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Relaxation Processes in Micromagnetics

微磁学中的弛豫过程

(影印版)

〔美〕祖尔 (H. Suhl) 著



北京大学出版社
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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学家的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究 and 积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套“中外物理学精品书系”的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任
中国科学院院士,北京大学教授

王恩哥

2010年5月于燕园

PREFACE

This book addresses the issue of translating a number of microscopic relaxation mechanisms into a language adapted to the peculiarities of the dynamics of the magnetization vector. It brings to light some previously unexplored, and some unexpected, features of that dynamics. Only modest mathematical equipment is employed throughout, but some familiarity with the simpler properties of ferromagnets at the level of advanced undergraduates or first year graduate students is assumed. The book should also be of some interest to professionals in the magnetic recording industry.

Magnetic phenomena have fascinated people for several millennia, but besides their early use for determining the magnetic north, really major applications date back no more than two or three centuries. A big advance in the harnessing of magnetic phenomena went hand in hand with the growing understanding of electromagnetism that culminated in the nineteenth century. More recent times have seen a development of comparable import: the utilization of magnetism in information technology. Magnetic recording and storing of immense amounts of information in a small space, and at (so far) steadily diminishing cost has been one of the two mainstays of information technology, with semiconductor progress the other. The steady progress in magnetic recording technology is based on material science on the one hand and on advances in engineering of very small structures and particle assemblies on the other. As in many other fields of applied science, empiricism holds sway, and one may ask what role, if any, theory plays in the advance of magnetic recording. The most helpful kind of theory, intended to provide direct support and guidance to experimentalists, is based on principles described by very plausible equations that have proved their worth in many contexts. Aided by computer power, equally plausible solutions have been obtained in many cases, and often provide a reasonable fit to observation. As a result, the provenance of these equations is rarely questioned. This is not at all unreasonable if the focus is on empirical advances in the recording industry. Nonetheless a critical examination of the origin of the equations is in order, if only as an insurance policy against unexpected aspects that may lurk in future developments. In particular, the manner in which various relaxation mechanisms enter the ultimate form of the equations deserves attention. In so far as this relaxation has to do with only transfer of energy of magnetic motion to nonmagnetic degrees of freedom of the substrate, it is obviously very inconvenient to carry along these degrees of freedom in a calculation designed to match theory to observation. Experiments almost always measure *only* the magnetization. Accordingly, part of this book will be concerned with elimination of the unwanted degrees of freedom. The result is an equation of motion for the magnetization vector alone. However, a price must be paid: the resulting equation in general

involves terms that are non-local in space and that involve the pre-history of the magnetization up to the time of interest. Asymptotic expansions of this result then determine the range of space and time variations over which these effects are of importance. In Chapter 3, five different relaxation mechanisms are considered in this light. Two of these describe the loss mechanism in magnetic metals: one (an obvious one) involves the eddy currents induced by a moving magnetization; the other, a quasithermodynamic one, is the result of ‘breathing’ of the Fermi surface owing to that motion. But there is one mechanism that has been left out in this work: direct relaxation of the magnetization in metallic ferromagnets owing to scattering of conduction electrons. In the opinion of this author, a credible account of this mechanism in transition metals must await clarification of the origin of their ferromagnetism. The same physics involved in damping of the motion of the magnetization must be involved in establishing the magnetization in the first place. Admittedly, it is tempting to describe the magnetization in terms of a sophisticated mean field theory, and to attribute the loss mechanism to inelastic electron scattering in this mean field, but to achieve a credible self-consistency would not be easy.

Magnetic recording by its nature involves large motions of the magnetization vector. Most purely analytic studies, on the other hand, deal with small motions that allow linearization of the equation of motion. In the absence of an adequate analytic theory far beyond the linearized limit and its lowest nonlinear corrections, recourse is had to extensive computer simulations. Undoubtedly the more thoughtful studies of this kind can yield important insights, but in this book, the emphasis is on analytic solutions of the nonlinear equations with only minimal computational assistance. To focus on the essential qualitative features of large motions, only the simplest non-trivial model will be treated in detail: a magnetic specimen with uniaxial anisotropy of crystalline and/or demagnetizing origin. Also, these solutions are restricted to only the lowest term in the above-mentioned asymptotic expansion. Only one chapter is devoted to small motions. Analyzed judiciously, these occasionally provide clues or correspondences with the nonlinear situation. But, more importantly, linear theory and the lowest nonlinear extensions thereof hold the promise of applications, such as magnetic delay lines. These studies, very briefly sketched in Chapter 2, are based on series expansions in powers of small motion amplitudes and have provided elegant demonstrations (both theoretical and in the laboratory) of soliton-like propagation of magnetic disturbances.

In this book, the magnetization field as a function of time and position is treated as a classical quantity. This classical field is a gross manifestation of the electronic spins and orbits and their largely quantum mechanical interactions in a magnetic medium, but these nonclassical features enter a coarse-grained description of the field only in the form of coefficients in its equations of motion. Except in specialized investigations employing scanning tunnelling microscopy, or magnetic force microscopy, of magnetic surfaces, measurements are coarse grained because of the limited resolution of equipment. The grains should be

considered large enough so that the spins in a grain centered on position \vec{x} say, are coupled to yield a large, essentially classical magnetic moment vector $\vec{M}(\vec{x}, t)$ of \vec{x} - and t -independent magnitude M_s well below the Curie temperature. This limits the description to excitations of energies sufficiently low so that the coupling of the spins within a grain is not significantly disrupted. Otherwise stated, this description is applicable only to phenomena of sufficiently gentle spatial variations. Just below the transition temperature, the magnitude of the magnetic moment is also variable and may have to be taken into account in so-called thermally assisted recording. A short section at the end of Chapter 1 addresses this matter. In theory, one can imagine quantum effects showing up even at the level of a coarse description. For example, one may consider spontaneous depinning of domain walls. However, the jury is still out on the question of submergence of such observations in extraneous effects.

Quite often, observations deal with a magnetization field averaged over distances orders of magnitude larger than the coarse-grained distances referred to above. Such a spatial average vector, call it \vec{M}' , is sometimes assumed to satisfy dynamic equations originally designed for special cases in which there is no significant distinction to be made between local and average values, as, for example, in magnetic particles smaller than a typical domain wall width. But, in general, \vec{M}' will satisfy equations with relaxation terms of a structure very different from those appropriate for the local \vec{M} field. The reason is that \vec{M}' is in contact with many other modes of magnetic motion of essentially zero spatial average that will detract from the magnitude of \vec{M}' . The process by which any one mode of magnetic motion loses energy through coupling to other modes of magnetic motion will be called distributive damping in these pages. In contrast, damping that results from transfer of energy to degrees of freedom of the host medium will be called intrinsic damping, and is treated in Chapter 3. In samples too small to support domain walls, it is the only allowed form of relaxation, but, in general, distributive and intrinsic relaxation will occur side by side.

Relaxation processes cannot be adequately discussed without reference to the closely related subject of fluctuations. This close relation is particularly evident in the famous fluctuation-dissipation theorem. It relates the dissipative part of the *linear* response of a system to the mean square fluctuations in the responding degrees of freedom. A rigorous proof has been given only for linearized equations of motion, classical or quantum mechanical. But for reasons that are not totally clear (at least not to this author), when a statistical ensemble of systems with infinitely many degrees of freedom is considered, a seemingly more general form of the fluctuation-dissipation theorem appears. In such a system, the role of the mean square fluctuations is played by the diffusion coefficients of the system. In this formulation, linear response of any particular one of these degrees of freedom does not appear to be a requirement. At least, this is the case if the motion of the ensemble distribution obeys the Fokker-Planck equation. Chapter 4 deals with these matters with special reference to ferromagnetic systems, but,

for the benefit of readers not especially familiar with this subject, some general background material is included.

Chapter 5 is devoted entirely to the question of magnetization reversal in small, effectively single-domain particles with given uniaxial anisotropy, and sufficiently sparse so that their interaction is neglected, even though this may not be quite justified in view of the long-range character of dipolar forces. The treatment is based on the Fokker-Planck equation, transformed to look like the Schroedinger equation of quantum mechanics. When the \vec{M} vector is constrained to move in a plane, that Schroedinger equation is the same as that for a particle in a one-dimensional periodic lattice, a thoroughly explored subject. The main interest from the point of view of magnetic recording is the graph of the field, the coercive field, necessary to reverse the magnetization in a given time. The main advantage of the method presented here is that it avoids the need for separate considerations for applied fields greater than or less than the anisotropy field. Most of the work assumes the easy magnetization direction to be parallel to the applied field, but one small section deals with the case of misalignment of these directions. The generalization to unconstrained rotation of \vec{M} is discussed, still for purely uniaxial anisotropy, using the fact that the azimuthal motion of \vec{M} about the field direction cannot seriously affect reversal times and may be averaged out. In another section, the relation of the work presented here to standard reaction rate theory and first passage time theories is outlined.

Chapter 6 consists of two parts. The first part examines the motion of a more dense array of single-domain particles, interacting by dipolar forces. It is found that, in applied fields less than the (still assumed uniaxial) anisotropy fields, the motion is in general chaotic. Applied fields substantially exceeding anisotropy fields tend to extinguish chaos, replacing it by 'clean', though multiperiodic, motion. The chapter begins by presenting a simple integrable case: just two particles with the line joining their centers parallel to the field applied along the easy direction. The slightest deviation from this lineup results in chaos except in sufficiently large fields. As one might expect, the motion of three such particles is shown to be even more prone to chaos. From these examples it is concluded that, in a sufficiently dense (but not too dense) array, general chaotic motion should prevail at low applied fields, and that its characteristic time scale may well be shorter than the time scale of intrinsic damping. Then chaos appears as an extreme case of distributive damping and we speculate that intrinsic relaxation may then be regarded as the 'small friction' scenario of Kramers' diffusion model of chemical reactions. In that scenario, only one variable is relaxing: the energy of the system, the detailed motion of its particles becoming irrelevant. If our speculation turns out to be valid, simplicity will have emerged from chaos.

The second part of the chapter deals with reversal in continuous media. As is well known, chaos in a system of discrete particles does not necessarily carry over to the limit of a system of infinitely small, but infinitely dense particles forming a continuum. Usually new and different physics arises. The question of irregular motion in a continuous medium must be considered *ab initio*. (The same may

apply even to discrete, but extremely dense particle arrays.) Examining conditions of integrability of the partial differential equations of the magnetization field is obviously very hard. One might suspect that the instabilities of small motions discussed in Chapter 2 portend stochastics of large motions, but no evidence to this effect, experimental or theoretical, has come to light so far. Therefore the rest of Chapter 6 is confined to purely deterministic methods.

In an infinite, perfect sample, magnetization reversal can occur by uniform rotation, or by domain wall motion. In the case of rotation, the magnetization vector everywhere turns from one easy direction to the opposite easy direction in a magnetic field applied along the latter direction. On the other hand, reversal by domain wall motion depends on the existence of a domain wall separating regions of opposite magnetization direction prior to the switch. In that case, some small region of the sample must already have its magnetization in the final direction to be reached by the entire sample as the applied field drives the wall along. In the case of rotation, the graph of magnetization versus field as that field is cycled between positive and negative values exceeding the coercive field is a rectangle: the hysteresis loop. In the case of reversal by domain wall motion, that loop shrinks to a line: no hysteresis occurs.

However, real samples are finite, and even if they are free of imperfections, these conclusions require major modifications, mainly owing to dipolar forces as manifested by demagnetizing fields. These tend to render perfectly uniform rotation unstable. A stable reversal process requires the magnetization field to assume a distinctive non-uniform pattern. This pattern minimizes the effect of the demagnetizing field, at the expense of introducing exchange torques set up by non-uniformity. But, on balance, stable magnetization reversal will result. Cycling the applied field still results in a hysteresis loop, but a more 'skinny' one than that corresponding to uniform rotation. In applied fields less than the field equivalent of the total anisotropy barrier energy (crystalline and/or demagnetizing), magnetization reversal can occur only with the assistance of thermal fluctuations. In this range of applied fields, the theory for the ideal sample with ideal boundary conditions parts company with a more realistic treatment. The activation energy needed to overcome the barrier in the finite, but idealized sample is proportional to the total sample volume, contrary to observation. Instead, the barrier is overcome only in the immediate vicinity of a nonuniformity in the underlying physical properties, particularly at a surface. The activation energy is proportional to the quite small volume of the imperfection. Once the magnetization on the far side of the imperfection is reversed, it provides the seed for further reversal by domain wall motion.

Thus the switching speed of a sample large enough to allow domain wall formation should be related to the speed of propagation of a domain wall. The only available theories are based on the assumption of a uniform wall velocity. As the result of this assumption, the partial differential equations describing the motion are replaced by ordinary nonlinear differential equations, which then describe the shape of the wall as viewed by an observer moving along with it.

Much of Chapter 6 is devoted to the motion of a Néel domain wall in these terms, with the rotation of the magnetization confined to a plane, as it might be in a thin film. This problem has some formal similarities with propagation problems in chemistry and biology. Permitted ranges of stable velocities are given in terms of applied and anisotropy fields, but the actual final steady value of the velocity cannot be determined without solving for the transient following launch of the wall. That transient obviously does not allow replacement of a time rate of change by a velocity times a spatial rate of change. Kolmogorov and co-workers have succeeded in relating a final steady velocity to a certain class of initial conditions. In the present problem, a plausible argument is presented for the speed of the wall shortly after launch; quite possibly that speed will persist. One weak point in the notion of a steady wave traveling with uniform velocity for ever is that it does not account for its fate upon arrival at its destination. The author is not aware of any literature addressing this question analytically. Here, an argument is presented suggesting that, as the state of complete magnetization reversal is approached, the speed of the wall steadily grows to a (formally) infinite value.

The moving Bloch wall is considerably harder to analyze, since it involves both the azimuthal and the polar angles of the magnetization vector. There is only one known analytic solution of this problem, that of L. R. Walker. It is a tour de force, both elegant and exact, and for these reasons it is presented in these pages. However, its singular character makes it hard to judge if it is in some sense an isolated, not widely applicable triumph.

The motion of actual domain walls, even in ideal media, conforms with these types of theories only in certain geometries and/or ranges of applied field strength. For applied fields well above saturation value, they should be more or less valid. In field values below saturation, the very concept of a wall velocity becomes dubious, except in special geometries. The final section of Chapter 6 discusses the behavior of domain walls in applied fields below saturation. No attempt is made to explore this subject in depth, and treatment is restricted to domain walls in the form of sheets that render the problem effectively two dimensional. In samples of dimensions larger than a typical domain wall, static domain walls form owing to demagnetizing effects alone, without significant involvement of the exchange field inside the walls. In the absence of an applied field, the magnetization distribution in the sample arranges itself so as to result in zero internal field also (any crystalline anisotropy field of course remains), resulting in a particular domain wall configuration. In a finite subsaturating applied field, the magnetization still attempts to shield the interior from the external field, reducing the interior field to zero as far as possible (some field penetration must occur at sharp edges or corners). As the applied field is increased adiabatically, the wall configuration moves in such a way as to increase the size of domains with magnetizations tending towards lineup with the field, pervading the entire sample when the applied field reaches saturation value. This process can be analyzed rigorously for a certain class of sample shapes, and from that analysis some physical conclusions may be drawn that allow a simple geometric construction

for the general situation. When the external field is increased dynamically, a purely analytic treatment of the wall motion becomes impossible, even for that class of sample shapes. However, it is proposed that, in the dynamic case, the development of the wall configuration, even if quite complex, the time dependence is nearly the same as that of a primitive case, for which an exact solution is presented. The reason why the usual concept of domain wall velocity is not applicable here is simple: in the subsaturated case there is no length scale; configurations and their movements depend only on sample shapes since exchange length is not significantly involved. Standard wall motion theory on the other hand depends on that length.

This book is focussed on analytic formulations of magnetic relaxation processes on the one hand, and on ways of solving the resulting equations in their full nonlinear form without significant resort to computer simulation on the other, at least for simple cases. Unfortunately, this narrow focus did not allow inclusion of certain highly topical subjects, such as relaxation processes in multilayer structures, the physics of exchange bias, magnetization fields in constricted structures, proximity effects and spin polarized tunneling into nonmagnetic media, etc. Hopefully, the methods adopted in this work will prove relevant in at least some of these areas.

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THE CLASSICAL MAGNETIZATION FIELD

1.1 Introduction

At the time of writing, all applications of magnetically ordered materials can safely avoid explicit reference to the quantum mechanical aspects of the underlying spin and orbital degrees of freedom of the constituent ions. Also, it is not normally necessary to keep track of the atomistic character of the material. The phenomena of interest are usually describable as spatial averages over many lattice spacings. In such a description, the microscopic quantum aspects appear only in the form of constants in the equations governing the statics and dynamics of the magnetization. There are exceptions (for example magnetic force microscopes) that probe the material on a more or less atomic scale. Also, with the advent of nanoscience, intrinsically quantum mechanical phenomena that have no classical limit (such as tunneling of the magnetization as such) may sooner or later require attention. This book will largely be confined to a macroscopic description in terms of a magnetization field as a continuous function of position, governed by classical equations of motion.

We begin with an account of the plausibility of this point of view, at least for magnetic insulators in which the ionic spins (or effective spins) may be considered localized. At first sight, the case of metals would seem to lead to the classical magnetization field more readily, since the itinerant character of the electrons makes a quantum mechanical spin density field operator a logical starting point. In fact, the transition to the classical field description of magnetic metals requires rather advanced theoretical methods outside the scope of this work. For early efforts in that direction, see Herring and Kittel (1951).¹ It is conceptually easier to consider the case of magnetic insulators with a localized set $\{\vec{S}_i\}$ of N spins. Let H denote their Hamiltonian, i.e. their total energy expressed in terms of the S_i . Their Heisenberg equations of motion are

$$i\hbar d\vec{S}_i/dt = \left(\vec{S}_i, H \right)_- \quad i = 1 \dots N \quad (1.1)$$

with the right hand side denoting the commutator. The Hamiltonian H is always a multinomial in the components S_i^α of \vec{S}_i , ($\alpha = 1, 2, 3$). The commutation

¹For a nonmagnetic electron gas, reducing the quantum mechanics to a more phenomenological description, such as Fermi liquid theory, has more recently been found possible using renormalization theory (Shankar, 1994). For possible extensions to magnetism, see Chubukov (2005) and Rech *et al.* (2006).

relations of these components S_i^α of \vec{S}_i are

$$S_i^\alpha S_j^\beta - S_j^\beta S_i^\alpha = i\hbar \delta_{ij} \epsilon_{\alpha\beta\gamma} S_j^\gamma \quad (1.2)$$

Consider, for example, the commutator $(S_i^\alpha, (S_i^\beta)^2)_-$ that might be encountered during the evaluation of the commutator in equation (1.1). It is equal to

$$\begin{aligned} (S_i^\alpha, S_i^\beta)_- S_i^\beta + S_i^\beta (S_i^\alpha, S_i^\beta)_- &= i\hbar \epsilon_{\alpha\beta\gamma} \left(S_i^\gamma S_i^\beta + S_i^\beta S_i^\gamma \right) \\ &= i\hbar \epsilon_{\alpha\beta\gamma} \left(2S_i^\gamma S_i^\beta + i\hbar \epsilon_{\beta\gamma\delta} S_i^\delta \right) \\ &= i\hbar \epsilon_{\alpha\beta\gamma} S_i^\gamma \left(d(S_i^\beta)^2 / dS_i^\beta + o(\hbar) \right). \end{aligned}$$

Similar results hold for commutators of the S_i^α with the higher powers and/or products of spin components that occur in H . From this it follows that

$$\frac{d\vec{S}_i}{dt} = -\vec{S}_i \times \frac{\partial H}{\partial \vec{S}_i} + o(\hbar) \quad (1.3)$$

As $\hbar \rightarrow 0$, this equation, though still in terms of operators, has the same form as the classical equations of motion of magnetic moments associated with the spins. Thus, (looking forward to eqn (1.4)), this obeys the correspondence principle, and becomes a classical equation in the limit of large spins. (For large spins, the right hand side of eqn (1.2) may be neglected compared with the quantities on the left. The latter then commute, i.e. behave classically.) However, this is no great comfort; in many materials the individual spins are not large. Thus, if we desire a classical description of a coarse-grained magnetization field, we need to examine the sense in which the spin may be considered large, even though the constituent spins may be small.

This is made plausible as follows: A full quantum mechanical treatment of the system would supply a correlation length of the spin system. At temperatures well below the Curie temperature, this correlation length will be large, comprising n lattice sites, say, around position \vec{x} . Then over a region n^3 , the spin orientations will hardly change from an average direction, $\langle \vec{S}(\vec{x}) \rangle$. This average may thus be viewed as a single, rigid spin of magnitude $n^3 S$, independent of \vec{x} , with only very small fluctuations around this value. Similarly, its magnitude should fluctuate in time with only small amplitude. A continuum field view is then justified if n is large enough to comprise many spins, yet small on the scale of distances over which the direction of the magnetization changes. The spin system may then be represented by a magnetization field $\vec{M}(\vec{x}, t)$ of constant magnitude $M(x, t)$ practically equal to the saturation magnetization M_s at the ambient temperature $T \ll T_c$. The mathematics needed to justify this qualitative argument is known as renormalization theory.