# PROGRESS IN OPTICS

## VOLUME VII

EDITED BY

E. WOLF

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## PREFACE

The present volume reflects the considerable advances made in recent years in several areas of modern optics. The first article reviews the theory of modes in open resonators and its relation to the theory of multiple beam interference. In the second article an account is given of various methods employed in the design of multilayer filters. The interesting phenomena of photon echoes, first observed about five years ago is discussed in the third article, which reviews the underlying theory and gives an account of the related experimental investigations. A great deal of work concerned with image formation in partially coherent light is discussed in the next article. The fifth article presents a review, written by two Soviet scientists, of the quasi-classical theory of laser radiation. This article should prove to be of special interest to workers in the West, as it contains accounts of many investigations previously published only in Soviet journals. The next article deals with photographic images, paying special attention to their optical quality and to the effects of granularity. In the concluding article investigations are described concerning the interaction between very intense light beams and free electrons. This subject has attracted a good deal of attention in recent years, because of the possibility of observing various interesting new nonlinear phenomena by experiments employing laser light.

The favorable reception accorded to this series of publications in the past, is undoubtedly in a very large measure due to the efforts of the members of the international board of editors of PROGRESS IN OPTICS, to secure articles of high standard. It is with sadness that I record here the death of Professor Hiroshi Kubota, one of the most active members of the Editorial Board. His helpful advice and participation, as well as the personal friendship that he has so generously extended to optical scientists throughout the world will be greatly missed.

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## CONTENTS OF VOLUME I (1961)



## CONTENTS OF VOLUME II (1963)



## CONTENTS OF VOLUME III (1964)



## CONTENTS OF VOLUME IV (1965)

## CONTENTS OF VOLUME V (1966)



## CONTENTS OF VOLUME VI (1967)

# CONTENTS

## I. MULTIPLE-BEAM INTERFERENCE AND NATURAL MODES IN OPEN RESONATORS

*by* G. Koppelman (Berlin)



## II. METHODS OF SYNTHESIS FOR DIELECTRIC MULTILAYER FILTERS

*by* E. Delano and R. J. Pegis (Rochester, N.Y.)

## III. ECHOES AT OPTICAL FREQUENCIES

*by* I. D. Abella (Chicago)



## IV. IMAGE FORMATION WITH PARTIALLY COHERENT LIGHT

*by* B. J. Thompson (West Mountain View, California)

## V. QUASI-CLASSICAL THEORY OF LASER RADIATION

*by* A. L. Mikaelian and M. L. Ter-Mikaelian (Erevan, USSR)

## VI. THE PHOTOGRAPHIC IMAGE

*by* S. Ooue (Saitama, Japan)



## VII. INTERACTION OF VERY INTENSE LIGHT WITH FREE ELECTRONS

*by* J. H. Eberly (Rochester, N.Y.)

I

# MULTIPLE-BEAM INTERFERENCE AND NATURAL MODES IN OPEN RESONATORS*

BY

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* Translated from German by Mrs. J. Welford, London.

## CONTENTS

# § 1. The Classical Concept of Multiple-Beam Interference

Let a plane monochromatic light wave be incident on two high-reflecting and slightly transmitting mirrors I, II (Fig. 1); as a result of the multiple reflections the wave will split into many partial waves which interfere with each other as they superimpose. In the classical treatment it is assumed that the mirrors and waves have an infinite extent and diffraction effects are ignored. The infinite plane waves can then be regarded as beams and therefore the term "multiple-beam interference" is used.†

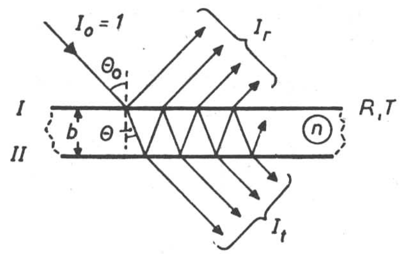


Fig. 1. Formation of multiple-beam interference.

## 1.1. THE AIRY DISTRIBUTION

Let the light intensity reflection factor of one mirror be $R$, the transmission $T$ and the absorption $A$, so that

$$R+T+A = 1. \tag{1.1}$$

For simplicity we suppose the mirrors I and II to be the same, and also we assume that the reflectivity is the same for light incident from either side of the reflecting surface.

† For details of principles and applications of multiple-beam interference we refer to Fabry [1923], Feussner and Janicki [1927], Tolansky [1948, 1960], Candler [1950] and Wilcock [1959].

By a simple geometrical construction the optical path difference between two successive partial waves in Fig. 1 is found to be $2nb \cos \theta$, where $b$ is the separation between the mirrors and $n$ and $\theta$ are the refractive index and the angle of incidence in the medium between the mirrors. There is an additional phase-change $\psi_r$ on reflection at the mirrors, so that the phase difference between two successive waves is

$$\zeta = (4\pi/\lambda_0)nb \cos \theta + 2\psi_r, \tag{1.2}$$

where $\lambda_0$ is the vacuum wavelength.

The amplitudes of all the reflected or all the transmitted partial waves are added when they superimpose. At each reflection or transmission the amplitude is attenuated by a factor $R^{\frac{1}{2}}$ or $T^{\frac{1}{2}}$, and so we obtain for the sum of the amplitudes of the transmitted partial waves the complex geometrical series

$$\begin{aligned} u_t &= T \exp\{i(\tfrac{1}{2}\zeta - \psi_r)\} [1 + R e^{i\zeta} + R^2 e^{i2\zeta} + \cdots] \\ &= T \exp\{i(\tfrac{1}{2}\zeta - \psi_r)\} [1 - R e^{i\zeta}]^{-1}, \end{aligned} \tag{1.3}$$

the amplitude of the incident wave being put equal to unity. The light intensity transmitted by the system is the squared modulus of the expression (1.3) and is given by the formula first derived by AIRY [1831],

$$I_t = I_{airy} = \left(\frac{T}{1-R}\right)^2 \left[1 + \frac{4R}{(1-R)^2} \sin^2 \tfrac{1}{2}\zeta\right]^{-1}. \tag{1.4}$$

The intensity of the light transmitted by the interferometer varies periodically with the parameter $\zeta$; the form of the curve depends essentially on the reflectivity $R$ of the mirrors (Fig. 2). The transmission maxima occur when $\zeta = 2\pi q$, i.e., when

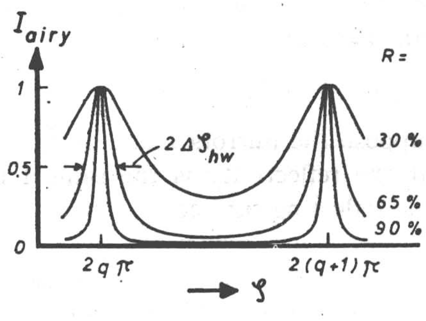


Fig. 2. Airy distribution for different reflection factors $R$.

$$\frac{2nb}{\lambda_0}\cos\theta + \frac{\psi_r}{\pi} = q \qquad \text{(interference condition)}. \tag{1.5}$$

The number $q$ ($= 0, 1, 2, \cdots$) is called the order of interference. Transmission minima occur when $q$ takes half integral values. From eq. (1.4) the maximum and minimum transmissions are

$$I_{t,\max} = T^2/(1-R)^2, \qquad I_{t,\min} = T^2/(1+R)^2. \tag{1.6}$$

The contrast $C$ of the fringes is therefore

$$C = I_{t,\max}/I_{t,\min} = (1+R)^2/(1-R)^2. \tag{1.7}$$

The sharpness of the fringes can be measured by their half-width $2\Delta\zeta_{hw}$ as a fraction of the separation between successive maxima $\Delta\zeta = 2\pi$. From the condition $I_t = \frac{1}{2}I_{t,\max}$ for $\zeta_{hw} = 2\pi q \pm \Delta\zeta_{hw}$, we obtain from eq. (1.4),

$$\frac{2\Delta\zeta_{hw}}{2\pi} = \frac{2}{\pi}\arcsin\frac{1-R}{2\sqrt{R}} \approx \frac{1-R}{\pi\sqrt{R}}, \tag{1.8}$$

where we have used the approximation that the reflectivity is large, i.e. $(1-R) \ll 1$. The greater the reflectivity, the sharper the fringes (see Fig. 2). The factor $F = 4R/(1-R)^2$ which occurs in the denominator of the Airy function (1.4) was called by Fabry the coefficient of finesse.

### 1.2. OBSERVATION OF THE INTERFERENCE FRINGES

In order to observe multiple-beam interferences the interferometer mirrors must be mounted and illuminated so that the phase $\zeta = (4\pi/\lambda_0)nb\cos\theta + 2\psi_r$ can be changed by continuous variation of one of the parameters $b$, $\theta$, $\lambda_0$ or $n$. There is then a variation of intensity corresponding to the Airy function (1.4), which can be observed as an interference pattern on a screen; alternatively if one of these parameters varies in time the temporal intensity variation can be recorded. Table 1 summarizes the best-known techniques of observation; further details are given in the references already quoted.

Interference effects can be observed by all techniques mentioned in Table 1 both in transmission and reflection. If the mirrors are absorption-free the corresponding intensity distributions $I_t(\zeta)$ and $I_r(\zeta)$ are complementary: $I_r = 1 - I_t$. Thus for high reflectivities there are bright fringes on a dark field in transmission and dark fringes on a bright field in reflection.

In discussing observation techniques it is generally assumed that

TABLE 1

Summary of multiple-beam interference techniques

| Variable | Constant | Designation | Examples of applications | Remarks |
|---|---|---|---|---|
| $b$ | $\lambda, n, \theta$ (usually $\theta = 0$) | Fringes of equal thickness, Fizeau fringes. | Wedge fringes, (modified) Newton's rings, surface topography, interference microscopy, film thickness measurement ($\Delta d < \frac{1}{2}\lambda$). | Fringes are contour lines with interval $\frac{1}{2}\lambda$; they are localized within or near the interferometer. The fringe spacing depends on the angle between the mirrors. |
| $\theta$ | $\lambda, n, b$ | Fringes of equal inclination, Haidinger fringes. | Fabry-Perot interferometer (wavelength measurement, length measurement) | Fringes are observed at infinity or in the focal plane of a lens. The interferometer is an angular filter. The fringes are not equidistant. |
| $\lambda$ or $b/\lambda$ | $n, \theta, (b)$ (usually $\theta = 0$) | Fringes of equal chromatic order. | Interference filter, interference microscopy, film thickness measurement (not restricted to $\Delta d < \frac{1}{2}\lambda$). | Fringes are formed in the spectrum and show a profile of the interference space with (in one dimension) a greatly increased scale. Fringe spacing depends on the distance between the mirrors. |
| $n$ | $\lambda, \theta, b$ | | Pressure scanned Fabry-Perot interferometer. | Fringes are recorded photo-electrically. |

only one parameter is varied, the others being held strictly constant; this means, however, that the light must be perfectly collimated and monochromatic or that the reflecting surfaces must be perfectly plane and parallel. The effects of departures from these ideal conditions on the properties of the fringes (fringe broadening and lowering of contrast) have been estimated by TOLANSKY [1948] and they can be calculated in detail by means of convolution integrals † (see, e.g.,

† In such a calculation, however, it is assumed that the beams in the imperfectly collimated illumination or the beams transmitted through different parts of the interferometer are incoherent, so that their intensities are added on superposition.

DUFOUR and PICCA [1945], CHABBAL [1953], BAYER-HELMS [1963]).

Another class of perturbations to multiple-beam interferences is connected with deviations from the simple form of the Airy sum (1.3). For example, if the light is incident obliquely on a parallel plate interferometer the multiply reflected beams gradually become laterally displaced (walk-off effect) which causes an additional energy loss. In an interferometer with plates inclined at an angle the multiply reflected beams gradually traverse increasing paths, so that the phase difference between successive beams is no longer constant. The effects on the fringes of such errors can also be calculated, or at least tolerances for effectively ideal fringe formation can be estimated (TOLANSKY [1948]).

### 1.3. THE SPHERICAL MIRROR INTERFEROMETERS

CONNES [1956, 1958] conceived and realized a multiple-beam interferometer with two confocal spherical mirrors (Fig. 3a; the radius of curvature $\rho$ is equal to $b$, the distance between the vertices of the

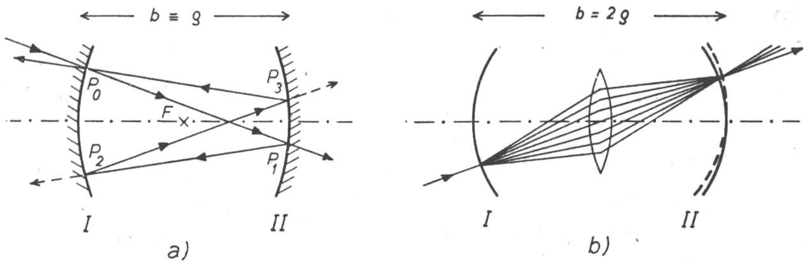


Fig. 3. Interferometers with spherical mirrors: (a) confocal system according to CONNES [1958], (b) concentric system with relay lens according to HERRIOTT [1966].

mirrors). It follows from geometrical optics in the Gaussian approximation that each incident ray emerges from the partially transmitting mirror II after four reflections at $P_1$, $P_2$, $P_3$ and $P_0$ both parallel to and at the same point ($P_1$) as the direct ray; this holds also for rays not in the plane of the diagram. The multiply-reflected partial waves all have the same path difference $4b$, independent of the angle of incidence and the point of incidence. Thus we have multiple-beam interference effects; they do not appear as fringes, but they can be recorded in time as a function of small changes in the separation between the mirrors. The fringe record will then correspond to that for a plane mirror system with double the mirror spacing. Connes showed that the confocal interference spectrometer has the advantages of greater light-