

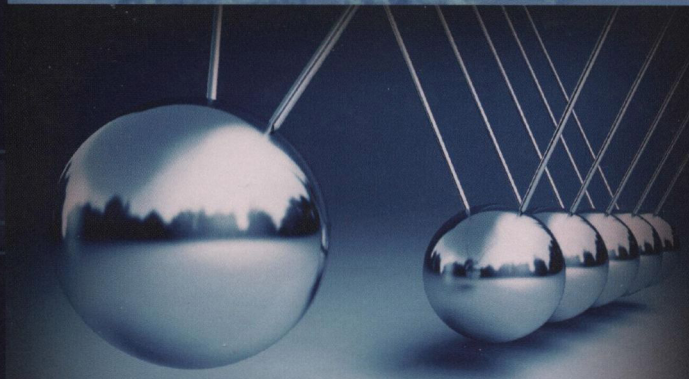


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Digital Control Applications

Illustrated with MATLAB[®]



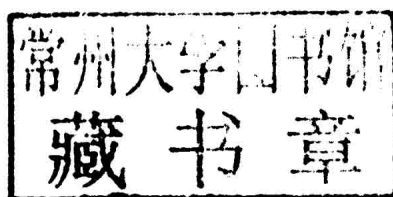
Hemchandra Madhusudan Shertukde

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Illustrated with MATLAB®

Hemchandra Madhusudan Shertukde

UNIVERSITY OF HARTFORD, CT, USA



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Digital Control Applications

Illustrated with MATLAB®

To all my professors at the Indian Institute of Technology, Kharagpur, India, who taught me control theory and electrical power engineering in my formative years in the early 1970s and later during my graduate study days at the University of Connecticut, Storrs, in the early 1980s.

Finally, to my lovely wife, Rekha; my dear and accomplished kids, Dr. Amola,

Karan, and Rohan; and, as always, my adopted loyal dog, Sheri

Above all this book is dedicated to the memory of my beloved parents; Late Shri Madhusudan

Gajanan Sherukde and Late Smt. Sulabha Madhusudan Shertukde; both were excellent teachers and as in Mahabharata my father taught me how to tell stories in four different ways!

May their souls rest in peace!

Preface

My association with the theory of controls in continuous time started during my studies at the Indian Institute of Technology, Kharagpur, India, in 1974 as an undergraduate student in the Controls and Power program. The initial introduction by Professors Kesavamurthy, Y. P. Singh, and Rajagopalan laid the foundation for a good basic understanding of the subject matter. This pursuit and further advanced study in the field of digital controls continued during my days as a graduate student in the Electrical and Systems Engineering Department at the University of Connecticut in Storrs, from 1983 to 1988.

The fundamentals and advances in control theory is a vast field to study. During my graduate studies, the knowledge imparted by Professors Charles Knapp, Peter Luh, Yaakov Bar-Shalom, David Jordan, and David Kleinman has been invaluable and priceless. Further my classmates Krishna Pattipati and Daniel Serafaty have guided and helped considerably in this arduous task.

In addition, the urge to start a graduate program was initiated in the Electrical and Computer Engineering Department at the University of Hartford to include controls as a specialty along with power in the graduate degree program has been a great motivation above all.

Since 1988, after being hired as an assistant professor in the Electrical and Computer Engineering (ECE) at the University of Hartford, I have personally pursued this dream of teaching every aspect of controls such as advanced controls, digital controls, optimal controls, and stochastic controls. The dream finally came true in 1992 when I started the graduate program as the director of Graduate Studies for the College of Engineering in 1993 and the chair of the ECE Department in 1994. For the last 22 years, this teaching process continued, and in 2010 the desire to write books in different areas of controls and power started very successfully, first with Verlag-Dr. Mueller company and later with CRC Press in 2013.

The book *Digital Controls and Applications Illustrated with MATLAB®* will be published by CRC Press with the support and agreement offered to me by Ms. Nora Konopka, publisher for Taylor & Francis Group. I am thankful to her for providing me this opportunity. This is my second book with this publisher and hope that this mutual understanding will continue for posterity's sake for years to come.

This book can be used as a text for two semester courses on digital controls at the senior undergraduate level or introductory level at the graduate level and the second one for an advanced level in digital controls with appropriate prerequisites at each level. Chapters 1 through 5 can be used for the first introductory course and Chapters 5 through 9 for the advanced course in digital controls. Chapter 1 explains the process of digital control, followed by a review of Z-transforms, feedback control concepts, and s to z plane conversions, mappings, signal sampling, and data reconstruction. This is followed by mathematical representations of discrete systems affected by use of advances in computing methodologies and the advent of computers.

Chapter 4 illustrates state-space representations and construction of transfer functions and their corresponding discrete equivalents. Chapter 5 then deals with the design approach and related design processes, followed by performance criteria evaluations through simulations and review of classical designs for comparison in Chapter 6.

Advances in the design of compensators using its discrete equivalent are studied in Chapter 7 and stability tests using transformations are illustrated in Chapter 8. Further, Chapter 9 deals with the computational aspects handled by the present PCs. With the advent of modern computers, hardware and software packages such as *FEEDBACK*® and *MATLAB*, both of which have been extensively used, make the understanding of the theory and its application more hands-on than the days when we toyed with dated equipment. I hope the use of such packages makes the simulation and hands-on experience fruitful.

I would like to acknowledge the cooperation of my publisher Ms. Nora Konopka of Taylor & Francis, Florida and the patience she has exhibited in the completion of this book. Further, the help and guidance of others in the project team, now Ms. Laurie Schlags and earlier Ms. Kate Gallo, is greatly appreciated.

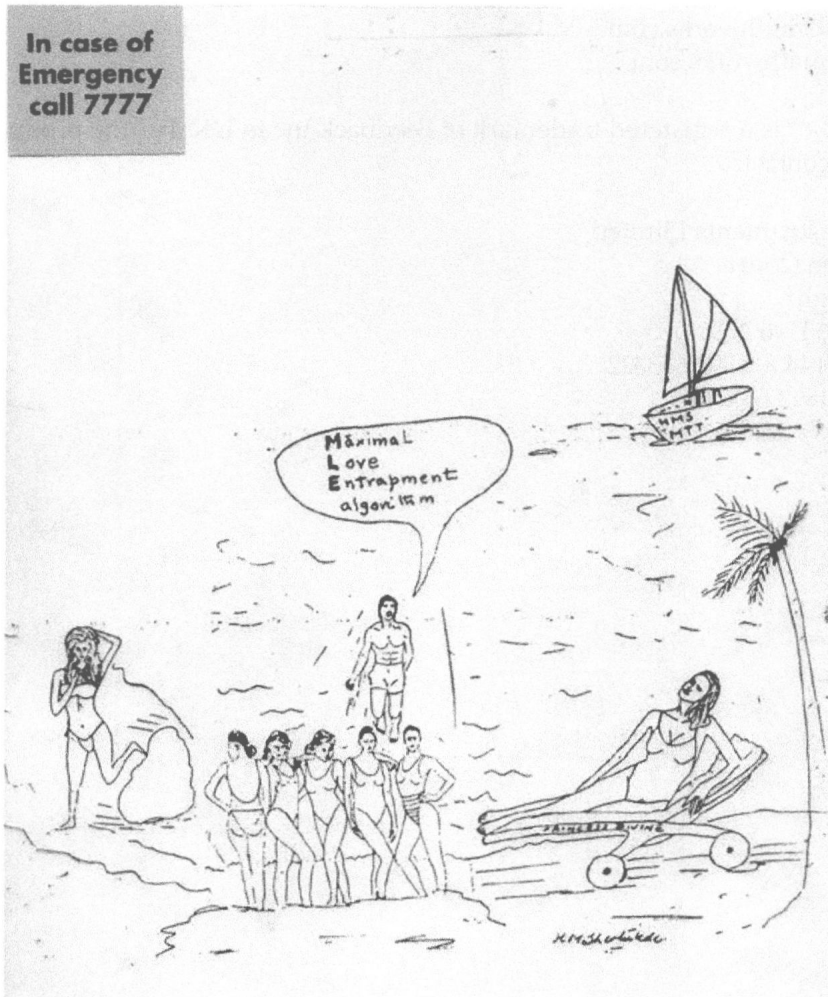
My first comical introduction to control theory was inspired by a famous cartoon by Dr. S. M. Joshi of the Dynamic Systems and Control Branch, NASA Langley Research Center, Hampton, Virginia, as shown below.



"Nice artwork, kiddo! I've got a gut feeling that a great many people will make a living off that third line some day!"

Observe carefully the third line as the cartoonist points out. Over the past five to six decades many have definitely made a mark and, in the process, made some money using it intelligently. In this effort, I have tried to make the complicated theory in digital controls

a little more palatable and understandable. My comical impressions and slightly amorous bend to control theory can be seen from my cartoon below, which I carefully crafted to deliver a message on a popular “Maximum Likelihood Estimate (MLE)” algorithm, by a name sake cartoon as illustrated. This was my bold opening slide at my Doctoral Defense at the University of Connecticut, Storrs on April 28, 1989. The MLE algorithm took a comical form as illustrated in my depiction of maximal love entrapment of my own life.



I would like to thank the Book Program at MathWorks Inc. for allowing me to adopt some of the MATLAB material. Similarly thanks to Professor Patricia Mellodge of the Electrical and Computer Engineering Department at the University of Hartford for permitting me to use the pictures of assemblies of workstations in the laboratory (D-319) using FEEDBACK \llcorner equipment. Also thanks to Mr. Iman Salehi, a graduate student in our department for helping me to understand the software and hardware complexities with this equipment. Finally, I would like to thank my wife, Rekha, for her patience in tolerating my mood changes during the completion of my book and that of my children, Amola, Karan, Rohan, and my loyal dog, Sheri.

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The book *DPV Grid Transformers* by CRC Press/Taylor & Francis Group has received international recognition and will be published in simple Chinese by China Machine Press in August 2016.

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