

# PROGRESS IN OPTICS

VOLUME VII

EDITED BY

E. WOLF

# PROGRESS IN OPTICS

(2) VOLUME VII

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(3) E. WOLF

*University of Rochester, N.Y., U.S.A.*

*Contributors*

G. KOPPELMANN, E. DELANO,  
R. J. PEGIS, I. D. ABELLA,  
B. J. THOMPSON, A. L. MIKAELIAN,  
M. L. TER-MIKAELIAN,  
S. OOUE, J. H. EBERLY



1969

NORTH-HOLLAND PUBLISHING COMPANY - AMSTERDAM · LONDON

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LIBRARY OF CONGRESS CATALOG CARD NUMBER: 61-19297

STANDARD BOOK NUMBER: 7204 1507 1

PUBLISHERS:

NORTH-HOLLAND PUBLISHING COMPANY - AMSTERDAM  
NORTH-HOLLAND PUBLISHING COMPANY, LTD. - LONDON

SOLE DISTRIBUTORS FOR THE WESTERN HEMISPHERE

WILEY INTERSCIENCE DIVISION  
JOHN WILEY & SONS, INC. - NEW YORK

PRINTED IN THE NETHERLANDS

## PREFACE

The present volume reflects the considerable advances made in recent years in several areas of modern optics. The first article reviews the theory of modes in open resonators and its relation to the theory of multiple beam interference. In the second article an account is given of various methods employed in the design of multilayer filters. The interesting phenomena of photon echoes, first observed about five years ago is discussed in the third article, which reviews the underlying theory and gives an account of the related experimental investigations. A great deal of work concerned with image formation in partially coherent light is discussed in the next article. The fifth article presents a review, written by two Soviet scientists, of the quasi-classical theory of laser radiation. This article should prove to be of special interest to workers in the West, as it contains accounts of many investigations previously published only in Soviet journals. The next article deals with photographic images, paying special attention to their optical quality and to the effects of granularity. In the concluding article investigations are described concerning the interaction between very intense light beams and free electrons. This subject has attracted a good deal of attention in recent years, because of the possibility of observing various interesting new nonlinear phenomena by experiments employing laser light.

The favorable reception accorded to this series of publications in the past, is undoubtedly in a very large measure due to the efforts of the members of the international board of editors of PROGRESS IN OPTICS, to secure articles of high standard. It is with sadness that I record here the death of Professor Hiroshi Kubota, one of the most active members of the Editorial Board. His helpful advice and participation, as well as the personal friendship that he has so generously extended to optical scientists throughout the world will be greatly missed.

*Department of Physics and Astronomy,  
University of Rochester,  
Rochester, New York, 14627  
April, 1969*

EMIL WOLF

## CONTENTS OF VOLUME I (1961)

I.	THE MODERN DEVELOPMENT OF HAMILTONIAN OPTICS, R. J. PEGIS . . . . .	1-29
II.	WAVE OPTICS AND GEOMETRICAL OPTICS IN OPTICAL DESIGN, K. MIYAMOTO. . . . .	31-66
III.	THE INTENSITY DISTRIBUTION AND TOTAL ILLUMINATION OF ABERRATION-FREE DIFFRACTION IMAGES, R. BAKARAT . . . .	67-108
IV.	LIGHT AND INFORMATION, D. GABOR. . . . .	109-153
V.	ON BASIC ANALOGIES AND PRINCIPAL DIFFERENCES BETWEEN OPTICAL AND ELECTRONIC INFORMATION, H. WOLTER . . . .	155-210
VI.	INTERFERENCE COLOR, H. KUBOTA . . . . .	211-251
VII.	DYNAMIC CHARACTERISTICS OF VISUAL PROCESSES, A. FIORENTINI. . . . .	253-288
VIII.	MODERN ALIGNMENT DEVICES, A. C. S. VAN HEEL. . . . .	289-329

## CONTENTS OF VOLUME II (1963)

I.	RULING, TESTING AND USE OF OPTICAL GRATINGS FOR HIGH-RESOLUTION SPECTROSCOPY, G. W. STROKE. . . . .	1-72
II.	THE METROLOGICAL APPLICATIONS OF DIFFRACTION GRATINGS, J. M. BURCH. . . . .	73-108
III.	DIFFUSION THROUGH NON-UNIFORM MEDIA, R. G. GIOVANELLI	109-129
IV.	CORRECTION OF OPTICAL IMAGES BY COMPENSATION OF ABERRATIONS AND BY SPATIAL FREQUENCY FILTERING, J. TSUJICHI. . . . .	131-180
V.	FLUCTUATIONS OF LIGHT BEAMS, L. MANDEL. . . . .	181-248
VI.	METHODS FOR DETERMINING OPTICAL PARAMETERS OF THIN FILMS, F. ABELÈS. . . . .	249-288

## CONTENTS OF VOLUME III (1964)

I.	THE ELEMENTS OF RADIATIVE TRANSFER, F. KOTTLER. . . .	1-28
II.	APODISATION, P. JACQUINOT AND B. ROIZEN-DOSSIER . . . .	29-186
III.	MATRIX TREATMENT OF PARTIAL COHERENCE, H. GAMO . . . .	187-332

## CONTENTS OF VOLUME IV (1965)

I.	HIGHER ORDER ABERRATION THEORY, J. FOCKE . . . . .	1-36
II.	APPLICATIONS OF SHEARING INTERFEROMETRY, O. BRYNGDAHL	37-83
III.	SURFACE DETERIORATION OF OPTICAL GLASSES, K. KINOSITA	85-143
IV.	OPTICAL CONSTANTS OF THIN FILMS, P. ROUARD AND P. BOUSQUET . . . . .	145-197
V.	THE MIYAMOTO-WOLF DIFFRACTION WAVE, A. RUBINOWICZ	199-240
VI.	ABERRATION THEORY OF GRATINGS AND GRATING MOUNTINGS, W. T. WELFORD . . . . .	241-280
VII.	DIFFRACTION AT A BLACK SCREEN, PART I: KIRCHHOFF'S THEORY, F. KOTTLER . . . . .	281-314

## CONTENTS OF VOLUME V (1966)

I.	OPTICAL PUMPING, C. COHEN-TANNOUDJI AND A. KASTLER. . . . .	1-81
II.	NON-LINEAR OPTICS, P. S. PERSHAN. . . . .	83-144
III.	TWO-BEAM INTERFEROMETRY, W. H. STEEL . . . . .	145-197
IV.	INSTRUMENTS FOR THE MEASURING OF OPTICAL TRANSFER FUNCTIONS, K. MURATA . . . . .	199-245
V.	LIGHT REFLECTION FROM FILMS OF CONTINUOUSLY VARYING REFRACTIVE INDEX, R. JACOBSSON . . . . .	247-286
VI.	X-RAY CRYSTAL-STRUCTURE DETERMINATION AS A BRANCH OF PHYSICAL OPTICS, H. LIPSON AND C. A. TAYLOR . . . . .	287-350
VII.	THE WAVE OF A MOVING CLASSICAL ELECTRON, J. PICT. . . . .	351-370

## CONTENTS OF VOLUME VI (1967)

I.	RECENT ADVANCES IN HOLOGRAPHY, E. N. LEITH AND J. UPAT- NIEKS . . . . .	1-52
II.	SCATTERING OF LIGHT BY ROUGH SURFACES, P. BECKMANN . . . . .	53-69
III.	MEASUREMENT OF THE SECOND ORDER DEGREE OF COHERENCE, M. FRANÇON AND S. MALLICK. . . . .	71-104
IV.	DESIGN OF ZOOM LENSES, K. YAMAJI . . . . .	105-170
V.	SOME APPLICATIONS OF LASERS TO INTERFEROMETRY, D. R. HERRIOTT . . . . .	171-209
VI.	EXPERIMENTAL STUDIES OF INTENSITY FLUCTUATIONS IN LASERS, J. A. ARMSTRONG AND A. W. SMITH . . . . .	211-257
VII.	FOURIER SPECTROSCOPY, G. A. VANASSE, H. SAKAI. . . . .	259-330
VIII.	DIFFRACTION AT A BLACK SCREEN, PART II: ELECTROMAGNETIC THEORY, F. KOTTLER . . . . .	331-377

# CONTENTS

## I. MULTIPLE-BEAM INTERFERENCE AND NATURAL MODES IN OPEN RESONATORS

by G. KOPPELMAN (Berlin)

1. THE CLASSICAL CONCEPT OF MULTIPLE-BEAM INTERFERENCE . . . . .	3
1.1 The Airy distribution . . . . .	3
1.2 Observation of the interference fringes . . . . .	5
1.3 The spherical mirror interferometers . . . . .	7
1.4 Standing waves and resonance enhancement . . . . .	8
1.5 The resonance and the Airy function . . . . .	10
2. THE CONCEPT OF NATURAL MODES . . . . .	12
2.1 Introduction . . . . .	12
2.2 Uniform waveguides . . . . .	13
2.3 Modes in hollow cylindrical waveguides . . . . .	14
2.4 Modes in particular waveguide cross-sections . . . . .	16
2.5 Modes in guides partly filled with dielectric media . . . . .	20
2.6 Non-uniform guides . . . . .	20
2.7 Modes in open structures . . . . .	21
2.8 Corresponding modes in waveguides and resonators . . . . .	24
2.9 Analogous modes in rectangular cavities and in plane mirror resonators . . . . .	26
3. THE PROPERTIES OF OPEN RESONATORS . . . . .	27
3.1 Diffraction and eigenmodes . . . . .	28
3.2 Fundamental formulae . . . . .	30
3.3 Similarity relations and the stability condition . . . . .	31
3.4 Field distributions of the modes . . . . .	32
3.5 Diffraction loss and resonance condition . . . . .	35
3.6 Experimental results . . . . .	38
3.7 Other open resonators . . . . .	41
4. THE RELATION BETWEEN EIGENMODES AND INTERFERENCE EFFECTS . . . . .	41
4.1 Introduction . . . . .	41
4.2 Multiple-beam interference and eigenmodes in plane-parallel mirror interferometers . . . . .	42
4.3 Multiple beam interference and modes in interferometers with a mirror step or a phase object . . . . .	50
4.4 A diffraction-based resolution limit in multiple-beam interferometry . . . . .	57
4.5 Conclusions . . . . .	62
REFERENCES . . . . .	62

## II. METHODS OF SYNTHESIS FOR DIELECTRIC MULTILAYER FILTERS

by E. DELANO and R. J. PEGIS (Rochester, N.Y.)

1. INTRODUCTION . . . . .	69
---------------------------	----

2. BASIC THEORY . . . . .	71
2.1 Reflection and transmission at a dielectric interface. . . . .	71
2.2 Reflection and transmission for a multilayer . . . . .	75
2.3 Fundamental recursion relations . . . . .	77
3. SURVEY OF SPECIAL METHODS . . . . .	80
3.1 Graphical methods . . . . .	80
3.2 Concept of equivalent layer . . . . .	83
3.3 Periodic multilayers . . . . .	85
3.4 Method of two effective interfaces . . . . .	88
3.5 Method of hyperbolic functions. . . . .	90
4. APPROXIMATE METHODS OF SYNTHESIS . . . . .	94
4.1 Vector method . . . . .	94
4.2 Fourier sampling method . . . . .	98
5. EXACT METHODS OF SYNTHESIS . . . . .	103
5.1 Synthesis by continued fractions . . . . .	103
5.2 Synthesis when $R/T$ is a perfect square . . . . .	108
5.3 Synthesis using radical factors . . . . .	114
5.4 Rational function synthesis . . . . .	118
6. METHODS OF DIFFERENTIAL CORRECTION . . . . .	123
6.1 General principles . . . . .	124
6.2 Classical matrix methods . . . . .	125
6.3 Design by evolution . . . . .	125
6.4 The orthonormal method . . . . .	126
7. APPENDICES . . . . .	126
A. The generation of truncated cosine series . . . . .	126
B. Uniqueness of denominator of $\mathcal{R}$ in continued fraction synthesis . . . .	128
C. Positive real functions . . . . .	129
D. Transmission line analogy . . . . .	130
E. Limiting case: inhomogeneous films. . . . .	134
REFERENCES . . . . .	135

### III. ECHOES AT OPTICAL FREQUENCIES

by I. D. ABELLA (Chicago)

1. INTRODUCTION . . . . .	141
2. THEORY OF SPIN ECHOES AND PHOTON ECHOES . . . . .	142
2.1 Echoes at radio-frequencies . . . . .	143
2.2 Extension to optical frequencies . . . . .	145
2.3 Time-dependent perturbation . . . . .	146
2.4 Large volumes . . . . .	147
2.5 Stimulated echoes . . . . .	149
3. EXPERIMENTAL OBSERVATIONS . . . . .	150
3.1 Source and sample temperatures . . . . .	152
3.2 Detection of echoes . . . . .	154
3.3 Relaxation time measurements . . . . .	159
3.4 Multiple echoes. . . . .	163
3.5 Magnetic field effects . . . . .	164
ACKNOWLEDGEMENTS . . . . .	167
REFERENCES . . . . .	167

### IV. IMAGE FORMATION WITH PARTIALLY COHERENT LIGHT

by B. J. THOMPSON (West Mountain View, California)

1. INTRODUCTION. . . . .	171
--------------------------	-----



2. DIFFRACTION THEORY OF IMAGE FORMATION . . . . .	172
2.1 Image of a point . . . . .	173
2.2 Incoherent object . . . . .	175
2.3 Coherent object . . . . .	176
2.4 The transfer function . . . . .	177
3. CONCEPTS OF THE THEORY OF PARTIAL COHERENCE . . . . .	180
4. IMAGE FORMATION WITH PARTIALLY COHERENT LIGHT . . . . .	183
4.1 Coherent limit . . . . .	186
4.2 Incoherent limit . . . . .	187
4.3 Two-lens imaging system . . . . .	188
4.4 Transilluminated objects . . . . .	188
5. IMAGE OF A TWO-POINT OBJECT . . . . .	191
5.1 Theoretical intensity distribution . . . . .	192
5.2 Resolution criteria . . . . .	194
5.3 Image intensity distribution . . . . .	197
6. IMAGE OF A SINE WAVE . . . . .	202
6.1 Sinusoidal amplitude transmittance . . . . .	203
6.2 Sinusoidal intensity transmittance . . . . .	208
7. IMAGE OF AN EDGE . . . . .	212
7.1 Image intensity distribution . . . . .	213
7.2 The apparent transfer function . . . . .	217
8. IMAGES OF OTHER OBJECTS . . . . .	220
8.1 Images of slit and bar objects . . . . .	220
8.2 Images of reflected light . . . . .	222
8.3 Imaging under controlled coherence conditions . . . . .	224
9. CONCLUSIONS . . . . .	227
ACKNOWLEDGEMENT . . . . .	229
REFERENCES . . . . .	229

## V. QUASI-CLASSICAL THEORY OF LASER RADIATION

by A. L. MIKAEKIAN and M. L. TER-MIKAEKIAN (Erevan, USSR)

1. INTRODUCTION . . . . .	233
2. APPROXIMATE THEORY OF GENERATION AND AMPLIFICATION . . . . .	234
2.1 Derivation of equations . . . . .	234
2.2 Stationary conditions of generation . . . . .	237
2.3 Non-stationary case, averaged along the length of a generating element . . . . .	238
2.4 The amplifier of travelling wave . . . . .	241
3. QUASI-CLASSICAL EQUATIONS . . . . .	244
3.1 Introduction . . . . .	244
3.2 Hamiltonian of interaction of two-level atoms with electromagnetic field . . . . .	248
3.3 Quantum-electrodynamic equations of the theory of generation and their quasi-classical limit . . . . .	252
3.4 The case of accurate resonance . . . . .	258
4. QUASI-CLASSICAL THEORY OF AMPLIFICATION . . . . .	261
4.1 Derivation of equations for the amplifier and their relation to approximate equations . . . . .	261
4.2 The case of high intensities . . . . .	266
5. QUASI-CLASSICAL THEORY OF THE GENERATOR . . . . .	281
5.1 Introduction . . . . .	281
5.2 Stationary conditions . . . . .	286
5.3 Non-stationary conditions . . . . .	288
REFERENCES . . . . .	295

## VI. THE PHOTOGRAPHIC IMAGE

by S. OOUÉ (Saitama, Japan)

1. INTRODUCTION . . . . .	301
2. THE OPTICAL PROPERTIES OF THE PHOTOGRAPHIC IMAGE . . . . .	302
2.1 The absorption of the image . . . . .	303
2.2 Light scattering by the image . . . . .	304
2.3 Unevenness of the optical paths of the photographic layer . . . . .	306
3. THE GRANULARITY . . . . .	309
3.1 Fourier analysis of the granular pattern . . . . .	310
3.2 Measurement of the autocorrelation function and the Wiener spectrum . . . . .	312
3.3 Various effects influencing the granularity . . . . .	318
3.4 The relation between the granularity and the graininess . . . . .	323
4. THE OPTICAL TRANSFER FUNCTION . . . . .	325
4.1 Fourier analysis of the photographic system . . . . .	326
4.2 Measurement of the OTF . . . . .	330
4.3 Various effects influencing the OTF . . . . .	339
4.4 Analysis and evaluation of the OTF . . . . .	348
5. CONCLUSION . . . . .	355
ACKNOWLEDGEMENT . . . . .	356
REFERENCES . . . . .	356

## VII. INTERACTION OF VERY INTENSE LIGHT WITH FREE ELECTRONS

by J. H. EBERLY (Rochester, N.Y.)

1. INTRODUCTION . . . . .	361
1.1 Elementary considerations and conventions . . . . .	362
1.2 Dimensional considerations . . . . .	364
1.3 Speculations with longer wavelengths . . . . .	367
2. ELECTRON IN A MONOCHROMATIC EXTERNAL FIELD . . . . .	368
2.1 Non-relativistic electron orbits . . . . .	368
2.2 The classical relativistic problem . . . . .	369
2.3 Quantum mechanical wave equations and exact wave functions . . . . .	372
2.4 The Green's function . . . . .	376
2.5 The electron self-energy . . . . .	378
3. ELECTRON REFLECTION AND REFRACTION . . . . .	382
3.1 Effective potentials . . . . .	382
3.2 Low energy electron reflection and refraction . . . . .	385
4. PHOTON-ELECTRON SCATTERING . . . . .	386
4.1 Thomson scattering and radiation reaction effects . . . . .	387
4.2 Compton scattering . . . . .	388
4.3 Non-linear Compton wavelength shift and observation of electron mass shift . . . . .	392
4.4 Production of harmonics, beats and electron-positron pairs . . . . .	395
4.5 Kapitza-Dirac scattering, theory and experimental results . . . . .	397
5. SCATTERING FROM BOUND SYSTEMS . . . . .	399
APPENDIX A. Coherent states in quantum field theory . . . . .	401
APPENDIX B. Feynman diagrams in intense-field electrodynamics . . . . .	403
ACKNOWLEDGEMENT . . . . .	410
REFERENCES . . . . .	410
AUTHOR INDEX . . . . .	416
SUBJECT INDEX . . . . .	423

I

**MULTIPLE-BEAM INTERFERENCE  
AND NATURAL MODES IN OPEN RESONATORS\***

BY

G. KOPPELMANN

*II. Physikalisches Institut, Technische Universität, Berlin, 1 Berlin 12*

\* Translated from German by Mrs. J. Welford, London.

## CONTENTS

	PAGE
§ 1. THE CLASSICAL CONCEPT OF MULTIPLE-BEAM INTERFERENCE. . . . .	3
§ 2. THE CONCEPT OF NATURAL MODES. . . . .	12
§ 3. THE PROPERTIES OF OPEN RESONATORS. . . . .	27
§ 4. RELATIONS BETWEEN EIGENMODES AND INTERFERENCE EFFECTS . . . . .	41
REFERENCES . . . . .	62

## § 1. The Classical Concept of Multiple-Beam Interference

Let a plane monochromatic light wave be incident on two high-reflecting and slightly transmitting mirrors I, II (Fig. 1); as a result of the multiple reflections the wave will split into many partial waves which interfere with each other as they superimpose. In the classical treatment it is assumed that the mirrors and waves have an infinite extent and diffraction effects are ignored. The infinite plane waves can then be regarded as beams and therefore the term "multiple-beam interference" is used. <sup>†</sup>

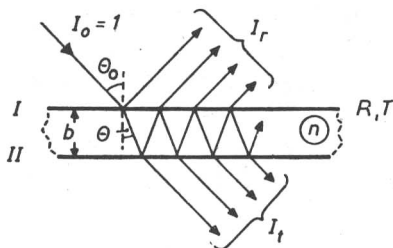


Fig. 1. Formation of multiple-beam interference.

### 1.1. THE AIRY DISTRIBUTION

Let the light intensity reflection factor of one mirror be  $R$ , the transmission  $T$  and the absorption  $A$ , so that

$$R + T + A = 1. \quad (1.1)$$

For simplicity we suppose the mirrors I and II to be the same, and also we assume that the reflectivity is the same for light incident from either side of the reflecting surface.

<sup>†</sup> For details of principles and applications of multiple-beam interference we refer to FABRY [1923], FEUSSNER and JANICKI [1927], TOLANSKY [1948, 1960], CANDLER [1950] and WILCOCK [1959].

By a simple geometrical construction the optical path difference between two successive partial waves in Fig. 1 is found to be  $2nb \cos \theta$ , where  $b$  is the separation between the mirrors and  $n$  and  $\theta$  are the refractive index and the angle of incidence in the medium between the mirrors. There is an additional phase-change  $\psi_r$  on reflection at the mirrors, so that the phase difference between two successive waves is

$$\zeta = (4\pi/\lambda_0)nb \cos \theta + 2\psi_r, \quad (1.2)$$

where  $\lambda_0$  is the vacuum wavelength.

The amplitudes of all the reflected or all the transmitted partial waves are added when they superimpose. At each reflection or transmission the amplitude is attenuated by a factor  $R^{\frac{1}{2}}$  or  $T^{\frac{1}{2}}$ , and so we obtain for the sum of the amplitudes of the transmitted partial waves the complex geometrical series

$$\begin{aligned} u_t &= T \exp \{i(\tfrac{1}{2}\zeta - \psi_r)\} [1 + R e^{i\zeta} + R^2 e^{i2\zeta} + \dots] \\ &= T \exp \{i(\tfrac{1}{2}\zeta - \psi_r)\} [1 - R e^{i\zeta}]^{-1}, \end{aligned} \quad (1.3)$$

the amplitude of the incident wave being put equal to unity. The light intensity transmitted by the system is the squared modulus of the expression (1.3) and is given by the formula first derived by AIRY [1831],

$$I_t = I_{\text{airy}} = \left( \frac{T}{1-R} \right)^2 \left[ 1 + \frac{4R}{(1-R)^2} \sin^2 \tfrac{1}{2}\zeta \right]^{-1}. \quad (1.4)$$

The intensity of the light transmitted by the interferometer varies periodically with the parameter  $\zeta$ ; the form of the curve depends essentially on the reflectivity  $R$  of the mirrors (Fig. 2). The transmission maxima occur when  $\zeta = 2\pi q$ , i.e., when

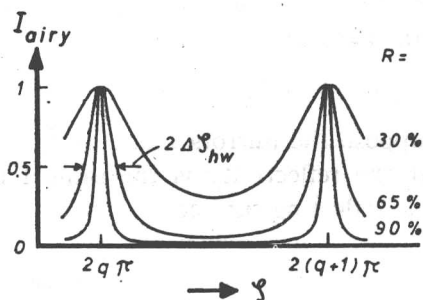


Fig. 2. Airy distribution for different reflection factors  $R$ .

$$\frac{2nb}{\lambda_0} \cos \theta + \frac{\psi_r}{\pi} = q \quad (\text{interference condition}). \quad (1.5)$$

The number  $q$  ( $= 0, 1, 2, \dots$ ) is called the order of interference. Transmission minima occur when  $q$  takes half integral values. From eq. (1.4) the maximum and minimum transmissions are

$$I_{t,\max} = T^2/(1-R)^2, \quad I_{t,\min} = T^2/(1+R)^2. \quad (1.6)$$

The contrast  $C$  of the fringes is therefore

$$C = I_{t,\max}/I_{t,\min} = (1+R)^2/(1-R)^2. \quad (1.7)$$

The sharpness of the fringes can be measured by their half-width  $2\Delta\zeta_{\text{hw}}$  as a fraction of the separation between successive maxima  $\Delta\zeta = 2\pi$ . From the condition  $I_t = \frac{1}{2}I_{t,\max}$  for  $\zeta_{\text{hw}} = 2\pi q \pm \Delta\zeta_{\text{hw}}$ , we obtain from eq. (1.4),

$$\frac{2\Delta\zeta_{\text{hw}}}{2\pi} = \frac{2}{\pi} \arcsin \frac{1-R}{2\sqrt{R}} \approx \frac{1-R}{\pi\sqrt{R}}, \quad (1.8)$$

where we have used the approximation that the reflectivity is large, i.e.  $(1-R) \ll 1$ . The greater the reflectivity, the sharper the fringes (see Fig. 2). The factor  $F = 4R/(1-R)^2$  which occurs in the denominator of the Airy function (1.4) was called by Fabry the coefficient of finesse.

## 1.2. OBSERVATION OF THE INTERFERENCE FRINGES

In order to observe multiple-beam interferences the interferometer mirrors must be mounted and illuminated so that the phase  $\zeta = (4\pi/\lambda_0)nb \cos \theta + 2\psi_r$  can be changed by continuous variation of one of the parameters  $b$ ,  $\theta$ ,  $\lambda_0$  or  $n$ . There is then a variation of intensity corresponding to the Airy function (1.4), which can be observed as an interference pattern on a screen; alternatively if one of these parameters varies in time the temporal intensity variation can be recorded. Table 1 summarizes the best-known techniques of observation; further details are given in the references already quoted.

Interference effects can be observed by all techniques mentioned in Table 1 both in transmission and reflection. If the mirrors are absorption-free the corresponding intensity distributions  $I_t(\zeta)$  and  $I_r(\zeta)$  are complementary:  $I_r = 1 - I_t$ . Thus for high reflectivities there are bright fringes on a dark field in transmission and dark fringes on a bright field in reflection.

In discussing observation techniques it is generally assumed that

TABLE 1  
Summary of multiple-beam interference techniques

Variable	Constant	Designation	Examples of applications	Remarks
$b$	$\lambda, n, \theta$ (usually $\theta = 0$ )	Fringes of equal thickness, Fizeau fringes.	Wedge fringes, (modified) Newton's rings, surface topography, interference microscopy, film thickness measurement ( $\Delta d < \frac{1}{2}\lambda$ ).	Fringes are contour lines with interval $\frac{1}{2}\lambda$ ; they are localized within or near the interferometer. The fringe spacing depends on the angle between the mirrors.
$\theta$	$\lambda, n, b$	Fringes of equal inclination, Haidinger fringes.	Fabry-Perot interferometer (wavelength measurement, length measurement)	Fringes are observed at infinity or in the focal plane of a lens. The interferometer is an angular filter. The fringes are not equidistant.
$\lambda$ or $b/\lambda$	$n, \theta, (b)$ (usually $\theta = 0$ )	Fringes of equal chromatic order.	Interference filter, interference microscopy, film thickness measurement (not restricted to $\Delta d < \frac{1}{2}\lambda$ ).	Fringes are formed in the spectrum and show a profile of the interference space with (in one dimension) a greatly increased scale. Fringe spacing depends on the distance between the mirrors.
$n$	$\lambda, \theta, b$		Pressure scanned Fabry-Perot interferometer.	Fringes are recorded photo-electrically.

only one parameter is varied, the others being held strictly constant; this means, however, that the light must be perfectly collimated and monochromatic or that the reflecting surfaces must be perfectly plane and parallel. The effects of departures from these ideal conditions on the properties of the fringes (fringe broadening and lowering of contrast) have been estimated by TOLANSKY [1948] and they can be calculated in detail by means of convolution integrals <sup>†</sup> (see, e.g.,

<sup>†</sup> In such a calculation, however, it is assumed that the beams in the imperfectly collimated illumination or the beams transmitted through different parts of the interferometer are incoherent, so that their intensities are added on superposition.



DUFOUR and PICCA [1945], CHABBAL [1953], BAYER-HELMS [1963]).

Another class of perturbations to multiple-beam interferences is connected with deviations from the simple form of the Airy sum (1.3). For example, if the light is incident obliquely on a parallel plate interferometer the multiply reflected beams gradually become laterally displaced (walk-off effect) which causes an additional energy loss. In an interferometer with plates inclined at an angle the multiply reflected beams gradually traverse increasing paths, so that the phase difference between successive beams is no longer constant. The effects on the fringes of such errors can also be calculated, or at least tolerances for effectively ideal fringe formation can be estimated (TOLANSKY [1948]).

### 1.3. THE SPHERICAL MIRROR INTERFEROMETERS

CONNES [1956, 1958] conceived and realized a multiple-beam interferometer with two confocal spherical mirrors (Fig. 3a; the radius of curvature  $\rho$  is equal to  $b$ , the distance between the vertices of the

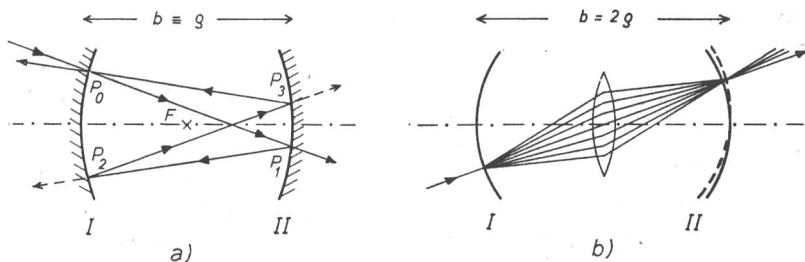


Fig. 3. Interferometers with spherical mirrors: (a) confocal system according to CONNES [1958], (b) concentric system with relay lens according to HERRIOTT [1966].

mirrors). It follows from geometrical optics in the Gaussian approximation that each incident ray emerges from the partially transmitting mirror II after four reflections at  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_0$  both parallel to and at the same point ( $P_1$ ) as the direct ray; this holds also for rays not in the plane of the diagram. The multiply-reflected partial waves all have the same path difference  $4b$ , independent of the angle of incidence and the point of incidence. Thus we have multiple-beam interference effects; they do not appear as fringes, but they can be recorded in time as a function of small changes in the separation between the mirrors. The fringe record will then correspond to that for a plane mirror system with double the mirror spacing. Connes showed that the confocal interference spectrometer has the advantages of greater light-