaofeng Zou for their professional judgment.

The Steering Committee has contributed greatly to the popularity of the Awards in society. Many students, teachers, and schools have participated in the competition. The Executive Committee, chaired by Thomas Au, is in debted to the secretariat led by Serena Yip and the assistance of Peony Wu and Simon Tse. The cooperation of Susan Wong, Joyce Kwong, Joyce Leung, and the newly on board Carolina Yip from Hang Lung Properties has been invaluable.

This volume will never be perfect without the commitment of Kung-Fu Ng who have led the editorial work. He would like to express his sincere gratitude to Chi-Ping Lau and Ping-Kwan Tam for their careful reading and comment on the winning papers. The technical work of Yat-Ming Cheung and Chun-Ngai Cheung and the coordination of Mavis Chan had also played a crucial role in the editorial process.

Hang Lung Mathematics Awards

Introduction The Hang Lung Mathematics Awards (HLMA), co-organized by Hang Lung Properties Ltd. and the Department of Mathematics at The Chinese University of Hong Kong, is a biennial research-based mathematics competition for secondary school students in Hong Kong. Founded in 2004 by Mr. Ronnie C. Chan, Chairman of Hang Lung Properties Ltd., and world-renowned mathematician Professor Shing-Tung Yau, a 1982 Fields Medalist and 2010 Wolf Prize recipient, the competition aims to stimulate creativity and to encourage intellectual discovery in mathematics and science among secondary school students in Hong Kong.

Schools are invited to form teams and, under the supervision of a lead teacher, the teams design and carry out a mathematics research project. Each team submits a project report summarizing the findings, which is evaluated by the Scientific Committee in a multi-step process similar to that for the selection of publication in a scientific journal. Short-listed teams are invited to participate in an oral defense of their project before members of the Scientific Committee. This final stage is modeled after a doctoral degree defense and comprises two parts: a public presentation of the research project followed by a closed-door inquiry. The winners of the HLMA will be decided after the oral defense.

Hang Lung Properties Ltd. donated over HK$2 million to each competition. The Department of Mathematics at The Chinese University of Hong Kong provides tuition scholarships to the teachers of the winning schools. The Department of Mathematics handles all administrative, operational, and educational aspects of the competition.

Participation Up to five students of the same secondary school may form a team to participate in the competition. The team shall be led by a teacher of the school. After a simple registration process which started in early 2007, each team performed their study and research on a topic selected by the team. Research reports were submitted in August 2008. There were 77 teams of 229 students from 50 schools participated and 50 of those teams submitted research reports.

Assessment To decide the winners, there are two stages of assessment: research report review and oral defense. The assessment in each stage is the jurisdiction of the Scientific Committee, with the support of the Screening Panel and external experts from the international mathematics community. The review spanned from early August to early October of 2008. In this stage, each research report has to pass an initial screening. Then it is sent to at least two referees of external experts. A shortlist will be selected by the Scientific Committee to proceed to the second stage of assessment, which is the oral defense. In this stage, each team will make a brief presentation of their research in front of the Scientific Committee. The presentation is open to the public. It is then followed by a closed door inquiry. The winners will be decided afterwards.

Awards Up to eight awards are presented to the mathematics projects that meet the highest academic standard in terms of research methodology and originality: Gold, Silver, Bronze, and up to five Honorable Mentions. Each award consists of four components: a "Student Education Award" to be shared equally among team members and applied towards their university studies; a "Teacher Leadership Award" for the supervising teacher; a "School Development Award" to promote mathematics education at the school; and a "Tuition Scholarship" for any teacher at the winning schools to earn a Master of Science (MSc) in Mathematics from The Chinese University of Hong Kong. The winning students and teachers also received a crystal trophy and a certificate while the school was presented with a crystal trophy.

Organization

The two principal committees of the Hang Lung Mathematics Awards are the Scientific Committee and the Steering Committee.

Scientific Committee, 2008

The Scientific Committee comprises world renowned mathematicians and is the academic and adjudicating body of the Hang Lung Mathematics Awards. The Scientific Committee oversees the whole process of assessment, including the review of the research project reports and conducting the oral defense. The committee upholds the academic integrity and high standard of the mathematics research competition as well as the awards.

List of members (affiliations at the time of the event)
Chair: Professor Shing-Tung Yau, Harvard University
Professor Tony F. Chan, University of California, Los Angeles
Professor David C. Chang, Polytechnic Institute of New York University
Professor Chong-Qing Cheng, Nanjing University
Professor John H. Coates, University of Cambridge
Professor Benedict H. Gross, Harvard University
Professor Ka-Sing Lau, The Chinese University of Hong Kong
Professor Jian-Shu Li, The Hong Kong University of Science and Technology
Professor Chang-Shou Lin, National University of Taiwan
Professor Jill P. Mesirov, Broad Institute of MIT and Harvard
Professor Kenneth C. Millett, University of California, Santa Barbara
Professor Ngaiming Mok, The University of Hong Kong
Professor Stanley J. Osher, University of California, Los Angeles
Professor Duong H. Phong, Columbia University
Professor Wilfried Schmid, Harvard University
Professor Tom Yau-Heng Wan, The Chinese University of Hong Kong
Professor Hung-Hsi Wu, University of California, Berkeley

Screening Panel is a subcommittee of the Scientific Committee. It handles the initial screening of each report, supervises external review process, and serves as a bridge between all referees and members of Scientific Committee.

Chair: Professor Tom Yau-Heng Wan, The Chinese University of Hong Kong
Professor Wing Sum Cheung, The University of Hong Kong
Professor Conan Nai Chung Leung, The Chinese University of Hong Kong

Steering Committee, 2008

The Steering Committee comprises mathematicians and representatives from different sectors of society and serves as the advisory body. The committee also includes mathematics department heads of major Hong Kong universities. Some members from the Scientific Committee and Executive Committee also serve in the Steering Committee so that it has an overall perspective of all the aspects.

List of members (affiliations at the time of the event)
Chair: Professor Sir James A. Mirrlees, 1996 Nobel Laureate in Economics
Professor Thomas Kwok-Keung Au, The Chinese University of Hong Kong
Professor Wing-Sum Cheung, The University of Hong Kong
Professor Ka-Sing Lau, Chairman, Mathematics Department, CUHK
Professor Jian-Shu Li, The Hong Kong University of Science and Technology
Professor Lo Yang, Chinese Academy of Sciences
Mr. Siu-Leung Ma, CEO, Fung Kai Public Schools
Mr. Chun-Kau Poon, Principal, The Hong Kong Federation of Youth Group Lee Shau Kee College
Ms. Susan Wong, Hang Lung Properties Limited
Mr. Chee-Tim Yip, Principal, Pui Ching Middle School

Executive Committee is a subcommittee of the Steering Committee. It is in charge of the administration, promotion, team registration, communication, and organization of events including the oral defense.

Chair: Professor Thomas Kwok-Keung Au, The Chinese University of Hong Kong
Dr. Ka-Luen Cheung, The Hong Kong Institute of Education
Dr. Leung-Fu Cheung, The Chinese University of Hong Kong
Dr. Charles Chun-Che Li, The Chinese University of Hong Kong
Secretariat: Ms. Serena Wing-Hang Yip, The Chinese University of Hong Kong

Gold, Silver, and Bronze

GOLD

Team member(s): Kwok Chung Li, Chi Fai Ng
School: Shatin Tsung Tsin Secondary School
Team Teacher: Mr. Wing Kay Chang
Topic: *Isoareal and Isoperimetric Deformation of Curves*

SILVER

Team member(s): Chi Yeung Lam, Yin Tat Lee
School: The Methodist Church Hong Kong Wesley College
Team Teacher: Mr. Chun Kit Ho
Topic: *Sufficient Condition of Weight-Balance Tree*

BRONZE

Team member(s): Fung Ming Ng, Chi Chung Wan, Wai Kwun Kung, Ka Chun Hong
School: S.K.H. Tsang Shiu Tim Secondary School
Team Teacher: Mr. Yiu Kwong Lau
Topic: *Fermat Point Extension – Locus, Location and Local Use*

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ISOAREAL AND ISOPERIMETRIC DEFORMATION OF CURVES

TEAM MEMBERS

KWOK-CHUNG LI, CHI-FAI NG[1]

SCHOOL

SHATIN TSUNG TSIN SECONDARY SCHOOL

ABSTRACT. In the report, we want to answer the following question: How to deform a curve such that the rate of change of perimeter is minimum while the area and the total kinetic energy are fixed? This means that the perimeter shrinks fastest when $\frac{dP}{dt} > 0$, increases most slowly when $\frac{dP}{dt} > 0$ First we work on isosceles triangle as a trial. Then we study smooth simple closed curve and obtain the following results:

1. The radial velocity of each point of the curve in polar coordinates. (3.1.6)
2. The magnitude of the velocity at each point of the curve along the normal direction is equal to standard score of the curvature at that point (3.2.2).
3. Application of the results on Isoperimetric inequality (3.3).
4. The velocity for the dual isoperimetric problem (4).

1. Introduction

Deformation is a change of an object in its size or shape. Only from this word, you may not know what our project is really about. Apparently, you may think that it is a study of Isoperimetric inequality [2]. But actually, we are doing a research about a similar problem which is totally a new idea. As we have searched for any problem similar to our project in the internet, by Yahoo and even Google. However, maybe our searching skill is bad, we found nothing like this!

---

[1]This work is done under the supervision of the author's teacher, Mr. Wing-Kay Chung

Before starting to study this project, we loved to play war type computer games and were interested by the army array. We thought that the best strategy is to increase the perimeter so the army can have a higher chance attacking the enemy. The first idea exists in our mind is to find out how the army should respond so as to increase the contact surface with the enemy. It is obvious that there will not be a final state for the army array. However, how can we modify such a vague thinking to a precise mathematic problem?

On one raining day, we were shocked by a slug near the window. To expel the slug, we were inspired by its reaction. It tried to turn its body to a ball shape to minimize its surface area. We then abandoned maximizing its perimeter, trying the opposite direction. Finally, we linked up our thinking with what we were seeing, creating the present research!

In nature, there are many cases that related to minimizing the perimeter or surface area in the fast rate, for instance, animals want to protect themselves by shrinking its body or plants want to prevent loss of water by reducing its surface area. So, it is meaningful for us to study it.

Isoperimetric inequality states that among all closed curves, circle has the minimum perimeter with an enclosed fixed area. We do not pursue to prove or doubt it, but were attracted by its variable process of deformation. If a curve wants to change to this final state as fast as possible, what process will it choose? In other words, how can it minimize its perimeter at the fastest rate? That's our project's aim.

Beside Isoperimetric inequality, we also got the idea from another classic problem that is Brachistochrone problem. We are also going to find the least time for a shape to minimize its perimeter with fixed area. (cf. **Reviewer's Comments** 1) The difference between our problem and the Brachistochrone problem is that Brachistochrone curve describes the process of a point to another point; our problem is to describe the deformation process of a curve.

In chapter 1, we study isosceles triangle as it is the simplest polygon. Using simple algebra and calculus, we calculate how the triangle should be deformed so that the rate of change of perimeter is maximum. We also find the locus of the vertices by solving a suitable differential equation.

In chapter 2, we use two methods, Euler Lagrange equation[3] and inner product[5], to find the velocity of the points of a smooth simple closed curve under isoareal deformation so that the rate of change of perimeter is maximum. Then we relate the velocity at each point along the normal direction with the curvature at that point. We show that the velocity is equal to the standard score of the curvature. Then we derive the analog formulae for isoperimetric deformation.

2. Triangle

In this part, we are going to study the deformation process of triangle. Since triangle is the simplest polygon, it allows us to have a taste of how to study our project. In the following parts, we are

Minimizing the rate of change of perimeter with the area of the triangle fixed.

2.1. Isosceles Triangle

We start the study by considering isosceles triangle as its symmetric property helps us simplify the calculation. We need to think of one side of the triangle only. It is because the movement of the two bottom vertices should be the same (cf. **Reviewer's Comments** 2), otherwise if we turn over the isosceles triangle, the movement will be different in spite of the same isosceles triangle.

Consider a symmetric triangle where the three points are $A(0,a)$, $B(b,0)$, $C(-b,0)$.

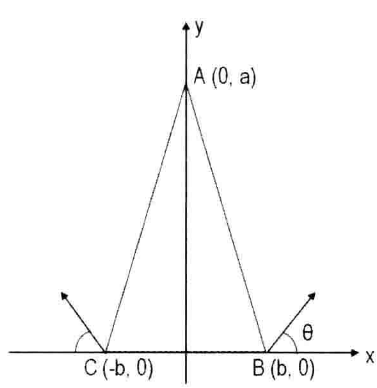

FIGURE 1

Let
A be the area of the triangle at time t.

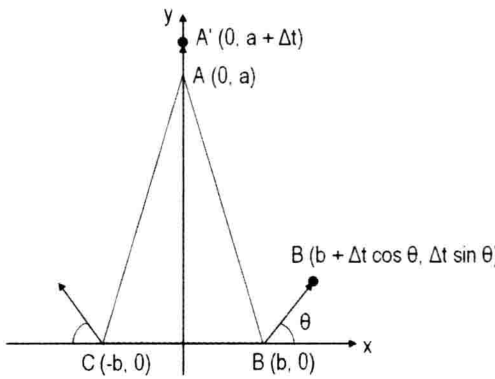

FIGURE 2

P be the perimeter of the triangle at time t.
θ be the angle made between the horizontal and the instantaneous direction of the movement of the bottom vertices where $\theta \in [0, 2\pi]$.

We assume that the vertices move with constant velocity 1 unit/second.

At time $t = 0$, the area $A_0 = \frac{1}{2}a(2b) = ab$.

At time $t = 0$, the perimeter $P_0 = 2b + 2\sqrt{a^2 + b^2}$.

Because of the symmetry property of isosceles triangle, the top vertex $A(0, a)$ is also forced to move either up or down, so we study the problem in two cases.

Case 1: The point $A(0, a)$ moves upwards

After time interval Δt, the area

$$A_{\Delta t} = \frac{1}{2}(a + \Delta t - \Delta t \sin\theta)[2(b + \Delta t \cos\theta)]$$

Hence

$$\begin{aligned}
\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} &= \lim_{\Delta t \to 0} \frac{A_{\Delta t} - A_0}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{(a + \Delta t - \Delta t \sin\theta)(b + \Delta t \cos\theta) - ab}{\Delta t} \\
&= b(1 - \sin\theta) + a\cos\theta
\end{aligned}$$

As the area of the triangle is fixed,

$$\begin{aligned}
\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} &= 0 \\
\frac{a}{b} &= \frac{\sin\theta - 1}{\cos\theta} = \frac{\sin\frac{\theta}{2} - \cos\frac{\theta}{2}}{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}} && (1) \\
\frac{a}{b} &= \tan\left(\frac{a}{2} - \frac{\pi}{4}\right) && (2)
\end{aligned}$$

Refer to 1. If $a > \sqrt{3}b$, then

$$\tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right) > \sqrt{3} = \tan\frac{\pi}{3}$$

$$\frac{7\pi}{6} < \theta < \frac{3\pi}{2}$$

If $a < \sqrt{3}b$, then

$$\tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right) < \sqrt{3} = \tan\frac{\pi}{3}$$

$$\frac{\pi}{2} < \theta < \frac{7\pi}{6}$$

The above results mean that the two bottom vertices, B and C, will move differently, depending on what the original triangle is.

If it is thinner than an equilateral triangle, i.e. $a > \sqrt{3}b$

$$\frac{7\pi}{6} < \theta < \frac{3\pi}{2}$$

If it is fatter than an equilateral triangle, i.e. $a < \sqrt{3}b$

$$\frac{\pi}{2} < \theta < \frac{7\pi}{6}$$

Also, from (2), we can see that the vertices B and C has only one way to move for particular values of a and b.