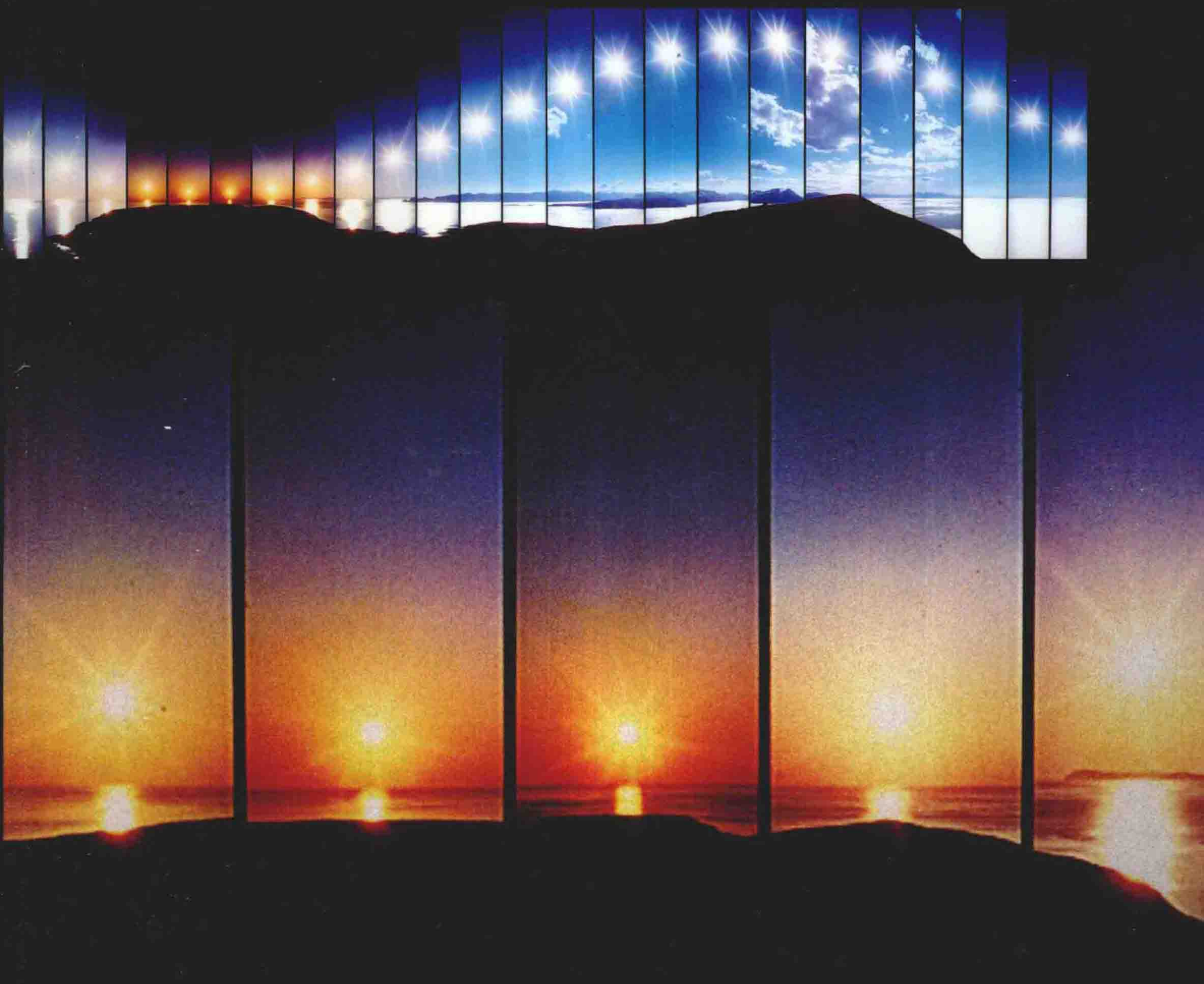
PRECALCULUS

CONCEPTS IN CONTEXT



MORAN · DAVIS · MURPHY

PRECALCULUS
CONCEPTS IN CONTEXT

Judith Flagg Moran
Trinity College

Marsha Davis
Eastern Connecticut State University

Mary Murphy
Smith College

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For more information contact:

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20 Park Plaza
Boston, MA 02116

International Thomson Publishing Europe
Berkshire House I68-I73
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England

Thomas Nelson Australia
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2-2-1 Hirakawacho
Chiyoda-ku, Tokyo 102
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PREFACE

"One of our goals in writing *Precalculus: Concepts in Context* was to dispel the notion that only certain people can succeed in mathematics."

At first, we thought, the world does not need another precalculus text.

Anyone looking at today's textbook market knows that there is an abundance of precalculus books available, and that many of them are very good. However, we have been teaching precalculus mathematics to college students for years, and we've given a lot of thought to what the course ought to entail. In attempting to integrate graphing technology and laboratory experiences with classwork, we realized that the elements and the organization of a conventional precalculus course were no longer working effectively for us and our students. We decided to reexamine, not only the way in which precalculus is taught, but also what precalculus *means.*

Precalculus: Concepts in Context is a completely new kind of textbook. Not only does it take a fresh look at the content of precalculus, but it offers students a different approach to learning mathematics.

Starting from scratch, we have constructed a one-semester or two-quarter course in elementary functions that emphasizes the mathematical concepts needed for calculus and incorporates today's graphing technology as an essential learning tool.

We have written a book that is truly interactive. It engages students by asking them to respond on nearly every page, and the text is not complete until the students do their part. It involves students with their classmates through frequent activities that require collaboration.

Our goals in writing *Precalculus Concepts* are these:

- To present mathematics as both a symbolic language (algebra) and as a visual language (graphs), and to assist students in becoming fluent users of both languages and adept translators to and from the languages of algebra, graphs, and English.
- To enable students to recognize mathematical principles in the natural world and in everyday life.
- To engage every student actively in the process of learning and doing mathematics.
- To foster a spirit of collaboration rather than competition inside and outside the classroom.
- To use writing about mathematics as a means of promoting a deeper understanding of patterns and concepts.
- To dispel the notion that only certain people can succeed in mathematics.
- To promote the use of calculators and computers as tools for investigating mathematics, ones that reduce the time spent on numerical calculations and point-plotting and support the student in accomplishing and understanding *more* rather than less.
- To emphasize mathematical reasoning as well as numerical results and to encourage a variety of methods for reaching a conclusion.
- To persuade students that independent thinking, rather than mimicry of a worked-out example, is a problem-solving approach that will serve them best in the long run.
- To prepare students for calculus, particularly for any of the reformed calculus courses.

Computers and calculators change both the way students learn and the way instructors teach. Many ideas that once required calculus can now be understood very

"Instead of starting with abstract mathematical ideas and looking for applications that illustrate them, we begin with the world of experience—music, commerce, psychology, natural science, and the daily news—and uncover the mathematics already present."

well at the precalculus level with the aid of a graphing utility. There is no need, for example, to wait until calculus to introduce (informally) the rate of change of a function. On the other hand, some topics to which we used to devote much time are no longer necessary. Algebraic techniques such as synthetic division to find roots of polynomials, for instance, are not essential these days because technology gives us very good estimates of those roots. Moreover, because we can use technology to perform complicated calculations, we can situate our mathematics in authentic real-world contexts, despite their tendency to be numerically "messy." Students, instead of having to wait two or three more semesters for honest applications, get to experience first-hand the relevance of mathematics.

Adding topics to an already overpacked traditional curriculum isn't a good way to adapt to the electronic age. Many in the mathematics community see the need to change what precalculus *is,* not simply the way in which it is taught. We have attempted a "re-vision" of the precalculus curriculum, starting afresh with only those topics and techniques a student needs for a solid understanding of elementary functions and tossing out (or tossing into an appendix) everything else. We have made efforts to dovetail our changes with those of the leading calculus reform projects.

Precalculus: Concepts in Context isn't for every school. Instructors looking for a traditional precalculus text will be disappointed, if not aghast, at the informal appearance of this book. To help students feel at ease with their text, we use humor, gentle explanations, and immediate reinforcement. We try, though, to encourage rigorous thinking without the formal "definition, theorem, proof" format appropriate for more advanced courses. The text sows the seeds for important calculus concepts as it develops the skills and mathematical maturity needed for calculus and for life itself. A course based on this text will succeed only with an instructor committed to the goals we have outlined above. This isn't a "teacher-proof" text.

We are thoroughly convinced of the value of learning through cooperation, and therefore we ourselves have made the writing of this book a collaborative effort. Each chapter was written by one of the three authors, reviewed by the other two, class-tested in several precalculus courses with a variety of students who were not shy about suggesting changes, and then completely revised by all three authors and rewritten by one of the two who had not written it originally. We encourage this sort of collaboration on a smaller scale among the students who use this text, particularly when they write and edit a group lab report.

AUDIENCE

Precalculus: Concepts in Context is intended for a one-semester or two-quarter course in precalculus mathematics. Students should have studied, but not necessarily mastered, the equivalent of two years of high-school algebra and a year of geometry. Many students who enroll in a college precalculus course have already taken a fourth year of high-school mathematics (that is, a course beyond Algebra II, which might even have been called "precalculus"), but they feel unready for calculus. Using this textbook will allow those students a fresh view of the material they saw in their earlier course and an opportunity to understand thoroughly the rules that they might simply have memorized the first time around. The approach is sufficiently different from that of a standard precalculus text that such students will not feel defeated from the beginning. (We want to avert the initial reaction, "This is the same stuff I couldn't understand last year; I won't understand it now, either.") At the same time, the text neither condescends to nor overwhelms those students who are seeing the material for the first time.

The text is suitable for a three- or four-semester-hour course. There is enough material for a six-credit course that stretches over two quarters or two trimesters. Specific suggestions for a variety of paths through the text will be found in the *Instructor's Guide.*

The text has been used in small (fifteen to thirty student) classes and could be used in large lecture courses if the students have frequent access to a graphing utility and regular opportunities to meet in small groups.

As its name implies, *Precalculus: Concepts in Context* is designed to prepare a student for calculus. At the same time, we recognize that precalculus is a terminal course for many students, and we have taken pains to design a course that has its own integrity and is worth studying in its own right.

ORGANIZATION

The mathematical concept of *function* is the unifying idea of this text. We picture the book as a mathematical tapestry, in which the principal topics form the horizontal strands and the recurring themes form the vertical strands.

The topics of a standard precalculus course constitute the woof strands of the tapestry. In that sense, this text is organized in a conventional manner; topics appear in the expected order. Students meet functions in general and various ways of representing them and then move to specific classes of functions, ranging from linear to exponential and periodic. Along the way, they learn about various transformations of functions and they become adept at interpreting graphs.

The major themes of the book constitute the warp strand of the tapestry:

- mathematical modeling of real-life phenomena
- distinctions between a function as an abstract mathematical entity and the same function used as a mathematical model
- rates of change
- the effects of scale
- different views of a graph
- connections between the algebraic and the geometric representation of a mathematical idea
- mathematical language, or the connections between mathematical symbols and English words ("What does it mean when a mathematician says . . . ?")

These themes are woven through the standard topics so that they become visible at every opportunity. The interwoven themes encourage students to make connections between new material and what they already know, thus strengthening their understandings of both.

HOW TO USE THIS TEXT

The book is written primarily for the student. We class-tested early versions of the text and rewrote every single page that students found confusing. In many cases, we now employ the very phrasing that our students used when they finally grasped an idea. Here's an example: one student who had never studied functions was finding function notation frustratingly complicated. At last, she said, "Now I get it. The sentence $y = F(x) = 4x - x^3$ means that y depends upon x *in this particular way*, which we call F." The rest of the class nodded in agreement. We knew, therefore, that her sentence belonged in the text. Every student who is willing to make the effort should be able to read and understand most of the material, and therefore it should not be necessary for the instructor to cover a section in class before assigning it. Each section contains opportunities for the student to verify that he or she is on the right track before proceeding.

At the same time, there will be inevitable misapprehensions and points of confusion. Students might think they understand a topic, only to find that they cannot do the associated exercises or that they're coming up with incorrect answers. Or they will feel stuck partway through a section, unable to grasp fully the main idea. Therefore, it is essential to use class time for discussion, clarification, and expansion of the ideas presented in the reading.

As students work their way through the text, they may need some review of algebra and trigonometry. The Algebra Appendix and the Trig Appendix are intended to be used as resources, much like a dictionary, to brush up on techniques that students haven't used recently. The Algebra Appendix includes practice exercises for each section.

Mathematics is best studied in an atmosphere that encourages collaboration, exploration, and creativity. The text lends itself well to collaborative learning by presenting some questions for which the correct response can be found via a variety of approaches or for which several different answers might be equally valid. We find it effective to involve all the students by having them discuss such questions during class in small groups (two to five students). Everyone has something to offer, and the conclusions of a group will often represent contributions from each of its members.

The laboratories are group activities on a grand scale. The instructor remains on the sidelines, allowing each group to make its own blunders and discoveries and to move at its own pace. In the process of negotiating agreements about how to interpret results, of constructing meaning from the work of the laboratory, and of organizing their conclusions for presentation in the report, group members will develop for themselves a solid understanding of the fundamental ideas of precalculus mathematics. Many instructors set aside one generous hour per week as lab time; others incorporate the lab work into their regular classes. The *Instructor's Guide* offers suggestions about when to use a particular lab and which labs are more crucial than others.

The text lends itself to collaborative work outside the classroom as well. Many of our students form study groups to do homework together, particularly when the homework includes exploratory exercises with a graphing utility. We encourage students to discuss their reading with one another and to compare with each other the responses they have written and the graphs they have drawn.

PEDAGOGICAL ELEMENTS

Precalculus: Concepts in Context uses a variety of pedagogical tools because different students learn in different ways and because every student can benefit from several approaches to the same idea.

- *Context.* Instead of starting with abstract mathematical ideas and then looking for applications that illustrate them, we begin with the world of experience—music, commerce, psychology, natural science, and the daily news—and uncover the mathematics already present. As much as possible, we begin the study of each new mathematical topic by examining the idea in a context from which that topic naturally arises. A problem involving gravitational forces provides a reason to study rational functions; a news report on world population projections motivates a comparison of linear growth and exponential growth.
- *Write-in text.* Because students learn best when they are actively engaged, we have tried every trick we can think of to keep students productively occupied while they study. The responses that they are asked to write are not lengthy or especially difficult, but they serve to focus attention on the material and to help students realize whether or not they understand it. The pages of the text are incomplete until the student has written his or her responses.
- *Check Your Understanding.* Because students benefit from some immediate feedback while they are studying, we provide several opportunities in each chapter for them to

try out their skills or to respond to key questions. There is space in the book for them to write their answers, and the correct answers are printed at the end of the chapter.

- *Stop and Think.* Because not every mathematical question has a short answer, because not all of the implications of a new concept will be crystal-clear from the start, and because connections always need to be made with other material, we present questions that students will not necessarily be able to answer right away, but which are worth thinking about. No space is provided for writing, and no answers are printed. These questions make good discussion-starters, especially if the students have had an opportunity to think about them before coming to class.
- *Graphs and the use of a graphing utility.* We assume that every student either owns a graphing calculator or has frequent access to a computer with a graphing package. We encourage students to use the graphing utility while they study and while they're doing homework exercises. Students should use a graphing utility whenever it would help them to understand the material; they should not await specific instructions. As often as possible, the text shows the relationship between the graphical representation of a problem and its algebraic formulation; students should be encouraged to make their own connections as well. Students are also asked to sketch their own graphs by interpreting (rather than merely copying) the information they receive from their graphing device.
- *Exercises.* There is an abundance of exercises, more than an instructor could reasonably assign and more than a student could be expected to complete, at the conclusion to each chapter. We have deliberately grouped the exercises at the end of the chapter rather than after each section to help overcome the tendency of students to pigeonhole their math problems and to go directly to the relevant section, looking for a template example. (The *Instructor's Guide*, however, indicates the sections that need to be covered before specific exercises are assigned.) Some exercises are straightforward applications of the material of the text; others require a deeper understanding or a synthesis of ideas. Exercises that would serve well as group activities or as test questions are identified in the *Instructor's Guide*.
- *What's the Big Idea?* As its name implies, this feature at the end of each chapter highlights the main ideas of the chapter.
- *Progress Check.* This feature follows the "Big Idea" and lists specific skills that each student should possess at that point in the course.
- *Key Algebra Skills.* At the end of each chapter, the most essential algebra skills used in that chapter are listed with page references to the Algebra Appendix.
- *Labs.* Each chapter contains one or more major activities, which serve to solidify student understanding of the Big Ideas. A "preparation" section precedes the actual lab pages and should be assigned in advance. The timing of the lab period can be somewhat flexible, though it should probably occur early in the chapter rather than after students have seen most of the material. For instance, Lab 1 can be done with almost no advance instruction on functions. Lab 2A can be done, if the schedule requires, even before the students begin Chapter 2. The *Instructor's Guide* provides detailed suggestions for organizing a precalculus laboratory and evaluating the results, as well as a flowchart indicating where in the syllabus each lab belongs.
- *Expository Writing.* We ask that students present what they have learned in each lab by writing a report in the form of an essay, complete with graphs to illustrate their discussion. Few students will have had any previous experience in doing this with mathematics, but nearly all of our students, by the end of the semester, have decided that the work that they did in the lab and wrote about in their reports was the work from which they learned the most.
- *Projects.* Following the lab(s) in each chapter are several projects, some of which extend the ideas of the lab and some of which stand alone as investigations or

explorations. Most, but not all, of the projects require a graphing utility. The *Instructor's Guide* suggests ways to use each project and indicates which ones are essential.

- *Explorations.* Included in the exercises and projects are several guided explorations, designed to enhance students' ability to learn mathematics with the graphing device and to improve their understanding of certain types of functions. There's no single "correct" way to conduct an exploration and students need not worry about making mistakes. The purpose of an exploration is discovery.

SUPPLEMENTS

- *Instructor's Guide.* Because *Precalculus: Concepts in Context* is sufficiently different from a conventional precalculus text, instructors will benefit from this detailed *Instructor's Guide.* In addition to answers to all text exercises, specific guidance on how to use the text is provided, including:
  - suggestions for structuring a course, using the class hour, and conducting a lab
  - techniques for implementing the discovery approach and collaborative learning
  - advice on assessment
  - section-by-section comments
- *Graphing Calculator Guidebook.* For the TI-82 and TI-85. Each part consists of a basic tutorial, followed by specific instruction on how to use the graphing calculator to work through each chapter of the text.
- *Transparencies.* 36 full-color transparencies.
- *Assessment Materials.* Sample tests and projects are provided for each chapter.
- *Computerized Testing System for DOS, Windows, and Macintosh.* Users can edit and scramble test items or create their own. Fast and highly flexible system.

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J.F.M.
M.D.
M.M.

TO THE STUDENT

HOW TO USE THIS BOOK

This book is probably quite different in style and format from other math textbooks you have used. The two major differences are the keys to using the book successfully.

- First, this is a book to **read**.

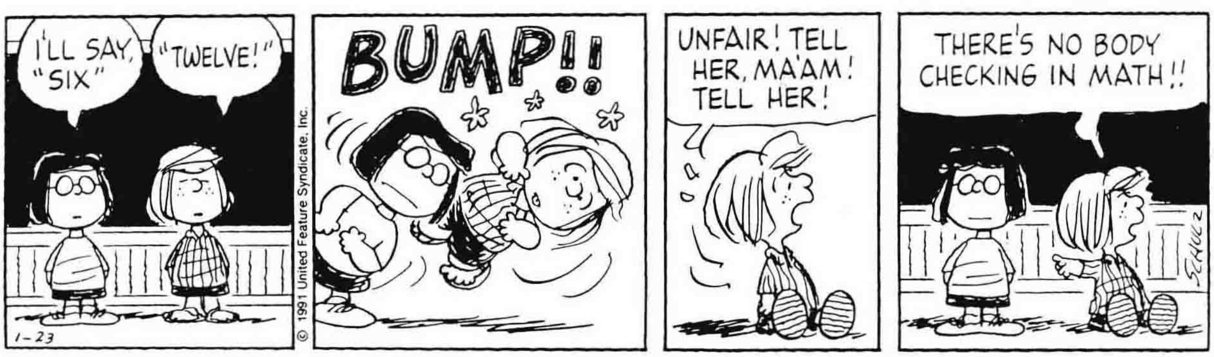
  You may be used to math classes where the teacher presents a technique, illustrates how to use it to solve sample problems, and then assigns very similar problems as homework exercises. Many students use the body of their text only as a reference, if at all.

  This book is different. It is written to and for *you*, not your instructor. We've tried to make the prose informal and clear. In the Preface (which *is* written primarily for the teacher) we suggest that students be asked to read the material before it is discussed in class, and we urge that class time be used to make sure everyone understands the reading. The class hour is a time to ask questions, not just to listen to the "sage on the stage."

- Second, this is a book to **write in**.

  You will learn best when you are actively engaged in the process of learning. This text is incomplete until you supply some of the material. The responses we ask you to write are not long or particularly difficult, but they will help you realize whether or not you understand what you have read. The class period is also a good time to check your write-in responses with those of your colleagues (and to ask your instructor, if necessary) to make sure you're on the right track.

We use the term *colleagues* deliberately. This text is the collaborative effort of three authors, with help from other instructors who class-tested our notes and students who served as our test drivers. Included in each chapter are one or more major investigations called "laboratories" which are designed as group activities. However, we hope that you will find teamwork so valuable that you will choose to collaborate on the readings and other assignments as well as on the labs. We expect you to find that talking



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about the math you're learning helps to clarify your own understanding. In a group, different students contribute a variety of skills and talents. Good questions are as important as good answers; in fact, they often help others to refine their thinking and thereby their answers.

Although many of the problems in the body of the text have brief, definite solutions (particularly those in the sections called Check Your Understanding), other questions are open-ended and have no single correct answer. Still others—those we designate as Stop and Think—are posed as questions that you may not be able to answer right away, but which are worth thinking about and discussing with your colleagues in a study group or in class.

This might not feel like mathematics to you. As one student, accustomed to cut-and-dried algebra, protested early in the semester, "This isn't math! You have to think about it!" We hope that by the end of the course you will have come to appreciate that, in fact, mathematics does ask you to think: not only to reason well and thoughtfully but also to explore and to try things out for yourself. One of the best ways in which a student of mathematics, or of any subject, can start a sentence is "What if . . .?"

HOW TO WRITE A GROUP LAB REPORT

The weekly laboratory report is one of the most important components of your precalculus course. It is an excellent means of gauging how well you and the other members of your group understand the material and are able to use the concepts you're learning. Each report represents the joint work and conclusions of the group, and is written by the "scribe" for that week. The office of scribe rotates among the group members so that each student ends up writing approximately the same number of lab reports during the semester.

As soon as the first draft of the report is written, the scribe makes a copy for each member of the group. The others review the draft, making corrections and suggestions. The scribe then revises the report to reflect the consensus of the group. Each member of the group should also have a copy of the final version of the report after the instructor has evaluated it. These reports will form an important body of notes to which you will refer during the semester and from which you will want to study at exam time.

Each lab group will design its own variation on the above procedure. The important thing is that responsibility be shared and that every member be fully apprised of what the group is doing.

On the last page of each lab project you will find instructions specific to that particular lab. Here are some general guidelines that apply to every lab.

A lab report is *not* a list of answers to the questions posed by the lab sheets. It is an *essay* showing what the group members have learned and what they now understand about the project. **Anyone should be able to pick up your report and understand what the project was about.** Think of your audience as a person who is familiar with precalculus mathematics but who has not read the lab sheets. You need to write in complete sentences and explain enough so the reader will understand. This does not mean that the report needs to be very long; it does mean that it must be able to stand on its own.

A good lab report contains the following:

- an introduction for your reader (What was this lab about?)
- the mathematical concepts involved
- an outline of your procedure
- anything you learned along the way
- conclusions.

Graphs can do much to clarify what you're saying. They needn't be elaborate, but the axes should always be labeled and the units, if appropriate, should be specified. Graphs can be woven into the text or gathered at the end of the report. There is room for plenty of creativity, because you get to decide the most effective way to present your ideas. Take the opportunity to read other groups' reports; you'll notice that no two are the same (nor should they be).

One tendency you'll probably have at first is to go through the lab sheets, highlighting the questions you find, then go about the business of answering those questions. Fine. But that, in itself, doesn't make a lab report. Imagine yourself a tailor: you cut out the pieces of fabric and put them together with large basting stitches; then you sew the seams and remove the bastings. Your final suit doesn't have (we hope) any basting stitches in it. Some of the questions in the lab sheets are like basting stitches: answering them helps to shape your work and keep you from straying too far off the track. The finished product doesn't need even to mention those questions; the fact that you produced an elegant piece of work shows that you put things together correctly.

Other questions, however, do need to be answered in the lab report. How can you tell which questions are basting stitches and which are essential construction details? Experience will help. So will the specific guidelines at the end of each lab. If a lab gives you a concrete example to calculate (for example, "what is the volume when L = 44 in.?"), that's probably just an illustration for your benefit, and the answer ("V = 7744 cu. in.," or whatever) doesn't need to be in the report, unless you're using it to illustrate a concept. If, however, the lab sheets ask, "Why does one shape yield more volume than the other?" your answer will form an essential part of the report and should be incorporated into your discussion.

The lab sheets help to lead you, by means of questions, from specific examples to general conclusions. Your lab report, in contrast, should begin with those conclusions and illustrate them with whatever examples and graphs you need. In other words, although you conduct your investigation in the order suggested by the lab sheets, you might write about it more effectively by switching things around.

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