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An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists

粒子物理学家用非阿贝尔离散对称导论

（影印版）

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序　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

"中外物理学精品书系"另一个突出特点是，在把西方物理的精华要义"请进来"的同时，也将我国近现代物理的优秀成果"送出去"。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的"硬实力"，也向世界主动传播我国科技文化领域不断创新的"软实力"，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，"中外物理学精品书系"还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是"只闻其声，未见其真"。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，"加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件"。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套"中外物理学精品书系"的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

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2010 年 5 月于燕园

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An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists

Springer

Preface

The purpose of these lecture notes is to introduce the basic framework of non-Abelian discrete symmetries, and to present some important applications in particle physics. Discrete non-Abelian groups have in fact played an important role in particle physics. However, they may not be so familiar to particle physicists as continuous non-Abelian symmetries. These lecture notes are written for particle physicists and differ in this respect from standard books on group theory. However, preliminary knowledge of group theory is not required to understand the non-Abelian discrete symmetries.

We hope our lecture notes will serve as a handbook for serious learners, and also as a helpful reference book for experts, as well perhaps as triggering future research.

It is pleasure to acknowledge fruitful discussions with H. Abe, T. Araki, K.S. Choi, Y. Daikoku, K. Hashimoto, J. Kubo, H.P. Nilles, F. Ploger, S. Raby, S. Ramos-Sanchez, M. Ratz, and P.K.S. Vaudrevange.

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Contents

Chapter 1
Introduction

These lecture notes aim to provide a pedagogical review of non-Abelian discrete groups and show some applications to physical issues. Symmetry constitutes a very important principle in physics. In particular, it has played an essential role in constructing the framework of particle physics. For example, continuous (and local) symmetries such as Lorentz, Poincaré, and gauge symmetries are crucial to understand several phenomena, such as the strong, weak, and electromagnetic interactions among particles. On the other hand, discrete symmetries such as C, P, and T are also vital concepts in particle physics. Abelian discrete symmetries, Z_N, are also often imposed in order to control allowed couplings for particle physics, in particular model-building beyond the standard model. In addition to Abelian discrete symmetries, non-Abelian discrete symmetries have also been applied for model-building in particle physics, in particular to understand the three-generation flavor structure.

There are many free parameters in the standard model, including its extension with neutrino mass terms. Most of them are Yukawa couplings of quarks and leptons to the Higgs boson. The quark and lepton sector is called the flavor sector. Flavor physics is a challenging aspect of the construction of the theory beyond the standard model. If a symmetry is imposed on the flavor sector, one can control the Yukawa couplings in the three generations, although the origin of the generations remains unknown. Therefore, quark masses and mixing angles have been studied from the standpoint of flavor symmetries.

In addition, the discovery of neutrino masses and neutrino mixing [1, 2] has stimulated work on flavor symmetries. Experiments on neutrino oscillations are now going into a new phase of precise determination of mixing angles and mass squared differences [3–7]. In particular, the recent long baseline neutrino experiment T2K is reaching the last neutrino mixing angle, so called θ_{13} [8]. The Double Chooz collaboration has also reported indications of non-zero θ_{13} [9]. Reactor neutrino experiments, Reno and Daya Bay are also attempting to observe it. Global analyses of neutrino data indicate the special neutrino mixing pattern, which is called tri-bimaximal mixing for three flavors in the lepton sector [10–13]. These large mixing angles are completely different from the quark mixing ones. Therefore, it is very important to find a natural model that leads to these mixing patterns of quarks and leptons with good accuracy.

Non-Abelian discrete symmetries are considered to be the most attractive choice for the flavor sector. Model builders have tried to derive experimental values of quark/lepton masses and mixing angles by assuming non-Abelian discrete flavor symmetries of quarks and leptons. In particular, lepton mixing has been intensively discussed in the context of non-Abelian discrete flavor symmetries, as seen, e.g., in the reviews [14, 15].

Particle physicists may be interested in the origin of the non-Abelian discrete symmetry for flavors. One of the most interesting is a higher dimensional spacetime symmetry. After it has been broken down to the 4D Poincaré symmetry through compactification, e.g., via orbifolding, a remnant symmetry appears in the flavor sector. This remnant symmetry is often a non-Abelian symmetry. Actually, it has been shown how the flavor symmetry A_4 (or S_4) can arise if the three fermion generations are taken to live on the fixed points of a specific 2D orbifold [16]. Further non-Abelian discrete symmetries can arise in a similar setup [17] (see also [18]).

Superstring theory is a promising candidate for a unified theory including gravity. Certain string modes correspond to gauge bosons, quarks, leptons, Higgs bosons, and gravitons as well as their superpartners. Superstring theory predicts six extra dimensions. Certain classes of discrete symmetries can be derived from superstring theories. A combination among geometrical symmetries of a compact space and stringy selection rules for couplings enhances discrete flavor symmetries. For example, D_4 and $\Delta(54)$ flavor symmetries can be obtained in heterotic orbifold models [19–21]. In addition to these flavor symmetries, the $\Delta(27)$ flavor symmetry can be derived from magnetized/intersecting D-brane models [22–24].

There is another possibility, namely that non-Abelian discrete groups may originate from the breaking of continuous (gauge) flavor symmetries [25–27].

Thus, a non-Abelian discrete symmetry can arise from the underlying theory, e.g., string theory or compactification via orbifolding. In addition, non-Abelian discrete symmetries are interesting tools for controlling flavor structure in model building using the bottom-up approach. Hence, non-Abelian flavor symmetries could provide a bridge between the low-energy physics and the underlying theory. It is thus quite important to understand the properties of non-Abelian groups for particle physics.

Continuous non-Abelian groups are well-known, and of course there are several good reviews and books. On the other hand, discrete non-Abelian symmetries may not be so familiar to particle physicists as continuous non-Abelian symmetries. However, discrete non-Abelian symmetries have become important tools for model building, as discussed above, in particular in the context of flavor physics. The purpose of these lecture notes is therefore to provide a pedagogical review of non-Abelian discrete groups with particle phenomenology in mind, and to exhibit the group-theoretical aspects of many concrete groups explicitly, including, for example, representations and their tensor products [15, 28–34]. We present these aspects in detail for the groups S_N [35–132], A_N [133–243], T' [33, 244–263], D_N [264–285], Q_N [286–300], QD_{2N}, $\Sigma(2N^2)$ [301], $\Delta(3N^2)$ [302–313], T_N [302–304, 312, 314–323], $\Sigma(3N^3)$ [315, 324], and $\Delta(6N^2)$ [302–304, 312, 325–330].

We explain pedagogically how to derive conjugacy classes, characters, representations, and tensor products for these groups (with a finite number) when algebraic relations are given. Thus, it will be straightforward for readers to apply this to other groups.

In applications to particle physics, the breaking patterns of discrete groups and decompositions of multiplets are often required to understand low energy phenomena. Such aspects are given in these notes.

Symmetries at the tree level are not always symmetries in quantum theory. If symmetries are anomalous, breaking terms are induced by quantum effects. Such anomalies are important in applications for particle physics. Here, we study such anomalies for discrete symmetries [331–344] and show anomaly-free conditions explicitly for the above concrete groups. If flavor symmetries are stringy symmetries, these anomalies may also be controlled by string dynamics, i.e., anomaly cancellation.

We also present flavor models with non-Abelian discrete symmetry as typical examples. One can see how to use the non-Abelian discrete symmetry for flavors. A lot of references are available to understand the model building here.

On the other hand, discrete subgroups of $SU(3)$ would also be interesting from the standpoint of phenomenological applications to flavor physics [345–349]. Here, most of them are shown for subgroups including doublets or triplets as the largest dimensional irreducible representations (for other groups see [29, 31, 34, 241, 350–354]).

The book is organized as follows. In Chap. 2, we summarize the basic group-theoretical aspects used in subsequent chapters, and also present some examples to provide a more concrete understanding. Readers familiar with group theory can skip Chap. 2. In Chaps. 3 to 13, we present the non-Abelian discrete groups S_N, A_N, T', D_N, Q_N, QD_{2N}, $\Sigma(2N^2)$, $\Delta(3N^2)$, T_N, $\Sigma(3N^3)$, and $\Delta(6N^2)$. In each chapter, groups with specific values of N are also discussed for typical examples. Chapter 14 discusses the breaking patterns of the non-Abelian discrete groups. In Chap. 15, we review the anomalies of non-Abelian flavor symmetries, which is an important topic in particle physics, and exhibit the anomaly-free conditions explicitly for the above concrete groups. Chapter 16 presents typical flavor models with the non-Abelian discrete symmetries A_4 and S_4.

Appendix A gives some useful theorems on finite group theory, while Appendices B, C, D, and E provide the representation bases of S_4, A_4, A_5, and T', which are different from those in Chaps. 3, 4, and 5. Appendix F presents other smaller groups in detail.

Note Added in Proof Finally, θ_{13} has been observed by Daya Bay [355] and Reno [356].

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