



国家出版基金项目
NATIONAL PUBLICATION FOUNDATION

中外物理学精品书系

引进系列 · 63

An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists

粒子物理学家用非阿贝尔离散对称导论

(影印版)

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著作权合同登记号 图字:01-2014-4298

图书在版编目(CIP)数据

粒子物理学家用非阿贝尔离散对称导论 = An introduction to non-Abelian discrete symmetries for particle physicists: 英文/(日)石森一等著. —影印本. —北京: 北京大学出版社, 2014. 12

(中外物理学精品书系)

ISBN 978-7-301-25184-3

I. ①粒… II. ①石… III. ①粒子物理学—对称—研究—英文 IV. ①O572.2

中国版本图书馆 CIP 数据核字(2014)第 278927 号

Reprint from English language edition:

An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists

by Hajime Ishimori, Tatsuo Kobayashi, Hiroshi Ohki, Hiroshi Okada, Yusuke Shimizu, Morimitsu Tanimoto

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书 名: **An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists**(粒子物理学家用非阿贝尔离散对称导论)(影印版)

著作责任者:〔日〕石森一(H. Ishimori) 〔日〕小林达夫(T. Kobayashi) 〔日〕大木洋(H. Ohki) 〔日〕冈田宽(H. Okada) 〔日〕清水勇介(Y. Shimizu) 〔日〕谷本盛光(M. Tanimoto) 著

责任编辑:刘 啸

标准书号:ISBN 978-7-301-25184-3/O·1055

出版发行:北京大学出版社

地 址:北京市海淀区成府路 205 号 100871

网 址: <http://www.pup.cn>

新浪微博: @北京大学出版社

电子信箱: zpup@pup.cn

电 话: 邮购部 62752015 发行部 62750672 编辑部 62752038 出版部 62754962

印 刷 者: 北京中科印刷有限公司

经 销 者: 新华书店

730 毫米×980 毫米 16 开本 19 印张 362 千字

2014 年 12 月第 1 版 2014 年 12 月第 1 次印刷

定 价: 51.00 元

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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子亲身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套“中外物理学精品书系”的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任

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2010年5月于燕园

Hajime Ishimori • Tatsuo Kobayashi •
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Morimitsu Tanimoto

An Introduction
to Non-Abelian
Discrete
Symmetries for
Particle Physicists

Preface

The purpose of these lecture notes is to introduce the basic framework of non-Abelian discrete symmetries, and to present some important applications in particle physics. Discrete non-Abelian groups have in fact played an important role in particle physics. However, they may not be so familiar to particle physicists as continuous non-Abelian symmetries. These lecture notes are written for particle physicists and differ in this respect from standard books on group theory. However, preliminary knowledge of group theory is not required to understand the non-Abelian discrete symmetries.

We hope our lecture notes will serve as a handbook for serious learners, and also as a helpful reference book for experts, as well perhaps as triggering future research.

It is a pleasure to acknowledge fruitful discussions with H. Abe, T. Araki, K.S. Choi, Y. Daikoku, K. Hashimoto, J. Kubo, H.P. Nilles, F. Ploger, S. Raby, S. Ramos-Sanchez, M. Ratz, and P.K.S. Vaudrevange.

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Contents

1	Introduction	1
	References	3
2	Basics of Finite Groups	13
	References	20
3	S_N	21
3.1	S_3	21
3.1.1	Conjugacy Classes	21
3.1.2	Characters and Representations	22
3.1.3	Tensor Products	22
3.2	S_4	25
3.2.1	Conjugacy Classes	27
3.2.2	Characters and Representations	27
3.2.3	Tensor Products	29
	References	30
4	A_N	31
4.1	A_4	31
4.2	A_5	34
4.2.1	Conjugacy Classes	35
4.2.2	Characters and Representations	35
4.2.3	Tensor Products	37
	References	41
5	T'	43
5.1	Conjugacy Classes	43
5.2	Characters and Representations	44
5.3	Tensor Products	47
6	D_N	51
6.1	D_N with N Even	51
6.1.1	Conjugacy Classes	52

	6.1.2	Characters and Representations	52
	6.1.3	Tensor Products	54
	6.2	D_N with N Odd	56
	6.2.1	Conjugacy Classes	56
	6.2.2	Characters and Representations	56
	6.2.3	Tensor Products	57
	6.3	D_4	58
	6.4	D_5	59
7	Q_N	61
	7.1	Q_N with $N = 4n$	61
	7.1.1	Conjugacy Classes	62
	7.1.2	Characters and Representations	62
	7.1.3	Tensor Products	62
	7.2	Q_N with $N = 4n + 2$	64
	7.2.1	Conjugacy Classes	64
	7.2.2	Characters and Representations	64
	7.2.3	Tensor Products	65
	7.3	Q_4	66
	7.4	Q_6	67
8	QD_{2N}	69
	8.1	Generic Aspects	69
	8.1.1	Conjugacy Classes	70
	8.1.2	Characters and Representations	70
	8.1.3	Tensor Products	71
	8.2	QD_{16}	72
9	$\Sigma(2N^2)$	75
	9.1	Generic Aspects	75
	9.1.1	Conjugacy Classes	75
	9.1.2	Characters and Representations	76
	9.1.3	Tensor Products	77
	9.2	$\Sigma(18)$	78
	9.3	$\Sigma(32)$	80
	9.4	$\Sigma(50)$	84
10	$\Delta(3N^2)$	87
	10.1	$\Delta(3N^2)$ with $N/3 \neq \text{Integer}$	87
	10.1.1	Conjugacy Classes	88
	10.1.2	Characters and Representations	89
	10.1.3	Tensor Products	89
	10.2	$\Delta(3N^2)$ with $N/3 \text{ Integer}$	91
	10.2.1	Conjugacy Classes	91
	10.2.2	Characters and Representations	92
	10.2.3	Tensor Products	93
	10.3	$\Delta(27)$	94
	References	95

- 11 T_N** 97
 - 11.1 Generic Aspects 97
 - 11.1.1 Conjugacy Classes 98
 - 11.1.2 Characters and Representations 99
 - 11.1.3 Tensor Products 99
 - 11.2 T_7 100
 - 11.3 T_{13} 102
 - 11.4 T_{19} 104
 - References 108
- 12 $\Sigma(3N^3)$** 109
 - 12.1 Generic Aspects 109
 - 12.1.1 Conjugacy Classes 110
 - 12.1.2 Characters and Representations 111
 - 12.1.3 Tensor Products 112
 - 12.2 $\Sigma(81)$ 113
 - References 121
- 13 $\Delta(6N^2)$** 123
 - 13.1 $\Delta(6N^2)$ with $N/3 \neq$ Integer 123
 - 13.1.1 Conjugacy Classes 123
 - 13.1.2 Characters and Representations 126
 - 13.1.3 Tensor Products 128
 - 13.2 $\Delta(6N^2)$ with $N/3$ Integer 131
 - 13.2.1 Conjugacy Classes 131
 - 13.2.2 Characters and Representations 133
 - 13.2.3 Tensor Products 134
 - 13.3 $\Delta(54)$ 138
 - 13.3.1 Conjugacy Classes 138
 - 13.3.2 Characters and Representations 139
 - 13.3.3 Tensor Products 141
 - References 145
- 14 Subgroups and Decompositions of Multiplets** 147
 - 14.1 S_3 147
 - 14.1.1 $S_3 \rightarrow Z_3$ 148
 - 14.1.2 $S_3 \rightarrow Z_2$ 148
 - 14.2 S_4 149
 - 14.2.1 $S_4 \rightarrow S_3$ 150
 - 14.2.2 $S_4 \rightarrow A_4$ 151
 - 14.2.3 $S_4 \rightarrow \Sigma(8)$ 151
 - 14.3 A_4 152
 - 14.3.1 $A_4 \rightarrow Z_3$ 152
 - 14.3.2 $A_4 \rightarrow Z_2 \times Z_2$ 153
 - 14.4 A_5 153
 - 14.4.1 $A_5 \rightarrow A_4$ 153

14.4.2	$A_5 \rightarrow D_5$	153
14.4.3	$A_5 \rightarrow S_3 \simeq D_3$	154
14.5	T'	154
14.5.1	$T' \rightarrow Z_6$	154
14.5.2	$T' \rightarrow Z_4$	155
14.5.3	$T' \rightarrow Q_4$	155
14.6	General D_N	155
14.6.1	$D_N \rightarrow Z_2$	156
14.6.2	$D_N \rightarrow Z_N$	157
14.6.3	$D_N \rightarrow D_M$	157
14.7	D_4	158
14.7.1	$D_4 \rightarrow Z_4$	158
14.7.2	$D_4 \rightarrow Z_2 \times Z_2$	159
14.7.3	$D_4 \rightarrow Z_2$	159
14.8	General Q_N	159
14.8.1	$Q_N \rightarrow Z_4$	160
14.8.2	$Q_N \rightarrow Z_N$	161
14.8.3	$Q_N \rightarrow Q_M$	161
14.9	Q_4	162
14.9.1	$Q_4 \rightarrow Z_4$	162
14.10	QD_{2N}	162
14.10.1	$QD_{2N} \rightarrow Z_2$	163
14.10.2	$QD_{2N} \rightarrow Z_N$	163
14.10.3	$QD_{2N} \rightarrow D_{N/2}$	163
14.11	General $\Sigma(2N^2)$	164
14.11.1	$\Sigma(2N^2) \rightarrow Z_{2N}$	164
14.11.2	$\Sigma(2N^2) \rightarrow Z_N \times Z_N$	164
14.11.3	$\Sigma(2N^2) \rightarrow D_N$	165
14.11.4	$\Sigma(2N^2) \rightarrow Q_N$	166
14.11.5	$\Sigma(2N^2) \rightarrow \Sigma(2M^2)$	166
14.12	$\Sigma(32)$	167
14.13	General $\Delta(3N^2)$	168
14.13.1	$\Delta(3N^2) \rightarrow Z_3$	169
14.13.2	$\Delta(3N^2) \rightarrow Z_N \times Z_N$	169
14.13.3	$\Delta(3N^2) \rightarrow T_N$	170
14.13.4	$\Delta(3N^2) \rightarrow \Delta(3M^2)$	170
14.14	$\Delta(27)$	172
14.14.1	$\Delta(27) \rightarrow Z_3$	172
14.14.2	$\Delta(27) \rightarrow Z_3 \times Z_3$	172
14.15	General T_N	173
14.15.1	$T_N \rightarrow Z_3$	173
14.15.2	$T_N \rightarrow Z_N$	173
14.16	T_7	174
14.16.1	$T_7 \rightarrow Z_3$	174
14.16.2	$T_7 \rightarrow Z_7$	175

14.17	General $\Sigma(3N^3)$	175
14.17.1	$\Sigma(3N^2) \rightarrow Z_N \times Z_N \times Z_N$	175
14.17.2	$\Sigma(3N^3) \rightarrow \Delta(3N^2)$	175
14.17.3	$\Sigma(3N^3) \rightarrow \Sigma(3M^3)$	176
14.18	$\Sigma(81)$	176
14.18.1	$\Sigma(81) \rightarrow Z_3 \times Z_3 \times Z_3$	177
14.18.2	$\Sigma(81) \rightarrow \Delta(27)$	177
14.19	General $\Delta(6N^2)$	178
14.19.1	$\Delta(6N^2) \rightarrow \Sigma(2N^2)$	179
14.19.2	$\Delta(6N^2) \rightarrow \Delta(3N^2)$	180
14.19.3	$\Delta(6N^2) \rightarrow \Delta(6M^2)$	180
14.20	$\Delta(54)$	181
14.20.1	$\Delta(54) \rightarrow S_3 \times Z_3$	182
14.20.2	$\Delta(54) \rightarrow \Sigma(18)$	182
14.20.3	$\Delta(54) \rightarrow \Delta(27)$	183
15	Anomalies	185
15.1	Generic Aspects	185
15.2	Explicit Calculations	189
15.2.1	S_3	189
15.2.2	S_4	190
15.2.3	A_4	190
15.2.4	A_5	191
15.2.5	T'	192
15.2.6	D_N (N Even)	193
15.2.7	D_N (N Odd)	194
15.2.8	Q_N ($N = 4n$)	194
15.2.9	Q_N ($N = 4n + 2$)	195
15.2.10	QD_{2N}	196
15.2.11	$\Sigma(2N^2)$	197
15.2.12	$\Delta(3N^2)$ ($N/3 \neq \text{Integer}$)	198
15.2.13	$\Delta(3N^2)$ ($N/3$ Integer)	199
15.2.14	T_N	200
15.2.15	$\Sigma(3N^3)$	201
15.2.16	$\Delta(6N^2)$ ($N/3 \neq \text{Integer}$)	202
15.2.17	$\Delta(6N^2)$ ($N/3$ Integer)	203
15.3	Comments on Anomalies	203
	References	204
16	Non-Abelian Discrete Symmetry in Quark/Lepton Flavor Models	205
16.1	Neutrino Flavor Mixing and Neutrino Mass Matrix	205
16.2	A_4 Flavor Symmetry	207
16.2.1	Realizing Tri-Bimaximal Mixing of Flavors	207
16.2.2	Breaking Tri-Bimaximal Mixing	209
16.3	S_4 Flavor Model	211
16.4	Alternative Flavor Mixing	219

16.5	Comments on Other Applications	222
16.6	Comment on Origins of Flavor Symmetries	223
	References	224
Appendix A	Useful Theorems	229
	References	235
Appendix B	Representations of S_4 in Different Bases	237
B.1	Basis I	237
B.2	Basis II	238
B.3	Basis III	240
B.4	Basis IV	242
	References	244
Appendix C	Representations of A_4 in Different Bases	245
C.1	Basis I	245
C.2	Basis II	245
	References	246
Appendix D	Representations of A_5 in Different Bases	247
D.1	Basis I	247
D.2	Basis II	253
	References	259
Appendix E	Representations of T' in Different Bases	261
E.1	Basis I	262
E.2	Basis II	263
	References	264
Appendix F	Other Smaller Groups	265
F.1	$Z_4 \times Z_4$	265
F.2	$Z_8 \times Z_2$	268
F.3	$(Z_2 \times Z_4) \times Z_2$ (I)	270
F.4	$(Z_2 \times Z_4) \times Z_2$ (II)	272
F.5	$Z_3 \times Z_8$	275
F.6	$(Z_6 \times Z_2) \times Z_2$	277
F.7	$Z_9 \times Z_3$	281
	References	283
Index	285

Chapter 1

Introduction

These lecture notes aim to provide a pedagogical review of non-Abelian discrete groups and show some applications to physical issues. Symmetry constitutes a very important principle in physics. In particular, it has played an essential role in constructing the framework of particle physics. For example, continuous (and local) symmetries such as Lorentz, Poincaré, and gauge symmetries are crucial to understand several phenomena, such as the strong, weak, and electromagnetic interactions among particles. On the other hand, discrete symmetries such as C , P , and T are also vital concepts in particle physics. Abelian discrete symmetries, Z_N , are also often imposed in order to control allowed couplings for particle physics, in particular model-building beyond the standard model. In addition to Abelian discrete symmetries, non-Abelian discrete symmetries have also been applied for model-building in particle physics, in particular to understand the three-generation flavor structure.

There are many free parameters in the standard model, including its extension with neutrino mass terms. Most of them are Yukawa couplings of quarks and leptons to the Higgs boson. The quark and lepton sector is called the flavor sector. Flavor physics is a challenging aspect of the construction of the theory beyond the standard model. If a symmetry is imposed on the flavor sector, one can control the Yukawa couplings in the three generations, although the origin of the generations remains unknown. Therefore, quark masses and mixing angles have been studied from the standpoint of flavor symmetries.

In addition, the discovery of neutrino masses and neutrino mixing [1, 2] has stimulated work on flavor symmetries. Experiments on neutrino oscillations are now going into a new phase of precise determination of mixing angles and mass squared differences [3–7]. In particular, the recent long baseline neutrino experiment T2K is reaching the last neutrino mixing angle, so called θ_{13} [8]. The Double Chooz collaboration has also reported indications of non-zero θ_{13} [9]. Reactor neutrino experiments, Reno and Daya Bay are also attempting to observe it. Global analyses of neutrino data indicate the special neutrino mixing pattern, which is called tri-bimaximal mixing for three flavors in the lepton sector [10–13]. These large mixing angles are completely different from the quark mixing ones. Therefore, it is very

important to find a natural model that leads to these mixing patterns of quarks and leptons with good accuracy.

Non-Abelian discrete symmetries are considered to be the most attractive choice for the flavor sector. Model builders have tried to derive experimental values of quark/lepton masses and mixing angles by assuming non-Abelian discrete flavor symmetries of quarks and leptons. In particular, lepton mixing has been intensively discussed in the context of non-Abelian discrete flavor symmetries, as seen, e.g., in the reviews [14, 15].

Particle physicists may be interested in the origin of the non-Abelian discrete symmetry for flavors. One of the most interesting is a higher dimensional spacetime symmetry. After it has been broken down to the 4D Poincaré symmetry through compactification, e.g., via orbifolding, a remnant symmetry appears in the flavor sector. This remnant symmetry is often a non-Abelian symmetry. Actually, it has been shown how the flavor symmetry A_4 (or S_4) can arise if the three fermion generations are taken to live on the fixed points of a specific 2D orbifold [16]. Further non-Abelian discrete symmetries can arise in a similar setup [17] (see also [18]).

Superstring theory is a promising candidate for a unified theory including gravity. Certain string modes correspond to gauge bosons, quarks, leptons, Higgs bosons, and gravitons as well as their superpartners. Superstring theory predicts six extra dimensions. Certain classes of discrete symmetries can be derived from superstring theories. A combination among geometrical symmetries of a compact space and stringy selection rules for couplings enhances discrete flavor symmetries. For example, D_4 and $\Delta(54)$ flavor symmetries can be obtained in heterotic orbifold models [19–21]. In addition to these flavor symmetries, the $\Delta(27)$ flavor symmetry can be derived from magnetized/intersecting D-brane models [22–24].

There is another possibility, namely that non-Abelian discrete groups may originate from the breaking of continuous (gauge) flavor symmetries [25–27].

Thus, a non-Abelian discrete symmetry can arise from the underlying theory, e.g., string theory or compactification via orbifolding. In addition, non-Abelian discrete symmetries are interesting tools for controlling flavor structure in model building using the bottom-up approach. Hence, non-Abelian flavor symmetries could provide a bridge between the low-energy physics and the underlying theory. It is thus quite important to understand the properties of non-Abelian groups for particle physics.

Continuous non-Abelian groups are well-known, and of course there are several good reviews and books. On the other hand, discrete non-Abelian symmetries may not be so familiar to particle physicists as continuous non-Abelian symmetries. However, discrete non-Abelian symmetries have become important tools for model building, as discussed above, in particular in the context of flavor physics. The purpose of these lecture notes is therefore to provide a pedagogical review of non-Abelian discrete groups with particle phenomenology in mind, and to exhibit the group-theoretical aspects of many concrete groups explicitly, including, for example, representations and their tensor products [15, 28–34]. We present these aspects in detail for the groups S_N [35–132], A_N [133–243], T' [33, 244–263], D_N [264–285], Q_N [286–300], QD_{2N} , $\Sigma(2N^2)$ [301], $\Delta(3N^2)$ [302–313], T_N [302–304, 312, 314–323], $\Sigma(3N^3)$ [315, 324], and $\Delta(6N^2)$ [302–304, 312, 325–330].

We explain pedagogically how to derive conjugacy classes, characters, representations, and tensor products for these groups (with a finite number) when algebraic relations are given. Thus, it will be straightforward for readers to apply this to other groups.

In applications to particle physics, the breaking patterns of discrete groups and decompositions of multiplets are often required to understand low energy phenomena. Such aspects are given in these notes.

Symmetries at the tree level are not always symmetries in quantum theory. If symmetries are anomalous, breaking terms are induced by quantum effects. Such anomalies are important in applications for particle physics. Here, we study such anomalies for discrete symmetries [331–344] and show anomaly-free conditions explicitly for the above concrete groups. If flavor symmetries are stringy symmetries, these anomalies may also be controlled by string dynamics, i.e., anomaly cancellation.

We also present flavor models with non-Abelian discrete symmetry as typical examples. One can see how to use the non-Abelian discrete symmetry for flavors. A lot of references are available to understand the model building here.

On the other hand, discrete subgroups of $SU(3)$ would also be interesting from the standpoint of phenomenological applications to flavor physics [345–349]. Here, most of them are shown for subgroups including doublets or triplets as the largest dimensional irreducible representations (for other groups see [29, 31, 34, 241, 350–354]).

The book is organized as follows. In Chap. 2, we summarize the basic group-theoretical aspects used in subsequent chapters, and also present some examples to provide a more concrete understanding. Readers familiar with group theory can skip Chap. 2. In Chaps. 3 to 13, we present the non-Abelian discrete groups S_N , A_N , T' , D_N , Q_N , QD_{2N} , $\Sigma(2N^2)$, $\Delta(3N^2)$, T_N , $\Sigma(3N^3)$, and $\Delta(6N^2)$. In each chapter, groups with specific values of N are also discussed for typical examples. Chapter 14 discusses the breaking patterns of the non-Abelian discrete groups. In Chap. 15, we review the anomalies of non-Abelian flavor symmetries, which is an important topic in particle physics, and exhibit the anomaly-free conditions explicitly for the above concrete groups. Chapter 16 presents typical flavor models with the non-Abelian discrete symmetries A_4 and S_4 .

Appendix A gives some useful theorems on finite group theory, while Appendices B, C, D, and E provide the representation bases of S_4 , A_4 , A_5 , and T' , which are different from those in Chaps. 3, 4, and 5. Appendix F presents other smaller groups in detail.

Note Added in Proof Finally, θ_{13} has been observed by Daya Bay [355] and Reno [356].

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