量子场论辅导材料

下　册

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目录

（下册）

第八章 散射矩阵和微扰论

§8·1 散射矩阵

§8·2 微扰论

◇ (8·15)

$$i\frac{\partial}{\partial t}u(t,t_0)=H_i(t)\,u(t,t_0)$$

设 $t-t_0\approx 0$ 则有 $i\frac{\partial}{\partial t}u(t,t_0)=H_i(t)$

积分之

$$u(t,t_0)=1-i\int_{t_0}^{t}H_i(t')dt'$$

◇ (8·16) ⟶ (8·17) ⟶ (8·19) ⟶ (8·21)

$$\int_{t_m}^{t_{m+1}}H_i(t)dt\cdot\int_{t_{m'}}^{t_{m'+1}}H_i(t')dt'$$

由于时间 $[t_{m+1},t_m]>[t_{m'+1},t_m]$

因此产生这样的求和

$$\Pi=\sum_{m=0}^{n-1}\sum_{m'=0}^{m-1}\int_{t_m}^{t_{m+1}}H_i(t)dt\int_{t_m}^{t_{m'+1}}H_i(t')dt'$$

$$=\sum_{m=0}^{n-1}\int_{t_m}^{t_{m+1}}H_i(t)dt\sum_{m'=0}^{m-1}\int_{t_m}^{t_{m'+1}}H_i(t')dt'$$

$$=\sum_{m=0}^{n-1}\int_{t_m}^{t_{m+1}}H_i(t)dt\int_{t_0}^{t_m}H_i(t')dt'$$

由于区间 $[t_{m+1},t_m]\approx 0$

所以 $\int_{t_m}^{t}\approx 0$ t在区间之内

$$\Pi=\sum_{m=0}^{n-1}\int_{t_m}^{t_{m+1}}H_i(t)dt\left(\int_{t_0}^{t_m}+\int_{t_m}^{t}\right)H_i(t')dt'$$

$$=\sum_{m=0}^{n-1}\int_{t_m}^{t_{m+1}}H_i(t)dt\underbrace{\int_{t_0}^{t}H_i(t')dt'}_{f(t)}$$

$$=\sum_{m=0}^{n-1}\int_{t_m}^{t_{m+1}}H_i(t)f(t)dt=\int_{t_0}^{t_n}H_i(t)f(t)dt=$$

⋮

$$=\int_{t_0}^{t_n}H_i(t)dt\int_{t_0}^{t}H_i(t')dt'$$

如果设 $T\left(H_i(t)H_i(t')\right)=\theta(t-t')H_i(t)H_i(t')+\theta(t'-t)H_i(t')H_i(t)$

则有 $\int_{t_0}^{t_n}dt\int_{t_0}^{t_n}dt'\,T\left(H_i(t)H_i(t')\right)$

$$=\int_{t_0}^{t_n}dt\int_{t_0}^{t_n}dt'\,\theta(t-t')H_i(t)H_i(t')+\int_{t_0}^{t_n}dt\int_{t_0}^{t_n}dt'\,\theta(t'-t)H(t')H_i(t)$$

$$=\int_{t_0}^{t_n}dt\int_{t_0}^{t}dt'H_i(t)H_i(t')+\int_{t_0}^{t_n}dt'\int_{t_0}^{t'}dt\,H_i(t')H_i(t)$$

$$=2!\int_{t_0}^{t_n}dt\int_{t_0}^{t}dt'H_i(t)H_i(t')$$

$$\Pi=\frac{1}{2!}\int_{t_0}^{t_n}dt\int_{t_0}^{t_n}dt'\,T\left(H_i(t)H_i(t')\right)$$

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§8.3 散射矩阵的简化

§8.4 正规乘积

$$N\left(\bar\varphi(y)\varphi(x)\right)+N\left(\varphi(x)\bar\varphi(y)\right)=\left\{u(x),v(y)\right\}+\left\{\overline{v(x)},\overline{v(y)}\right\}=0+0=0$$

令 (8·37)

$$*\ \varphi(x)\varphi(y)=\left(u(x)+\overline{v(x)}\right)\left(u(y)+\overline{v(y)}\right)$$

$$=u(x)u(y)+u(x)\bar v(y)+\bar v(x)u(y)+\bar v(x)\bar v(y)$$

$$N\left(\varphi(x)\varphi(y)\right)=N\left(u(x)u(y)\right)+N\left(u(x)\bar v(y)\right)+N\left(\bar v(x)u(y)\right)+\left(\bar v(x)\bar v(y)\right)$$

$$=u(x)u(y)-\bar v(y)u(x)+\bar v(x)u(y)+\bar v(x)\bar v(y)$$

由于 $\left\{\bar v(y),u(x)\right\}=0$

$$N\left(\varphi(x)\varphi(y)\right)=u(x)u(y)+u(x)\bar v(y)+\bar v(x)u(y)+\bar v(x)\bar v(y)$$

$$=\varphi(x)\varphi(y)$$

$$*\ \bar\varphi(x)\bar\varphi(y)=\bar u(x)\bar u(y)+\bar u(x)v(y)+v(x)\overline{u(y)}+v(x)v(y)$$

$$N\left(\bar\varphi(x)\bar\varphi(y)\right)=\overline{u(x)}\,\overline{u(y)}+\bar u(x)v(y)-\bar u(y)v(x)+v(x)v(y)$$

由于 $\left\{\bar u(y),v(x)\right\}=0$

$$N(\bar{\varphi}(x)\bar{\varphi}(y))=\bar{u}(x)\bar{u}(y)+\bar{v}(x)v(y)+\bar{v}(x)u(y)+v(x)v(y)$$
$$=\bar{\varphi}(x)\bar{\varphi}(y)$$

◇ (8·39)

$$J_\mu=\frac{ie}{2}[\bar{\varphi}\gamma_\mu\varphi-\tilde{\varphi}\tilde{\gamma}_\mu\tilde{\bar{\varphi}}]=\frac{ie}{2}(\bar{\varphi}_\alpha(\gamma_\mu)_{\alpha\beta}\varphi_\beta-\varphi_\beta(\tilde{\gamma}_\mu)_{\beta\alpha}\bar{\varphi}_\alpha)$$
$$=\frac{ie}{2}(\bar{\varphi}_\alpha(\gamma_\mu)_{\alpha\beta}\varphi_\beta-\varphi_\beta(\gamma_\mu)_{\alpha\beta}\bar{\varphi}_\alpha)=\frac{ie}{2}(\gamma_\mu)_{\alpha\beta}(\bar{\varphi}_\alpha\bar{\varphi}_\beta-\varphi_\beta\bar{\varphi}_\alpha)$$

$$2N(\bar{\varphi}_\alpha\varphi_\beta)+\{v_\alpha,\bar{v}_\beta\}-\{u_\beta,\bar{u}_\alpha\}$$
$$=2N((\bar{u}_\alpha+v_\alpha)+(u_\beta+\bar{v}_\beta))+\{v_\alpha,\bar{v}_\beta\}-\{u_\beta,\bar{u}_\alpha\}$$
$$=2[\bar{u}_\alpha u_\beta+\bar{u}_\alpha\bar{v}_\beta+v_\alpha u_\beta-\bar{v}_\beta v_\alpha]+v_\alpha\bar{v}_\beta+\bar{v}_\beta v_\alpha-u_\beta\bar{u}_\alpha-\bar{u}_\alpha u_\beta$$
$$=\bar{u}_\alpha u_\beta+2\bar{u}_\alpha\bar{v}_\beta+2v_\alpha u_\beta-\bar{v}_\beta v_\alpha+v_\alpha\bar{v}_\beta-u_\beta\bar{u}_\alpha$$
$$=\bar{u}_\alpha u_\beta+\bar{u}_\alpha\bar{v}_\beta+v_\alpha u_\beta+v_\alpha\bar{v}_\beta-\bar{v}_\beta\bar{u}_\alpha-u_\beta v_\alpha-\bar{v}_\beta v_\alpha-u_\beta\bar{u}_\alpha$$
$$=(\bar{u}_\alpha+v_\alpha)(u_\beta+\bar{v}_\beta)-(u_\beta+\bar{v}_\beta)(\bar{u}_\alpha+v_\alpha)=\bar{\varphi}_\alpha\varphi_\beta-\varphi_\beta\bar{\varphi}_\alpha$$

◇ (8·40) ⟶ (8·41)

1 $$(\gamma_\mu)_{\alpha\beta}\{u_\beta(x),\bar{u}_\alpha(x)\}=(\gamma_\mu)_{\alpha\beta}\left\{\sum_s a_s\varphi^{(+)}_{s\beta}(x),\sum_{s'}a^*_{s'}\bar{\varphi}^{(+)}_{s'\alpha}(x)\right\}$$
$$=(\gamma_\mu)_{\alpha\beta}\sum_s\varphi^{(+)}_{s\beta}(x)\bar{\varphi}^{(+)}_{s\alpha}(x)=\sum_s\bar{\varphi}^{(+)}_s(x)\gamma_\mu\varphi^{(+)}_s(x)$$

2. $$(\gamma_\mu)_{\alpha\beta}\{v_\alpha(x),\bar{v}_\beta(x)\}=(\gamma_\mu)_{\alpha\beta}\left\{\sum_s b_s\bar{\varphi}^{(-)}_{s\alpha}(x),\sum_{s'}b^*_{s'}\varphi^{(-)}_{s'\beta}(x)\right\}$$
$$=(\gamma_\mu)_{\alpha\beta}\sum_s\bar{\varphi}^{(-)}_{s\alpha}(x)\varphi^{(-)}_{s\beta}(x)=\sum_s\bar{\varphi}^{(-)}_s(x)\gamma_\mu\varphi^{(-)}_s(x)$$

利用

$$\varphi'(x)=C[\widetilde{\bar{\varphi}(x)}],\qquad \bar{\varphi}'(x)=\tilde{\varphi}(x)[\widetilde{C^{-1}}]$$
$$C^{-1}=C^+,\qquad \tilde{C}=-C,\qquad \tilde{\gamma}_\mu=-C^{-1}\gamma_\mu C$$

$$\sum_s\bar{\varphi}^{(-)}_s(x)\gamma_\mu\varphi^{(-)}_s(x)=\sum_{s'}\tilde{\varphi}^{(+)}_{s'}(x)[\widetilde{C^{-1}}]\gamma_\mu C[\widetilde{\bar{\varphi}^{(+)}_{s'}(x)}]$$
$$=\sum_{s'}\bar{\varphi}^{(+)}_{s'}(x)\tilde{C}\tilde{\gamma}_\mu C^{-1}\varphi^{(+)}_{s'}(x)=\sum_{s'}\bar{\varphi}^{(+)}_{s'}(x)\gamma_\mu\varphi^{(+)}_{s'}(x)$$

◇ (8·42)

$$J_\mu=ieN(\bar{\varphi}\gamma_\mu\varphi)$$

$$\mathcal{H}_i = -A_\mu J_\mu = -ieA_\mu N(\bar{\varphi}\gamma_\mu\varphi) = -ieN(\bar{\varphi}\hat{A}\varphi)$$

轻子与光子可以对易

◇ (8·44)

$$\bar{\varphi}\gamma_5\varphi = (\bar{u}+\bar{v})\gamma_5(u+\bar{v}) = (\bar{u}+v)_\alpha(\gamma_5)_{\alpha\beta}(u+\bar{v})_\beta$$

$$= (\gamma_5)_{\alpha\beta}[\bar{u}_\alpha u_\beta + \bar{u}_\alpha\bar{v}_\beta + v_\alpha u_\beta + v_\alpha\bar{v}_\beta]$$

$$N(\bar{\varphi}\gamma_5\varphi) = (\gamma_5)_{\alpha\beta}[\bar{u}_\alpha u_\beta + \bar{u}_\alpha\bar{v}_\beta + v_\alpha u_\beta - \bar{v}_\beta v_\alpha]$$

$$\bar{\varphi}\gamma_5\varphi - N(\bar{\varphi}\gamma_5\varphi) = (\gamma_5)_{\alpha\beta}(v_\alpha\bar{v}_\beta + \bar{v}_\beta v_\alpha) = (\gamma_5)_{\alpha\beta}\{v_\alpha, \bar{v}_\beta\}$$

$$= (\gamma_5)_{\alpha\beta}\left\{\sum_s b_s\bar{\varphi}^{(-)}_{s\alpha}(x), \sum_{s'} b^*_{s'}\varphi^{(-)}_{s'\beta}(x)\right\}$$

$$= (\gamma_5)_{\alpha\beta}\sum_s\bar{\varphi}^{(-)}_{s\alpha}(x)\varphi^{(-)}_{s\beta}(x) = \sum_s\bar{\varphi}^{(-)}_s(x)\gamma_5\varphi^{(-)}_s(x)$$

$$= \sum_s\sqrt{\frac{m\varepsilon(-E)}{-E}}\bar{u}_\Sigma(-p)e^{ipx}\gamma_5\sqrt{\frac{m\varepsilon(-E)}{-E}}u_\Sigma(-p)e^{-ipx}$$

$$= \sum_{\Sigma\vec{p}}\frac{m}{E}\bar{u}_\Sigma(-p)\gamma_5 u_\Sigma(-p) = \sum_{\Sigma\vec{p}}\frac{m}{E}\bar{u}_\Sigma(-\vec{e})\frac{i\hat{p}+m}{2m}\gamma_5\frac{i\hat{p}+m}{2m}u_\Sigma(\vec{e})$$

$$= \sum_{\Sigma\vec{p}}\frac{1}{4mE}\bar{u}_\Sigma(-\vec{e})(i\hat{p}+m)\gamma_5(i\hat{p}+m)u_\Sigma(\vec{e})$$

$$= \sum_{\Sigma\vec{p}}\frac{1}{4mE}\bar{u}_\Sigma(-\vec{e})(i\hat{p}+m)(-i\hat{p}+m)\gamma_5 u_\Sigma(\vec{e})$$

$$= \sum_{\Sigma\vec{p}}\frac{1}{4mE}\bar{u}_\Sigma(-\vec{e})(p^2+m^2)\gamma_5 u_\Sigma(\vec{e}) = 0$$

◇ (8·46)

$$\bar{\varphi}\vec{\tau}\gamma_5\varphi - N(\bar{\varphi}\vec{\tau}\gamma_5\varphi) = \sum_s\bar{\varphi}^{(-)}_s(x)\vec{\tau}\gamma_5\varphi^{(-)}_s(x)$$

$$= \sum_{\Sigma\vec{p}}\frac{1}{4mE}\bar{u}_\Sigma(-\vec{e})(i\hat{p}+m)\vec{\tau}\gamma_5(i\hat{p}+m)u_\Sigma(\vec{e})$$

$$= \sum_{\Sigma\vec{p}}\frac{1}{4mE}\bar{u}_\Sigma(-\vec{e})(i\hat{p}+m)(-i\hat{p}+m)\vec{\tau}\gamma_5 u_\Sigma(\vec{e}) = 0$$

$$\bar{\varphi}\vec{\tau}\gamma_5\varphi\cdot\vec{\varphi} = [N(\bar{\varphi}\vec{\tau}\gamma_5\varphi)]\cdot\vec{\varphi} = N(\bar{\varphi}\vec{\tau}\gamma_5\varphi\cdot\vec{\varphi})$$

$$\mathcal{H}_i = ig\bar{\varphi}\vec{\tau}\gamma_5\varphi\cdot\vec{\varphi} = igN(\bar{\varphi}\vec{\tau}\gamma_5\varphi\cdot\vec{\varphi})$$

◇ (8·47) β 衰变理论

$$\mathcal{L} = -\bar{\varphi}_p\left(\gamma_\mu\frac{\partial}{\partial x_\mu}+M\right)\varphi_p - \bar{\varphi}_N\left(\gamma_\mu\frac{\partial}{\partial x_\mu}+M\right)\varphi_N +$$

$$-\bar{\varphi}_e\left(\gamma_\mu \frac{\partial}{\partial x_\mu}+m\right)\varphi_e-\bar{\varphi}_\nu\gamma_\mu \frac{\partial}{\partial x_\mu}\psi_\nu$$

$$+\frac{G}{2}\left\{\bar{\varphi}_e\gamma_\mu(1+\gamma_5)\varphi_N\bar{\varphi}_e\gamma_\mu(1+\gamma_5)\psi_\nu+\bar{\varphi}_\nu\gamma_\mu(1+\gamma_5)\varphi_e\bar{\varphi}_N\gamma_\mu(1+\gamma_5)\varphi_p\right\}$$

$$\pi_p=\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_p}=i\bar{\varphi}_p\gamma_4 \quad , \quad \pi_N=\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_N}=i\bar{\varphi}_N\gamma_4$$

$$\pi_e=\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_e}=i\bar{\varphi}_e\gamma_4 \quad , \quad \pi_\nu=\frac{\partial \mathcal{L}}{\partial \dot{\psi}_\nu}=i\bar{\varphi}_\nu\gamma_4$$

$$\mathcal{H}=i\bar{\varphi}_p\gamma_4\dot{\varphi}_p+i\bar{\varphi}_N\gamma_4\dot{\psi}_N+i\bar{\varphi}_e\gamma_4\dot{\psi}_e+i\bar{\varphi}_\nu\gamma_4\dot{\psi}_\nu$$

$$=-\bar{\varphi}_p\gamma_4\frac{\partial}{\partial x_4}\psi_p-\bar{\varphi}_N\gamma_4\frac{\partial}{\partial x_4}\psi_N-\bar{\varphi}_e\gamma_4\frac{\partial}{\partial x_4}\psi_e+\bar{\varphi}_\nu\gamma_4\frac{\partial}{\partial x_4}\psi_\nu-\mathcal{L}$$

$$=\bar{\varphi}_e\left(\gamma_i\frac{\partial}{\partial x_i}+M\right)\psi_p+\bar{\varphi}_N\left(\gamma_i\frac{\partial}{\partial x_i}+M\right)\psi_N+\bar{\varphi}_e\left(\gamma_i\frac{\partial}{\partial x_i}+m\right)\varphi_e$$

$$+\bar{\varphi}_\nu\left(\gamma_i\frac{\partial}{\partial x_i}\right)\psi_\nu+\frac{G}{\sqrt{2}}\left\{\bar{\varphi}_p\gamma_\mu(1+\gamma_5)\psi_N\bar{\varphi}_e\gamma_\mu(1+\gamma_5)\psi_\nu\right.$$

$$\left.+\bar{\varphi}_\nu\gamma_\mu(1+\gamma_5)\bar{\varphi}_N\gamma_\mu(1+\gamma_5)\psi_p\right\}$$

$$\mathcal{H}_0=\psi_p^*(\vec{\alpha}\cdot\vec{p}+\beta M)\psi_p+\varphi_N^*(\vec{\alpha}\cdot\vec{p}+M)\varphi_N$$

$$+\varphi_e^*(\vec{\alpha}\cdot\vec{\beta}+\beta m)\varphi_e+\varphi_\nu^*\vec{\alpha}\cdot\vec{p}\,\psi_\nu$$

$$\mathcal{H}_i=\frac{G}{\sqrt{2}}\left\{\bar{\varphi}_p\gamma_\mu(1+\gamma_5)\psi_N\gamma_\mu(1+\gamma_5)\psi_\nu\right.$$

$$\left.+\bar{\varphi}_\nu\gamma_\mu(1+\gamma_5)\varphi_e\bar{\varphi}_N\gamma_\mu(1+\gamma_5)\psi_p\right\}$$

由于 P、N、e、ν 之间相互反对易，所以没有多余 δ 函数项接写成正规之积，换言之

$$\bar{\varphi}_p\varphi_N\varphi_e\psi_\nu=(\bar{u}_p+\bar{v}_p)(u_N+\bar{v}_N)(\bar{u}_e+v_e)(u_\nu+v_\nu)$$

$$=\bar{u}_p u_N\bar{u}_e u_\nu+\cdots\cdots$$

$$N(\bar{\varphi}_p\varphi_N\bar{\varphi}_e\varphi_\nu)=-\bar{u}_p\bar{u}_e u_N u_\nu+\cdots\cdots=\bar{u}_p u_N\bar{u}_e u_\nu+\cdots\cdots$$

$$=\varphi_p\varphi_N\bar{\varphi}_e\varphi_\nu$$

$$N\left(\bar{\varphi}_p\gamma_\mu(1+\gamma_5)\varphi_N\bar{\varphi}_e\gamma_\mu(1+\gamma_5)\varphi_\nu\right)$$

$$=[\gamma_\mu(1+\gamma_5)]_{\alpha\beta}[\gamma_\mu(1+\gamma_5)]_{\sigma\rho}N\left(\bar{\varphi}_{p\alpha}\varphi_{N\beta}\bar{\varphi}_{e\sigma}\varphi_{\nu\rho}\right)$$

$$=(\gamma_\mu(1+\gamma_5))_{\sigma\beta}(\gamma_\mu(1+\gamma_5))_{\sigma\rho}\bar{\varphi}_{p\alpha}\psi_{N\beta}\bar{\varphi}_{e\sigma}\psi_{\nu\rho}=$$

$$= \bar{\varphi}_p \gamma_\mu (1+\gamma_5)\varphi_N \bar{\varphi}_e \gamma_\mu (1+\gamma_5)\varphi_\nu$$

$$\mathcal{H} = \frac{G}{\sqrt{2}} N(\bar{\varphi}_p \gamma_\mu (1+\gamma_5)\varphi_N \bar{\varphi}_e \gamma_\mu (1+\gamma_5)\varphi_\nu)$$

$$+ \frac{G}{\sqrt{2}} N(\bar{\varphi}_\nu \gamma_\mu (1+\gamma_5)\varphi_e \bar{\varphi}_N \gamma_\mu (1+\gamma_5)\varphi_p$$

§8.5 维克的二 定理

◇ (8.51)

$$T(\varphi(x)\varphi(y)) = \theta(x_0-y_0)\varphi(x)\varphi(\ )-\theta(y_0-x_0)\varphi(y)\varphi(x)$$

由于 $\{\varphi(x),\varphi(y)\}=0$

$$T(\varphi(x)\varphi(y)) = \theta(x_0-y_0)\varphi(x)\varphi(y)+\ (y_0-x_0)\varphi(x)\varphi(y)$$

$$=[\theta(x_0-y_0)+\theta(y_0-x_0]\varphi(x)\varphi(y)$$

由于 $\theta(x)+\theta(-x) = \dfrac{1+\frac{x}{|x|}}{2} + \dfrac{1+}{} \dfrac{x}{x|} = 1$

$$T(\varphi(x)\varphi(y)) = \varphi(x)\varphi(y$$

同样

$$T(\bar{\varphi}(x)\bar{\varphi}(y)) = \theta(x_0-y_0)\bar{\varphi}(x)\bar{\varphi}(\ )-\theta(y_0-x_0)\bar{\varphi}(y_0)\bar{\varphi}(x_0)$$

$$=\theta(x_0-y_0)\bar{\varphi}(x)\bar{\varphi}(y)+\theta(y_0-x_0)\bar{\ }(x_0)\bar{\varphi}(y_0)$$

$$=[\theta(x_0-y_0)+\theta(y_0-x_0)]\bar{\varphi}(x_0)\bar{\varphi}(y_0 = \bar{\varphi}(x_0)\bar{\varphi}(y_0)$$

◇ (8.52)

$$T(\varphi(x)\varphi(y)) = \varphi(x)\varphi(y$$

$$N(\varphi(x)\varphi(y)) = \varphi(x)\varphi(y)$$

$$\frac{-)\qquad\qquad\qquad\qquad\qquad}{\varphi^{\cdot}(x)\varphi^{\cdot}(y) = (}$$

同样

$$T(\bar{\varphi}(x)\bar{\varphi}(y)) = \bar{\varphi}(x)\bar{\varphi}(y)$$

$$N(\bar{\varphi}(x)\bar{\varphi}(y)) = \varphi(x)\bar{\varphi}(y)$$

$$\frac{-)\qquad\qquad\qquad\qquad\qquad}{\bar{\varphi}^{\cdot}(x)\bar{\varphi}^{\cdot}(y) = 0}$$

◇ (8·53) — (8·58)

$$T(\varphi_\alpha(x)\bar{\varphi}_\beta(y)) = \theta(x_0-y_0)\varphi_\alpha(x)\bar{\varphi}_\beta(y) - \theta(y_0-x_0)\bar{\varphi}_\beta(y)\varphi_\alpha(x)$$

$$= \theta(x_0-y_0)\{u_\alpha(x)\bar{u}_\beta(y) + u_\alpha(x)v_\beta(y) + \bar{v}_\alpha(x)\bar{u}_\beta(y) + \bar{v}_\alpha(x)v_\beta(y)\}$$

$$-\theta(y_0-x_0)\{\bar{u}_\beta(y)u_\alpha(x) + \bar{u}_\beta(y)\bar{v}_\alpha(x) + v_\beta(y)u_\alpha(x) + v_\beta(y)\bar{v}_\alpha(x)\}$$

$$N(\varphi_\alpha(x)\bar{\varphi}_\beta(y)) = -\bar{u}_\beta(y)u_\alpha(x) + u_\alpha(x)v_\beta(y) + \bar{v}_\alpha(x)\bar{u}_\beta(y) + \bar{v}_\alpha(x)v_\beta(y)$$

$$= \theta(x_0-y_0)\{-\bar{u}_\beta(y)u_\alpha(x) + u_\alpha(x)v_\beta(y) + \bar{v}_\alpha(x)\bar{u}_\beta(y) + \bar{v}_\alpha(x)v_\beta(y)\}$$

$$+\theta(y_0-x_0)\{-\bar{u}_\beta(y)u_\alpha(x) + u_\alpha(x)v_\beta(y) + \bar{v}_\alpha(x)\bar{u}_\beta(y) + \bar{v}_\alpha(x)v_\beta(y)\}$$

相减获得其差为

$$\varphi_\alpha^{\cdot}(x)\bar{\varphi}_\beta^{\cdot}(y) = \theta(x_0-y_0)\{u_\alpha(x), \bar{u}_\beta(y)\} - \theta(y_0-x_0)\{v_\beta(y), \bar{v}_\alpha(x)\}$$

$$= -\frac{1}{2}S_{f,\alpha\beta}(x-y)$$

$$S_{f,\alpha\beta}(x-y) = -2\theta(x_0-y_0)\{u_\alpha(x)\ \bar{u}_\beta(y)\} + 2\theta(x_0-y_0)\{v_\beta(y), \bar{v}_\alpha(x)\}$$

◇ (8·59)

$$\bar{\varphi}_\alpha^{\cdot}(x)\varphi_\beta^{\cdot}(y) = T(\bar{\varphi}_\alpha(x)\varphi_\beta(y)) - N(\bar{\varphi}_\alpha(x)\varphi_\beta(y))$$

$$= -T(\varphi_\beta(y)\bar{\varphi}_\alpha(x)) + N(\varphi_\beta(y)\bar{\varphi}_\alpha(x))$$

$$= -\varphi_\beta^{\cdot}(y)\bar{\varphi}_\alpha^{\cdot}(x) = \frac{1}{2}S_{f,\beta\alpha}(y-x)$$

◇ (8·60) — (8·61)

$$T(A_\mu(x)A_\nu(y)) = \theta(x_0-y_0)A_\mu(x)A_\nu(y) + \theta(y_0-x_0)A_\nu(y)A_\mu(x)$$

$$= \theta(x_0-y_0)(w_\mu(x)+\bar{w}_\mu(x))(w_\nu(y)+\bar{w}_\nu(y))$$

$$+\theta(y_0-x_0)(w_\nu(y)+\bar{w}_\nu(y))(w_\mu(x)+\bar{w}_\mu(x))$$

$$= \theta(x_0-y_0)\{w_\mu(x)w_\nu(y) + w_\mu(x)\bar{w}_\nu(y) + \bar{w}_\mu(x)w_\nu(y)$$

$$+\bar{w}_\mu(x)\bar{w}_\nu(y)\} + \theta(y_0-x_0)\{w_\nu(y)w_\mu(x) + w_\nu(y)\bar{w}_\mu(x)$$

$$+\bar{w}_\nu(y)w_\mu(x) + \bar{w}_\nu(y)\bar{w}_\mu(x)\}$$

$$N(A_\mu(x)A_\nu(y)) = w_\mu(x)w_\nu(y) + \bar{w}_\nu(y)w_\mu(x) + \bar{w}_\mu(x)w_\nu(y) + \bar{w}_\mu(x)\bar{w}_\nu(y)$$

$$= \theta(x_0-y_0)\{w_\mu(x)w_\nu(y) + \bar{w}_\nu(y)w_\mu(x) + \bar{w}_\mu(x)w_\nu(y) + \bar{w}_\mu(x)\bar{w}_\nu(y)\}$$

$$-\theta(y_0-x_0)\{w_\mu(x)w_\nu(y) + \bar{w}_\nu(y)w_\mu(x) + \bar{w}_\mu(x)w_\nu(y) + \bar{w}_\mu(x)\bar{w}_\nu(y)\}$$

相减求其差

$$A_\mu^{\cdot}(x)A_\nu^{\cdot}(y)=\theta(x_0-y_0)[\overline{w}_\mu(x),\overline{w}_\nu(y)]+\theta(y_0-x_0)[w_\nu(y),\overline{w}_\mu(x)$$
$$=\frac{1}{2}\delta_{\mu\nu}D_f(x-y)$$

$$\delta_{\mu\nu}D_f(x-y)=2\theta(x_0-y_0)[w_\mu(x)\overline{w}_\nu(y)+2\theta(y_0-x_0)[w_\nu(y)\overline{w}_\mu(x)]$$

◇ (8·62)—(8·63)

$$T(\varphi_j(x)\varphi_k(y))=\theta(x_0-y_0)\varphi_j(x)\varphi_k(y)+\theta(y_0-x_0)\varphi_k(y)\varphi_j(x)$$
$$=\theta(x_0-y_0)(Z_j(x)+\overline{Z}_j(x))(Z_k(y)+\overline{Z}_k(y))$$
$$+\theta(y_0-x_0)(Z_k(y)+\overline{Z}_k(y))(Z_j(x)+\overline{Z}_j(x))$$
$$=\theta(x_0-y_0)(Z_j(x)Z_k(y)+Z_j(x)\overline{Z}_k(y)+\overline{Z}_j(x)Z_k(y)$$
$$+Z_j(x)\overline{Z}_k(y))+\theta(y_0-x_0)(Z_k(y)Z_j(x)+Z_k(y)\overline{Z}_j(x)$$
$$+\overline{Z}_k(y)Z_j(x)+\overline{Z}_k(y)\overline{Z}_j(x))$$

$$N(\varphi_j(x)\varphi_k(y))=Z_j(x)Z_k(y)+\overline{Z}_k(y)Z_j(x)+\overline{Z}_j(x)Z_k(y)+\overline{Z}_j(x)$$
$$=\theta(x_0-y_0)(Z_j(x)Z_k(y)+\overline{Z}_\beta(y)Z_j(x)+\overline{Z}_j(x)Z_k(y)+\overline{Z}_j(x)\overline{Z}_k(y))$$
$$+\theta(y_0-x_0)(Z_j(x)Z_k(y)+\overline{Z}_k(y)Z_j(x)+\overline{Z}_j(x)Z_k(y)+\overline{Z}_j(x)\overline{Z}_k(y))$$

相减求其差

$$\varphi_j^{\cdot}(x)\varphi_k^{\cdot}(y)=\theta(x_0-y_0)[Z_j(x),Z_k(y)]+\theta(y_0-x_0)[Z_k(y)\overline{Z}_j(x)]$$
$$=\frac{1}{2}\delta_{jk}\Delta_f(x-y)$$

$$\delta_{jk}\Delta_f(x-y)=2\theta(x_0-y_0)[Z_j(x)Z_k(y)]+2\theta(y_0-x_0)[Z_k(y)\overline{Z}_j(x)]$$

◇ (8·65)

1 $A_\mu^{\cdot}(x)\varphi_\alpha^{\cdot}(y)=0$

$$T(A_\mu(x)\varphi_\alpha(y))=\theta(x_0-y_0)A_\mu(x)\varphi_\alpha(y)+\theta(y_0-x_0)\varphi_\alpha(y)A_\mu(x)$$

由于 $[A_\mu(x),\varphi_\alpha(y)]=0$

$$T(A_\mu(x)\varphi_\alpha(y))=\theta(x_0-y_0)A_\mu(x)\varphi_\alpha(y)+\theta(y_0-x_0)A_\mu(x)\varphi_\alpha(y)$$
$$=[\theta(x_0-y_0)+\theta(y_0+x_0)]A_\mu(x)\varphi_\alpha(x)=A_\mu(x)\varphi_\alpha(y)$$

$$N(A_\mu(x)\varphi_\alpha(y))=N((w_\mu(x)+\overline{w}_\mu(x))(u_\alpha(y)+\overline{v}_\alpha(y))$$
$$=w_\mu(x)u_\alpha(y)+\overline{v}_\alpha(y)w_\mu(x)+\overline{w}_\mu(x)u_\alpha(y)+\overline{w}_\mu(x)\overline{v}_\alpha(y)$$

由于 $[W_\mu, \bar{v}_\alpha]=0$

$$=W_\mu(x)u_\alpha(y)+W_\mu(x)\bar{v}_\alpha(y)+\bar{W}_\mu(x)u_\alpha(y)+\bar{W}_\mu(x)\bar{v}_\alpha(y)$$

$$=A_\mu(x)\psi_\alpha(y)$$

相减其差

$$A^{\cdot}_\mu(x)\,\varphi^{\cdot}_\alpha(y)=0$$

2. $$A^{\cdot}_\mu(x)\,\bar{\varphi}^{\cdot}_\beta(y)=0$$

$$T(A_\mu(x)\bar{\varphi}_\beta(y))=\theta(x_0-y_0)A_\mu(x)\bar{\varphi}_\beta(y)+\theta(y_0-x_0)\bar{\varphi}_\beta(y)A_\mu(x)$$

由于 $[A_\mu(x),\bar{\varphi}_\beta(y)]=0$

$$T(A_\mu(x)\bar{\varphi}_\beta(y))=\theta(x_0-y_0)A_\mu(x)\bar{\varphi}_\beta(y)+\theta(y_0+x_0)A_\mu(x)\bar{\varphi}_\beta(y)$$

$$=A_\mu(x)\bar{\varphi}_\beta(y)$$

$$N(A_\mu(x)\bar{\varphi}_\beta(y))=N\big((W_\mu(x)+\bar{W}_\mu(x))(\bar{u}_\beta(y)+v_\beta(y))\big)$$

$$=\bar{u}_\beta(y)W_\mu(x)+W_\mu(x)v_\beta(y)+\bar{W}_\mu(x)\bar{u}_\beta(y)+\bar{W}_\mu(x)v_\beta(y)$$

由于 $[\bar{u}_\beta(y), W_\mu(x)]=0$

$$N(A_\mu(x)\bar{\varphi}_\beta(y))=W_\mu(x)\bar{u}_\beta(y)+W_\mu(x)v_\beta(y)+\bar{W}_\mu(x)\bar{u}_\beta(y)$$

$$+\bar{W}_\mu(x)v_\beta(y)=A_\mu(x)\bar{\varphi}_\beta(y)$$

相减求其差

$$A^{\cdot}_\mu(x)\,\bar{\varphi}^{\cdot}_\beta(y)=0$$

3. $$\varphi^{\cdot}_j(x)\,\psi^{\cdot}_\alpha(y)=0$$

$$T(\varphi_j(x)\varphi_\alpha(y))=\theta(x_0-y_0)\varphi_j(x)\varphi_\alpha(y)+\theta(y_0-x_0)\varphi_\alpha(y)\varphi_j(x)$$

由于 $[\varphi_\alpha(y)\varphi_j(x)]=0$

$$T(\varphi_j(x)\varphi_\alpha(y))=\theta(x_0-y_0)\varphi_j(x)\varphi_\alpha(y)+\theta(y_0-x_0)\varphi_j(x)\varphi_\alpha(y)$$

$$=\varphi_j(x)\varphi_\alpha(y)$$

$$N(\varphi_j(x)\varphi_\alpha(y))=N\big((Z_j(x)+\bar{Z}_j(x))(u_\alpha(y)+\bar{v}_\alpha(y))\big)$$

$$=Z_j(x)u_\alpha(y)+\bar{v}_\alpha(y)Z_j(x)+\bar{Z}_j(x)u_\alpha(y)+\bar{Z}_j(x)\bar{v}_\alpha(y)$$

由于 $[Z_j(x),\bar{v}_\alpha(y)]=0$

$$N(\varphi_j(x)\varphi_\alpha(y))=Z_j(x)u_\alpha(y)+Z_j(x)\bar{v}_\alpha(y)+\bar{Z}_j(x)u_\alpha(y)+\bar{Z}_j(x)\bar{v}_\alpha(y)$$

$$=\varphi_j(x)\varphi_\alpha(y)$$

相减求其差

$$\varphi_j^{\cdot}(x)\varphi_\alpha^{\cdot}(y)=0$$

4.
$$\varphi_j^{\cdot}(x)\bar{\varphi}_\beta^{\cdot}(y)=0$$

$$T(\varphi_j(x)\bar{\varphi}_\beta(y))=\theta(x_0-y_0)\varphi_j(x)\bar{\varphi}_\beta(y)+\theta(y_0-x_0)\bar{\varphi}_\beta(y)\varphi_j(x)$$

由于
$$[\varphi_j(x),\bar{\varphi}_\beta(y)]=0$$

$$T(\varphi_j(x)\bar{\varphi}_\beta(y))=\theta(x_0-y_0)\varphi_j(x)\bar{\varphi}_\beta(y)-\theta(y_0-x_0)\varphi_j(x)\bar{\varphi}_\beta(y)$$
$$=\varphi_j(x)\bar{\varphi}_\beta(y)$$

$$N(\varphi_j(x)\bar{\varphi}_\beta(y))=N((Z_j(x)+\bar{Z}_j(x))(\overline{u_\beta}(y)+v_\beta(y)))$$
$$=\bar{u}_\beta(y)Z_j(x)+Z_j(x)v_\beta(y)+\bar{Z}_j(x)\bar{u}_\beta(y)+\bar{Z}_j(x)v_\beta(y)$$

由于
$$[Z_j(x),\overline{u}_\beta(y)]=0$$

$$N(\varphi_j(x)\varphi_\beta(y))=Z_j(x)\bar{u}_\beta(y)+Z_j(x)v_\beta(y)+\bar{Z}_j(x)\bar{u}_\beta(y)+\bar{Z}_j(x)v_\beta(y)$$
$$=\varphi_j(x)\bar{\varphi}_\beta(y)$$

相减求其差
$$\varphi_j^{\cdot}(x)\bar{\varphi}_\beta^{\cdot}(y)=0$$

结论：两个对易，或反对易的场的收缩等于零

◇ (8·69)的分析

设 Z是最早的

则 $N(XY)Z=N(X^{\cdot}YZ^{\cdot})+N(XY^{\cdot}Z^{\cdot})+N(XYZ)$

1. X、Y、Z 是单个的产生或消失标符就够了。

如收缩是线性的

$$(A+B)^{\cdot}C^{\cdot}=T((A+B)C)-N((A+B)C)=T(AC+BC)-N(AC+BC)$$
$$=T(AC)+T(BC)-N(AC)-N(BC)=A^{\cdot}C^{\cdot}+B^{\cdot}C^{\cdot}$$

则可证如果分开各自成立

$$N(X_1Y)Z=N(X_1^{\cdot}YZ^{\cdot})+N(X_1Y^{\cdot}Z^{\cdot})+N(X_1YZ)$$
$$N(X_2Y)Z=N(X_2^{\cdot}YZ^{\cdot})+N(X_2Y^{\cdot}Z^{\cdot})+N(X_2YZ)$$

那么，合并相加也成立。

$$N(XY)Z=N(X^{\cdot}YZ^{\cdot})+N(XY^{\cdot}Z^{\cdot})+N(XYZ)$$

其中
$$X=X_1+X_2$$

2. Z是产生标符

* 如果 Z 是最早的，又是消失的，那末在 T、N 作用之下，都在最后，因此总有

$$X^{\cdot}Z^{\cdot}=T(XZ)-N(XZ)\doteq XZ-XZ=0$$

$$Y^{\cdot}Z^{\cdot}=T(YZ)-N(YZ)\doteq YZ-YZ=0$$

结论："和最早的，消失的协男的收缩等于零"

$$X^{\cdot}Z^{\cdot}=0$$

3. N(XY) 中的 X、Y 已按正规乘积的次序排列
如果没有排好请重排

$$\delta_{\rho}N(YX)Z=\delta_{\rho}N(YX^{\cdot}Z^{\cdot})+\delta_{\rho}N(Y^{\cdot}XZ^{\cdot})+\delta_{\rho}N(YXZ)$$

相互消去 δ_{ρ}

$$N(YX)Z=N(YX^{\cdot}Z^{\cdot})+N(Y^{\cdot}XZ^{\cdot})+N(YXZ)$$

可见一定能排成标序列

4. N(X、Y) 中的 X、Y 都是消灭标符
如果 X 是产生标符，那末

$$X^{\cdot}Z^{\cdot}=T(XZ)-N(XZ)=XZ-XZ=0$$

Z最早，X产生

贡献一个零项

◇ (8.83)

设时间上的迟早是这样的

产生　　　消失
(迟)　　　(早)

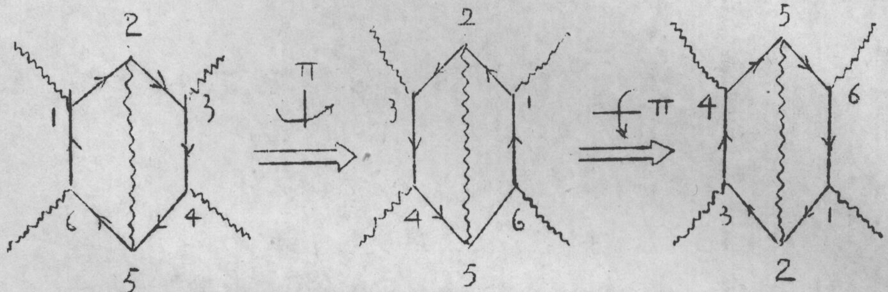
那么编时乘积(迟，早)和正规乘积(产生，消失)完全相同。

1、 $N(WXY)=\delta_{\rho}W'X'Y'=\delta_{\rho}T(W'X'Y')=T(WXY)$

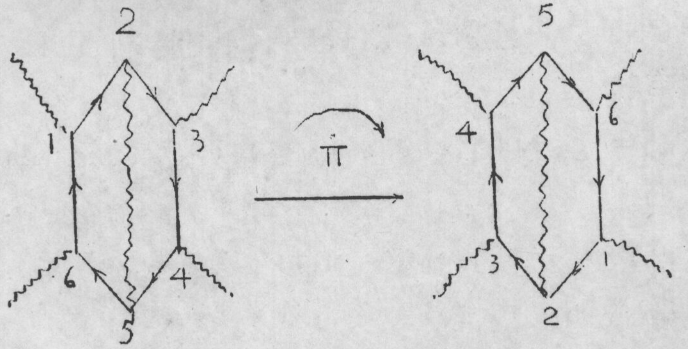
2. $W^{\cdot}X^{\cdot}=T(WX)-N(WX)=\delta_{\rho}W'X'-\delta_{\rho}W'X'=0$

§8·6 用图形表示正规乘积的方法

存在二个对称轴(不对，不是平面转动)



或存在一个对称轴（是平面转动）



产生之图形相同

234561 图形 = 561234 图形

∴ 排列之后可以获得

$$\frac{1}{2}\cdot 6\,!$$

个图形

如果一个图形可以由平面转动来达到，就认为正规乘积.

§8·7 正规乘积所代表的物理性质

◇ (8·94)

$$N={}_0\langle| C_{n_f} a_{g_f} N(\bar{\varphi}\hat{A}\varphi^{\cdot}\bar{\varphi}^{\cdot}\hat{A}\psi) a^*_{g_i} C^*_{\eta_i}|\rangle_0$$

$$={}_0\langle| C_{\eta_f} a_{g_f} N(\bar{\varphi}_\sigma(x_2)\hat{A}_{\sigma\rho}(x_2)\varphi^{\cdot}_\rho(x_2)\bar{\varphi}^{\cdot}_r(x_1)\hat{A}_{rs}(x_1)\varphi_s(x_1)) a^*_{g_i} C^*_{\eta_i}|\rangle_0$$

$$=\varphi^{\cdot}_\rho(x_2)\bar{\varphi}^{\cdot}_r(x_1)\,{}_0\langle| C_{\eta_f} a_{g_f} N(\bar{\varphi}_\sigma(x_2)\hat{A}_{\sigma\rho}(x_2)\hat{A}_{rs}(x_1)\varphi_s(x_1)) a^*_{g_i} C^*_{\eta_i}|\rangle_0$$

$$=(-\tfrac{1}{2}) S_{f,\rho r}(x_2-x_1)\,{}_0\langle| C_{n_f} a_{g_f} N(\bar{\varphi}_\sigma(x_2)\varphi_s(x_1)\cdot$$

$$\cdot \hat{A}_{\sigma\rho}(x_2)\hat{A}_{rs}(x_1))a^*_{s_i}C^*_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle|C_{\eta_f}a_{s_f}N\Big((\bar{u}_\sigma(x_2)+\bar{v}_\sigma(x_2))(u_s(x_1)+\bar{v}_s(x_1))\cdot$$

$$\cdot(\hat{w}_{\sigma\rho}(x_2)+\hat{\hat{w}}_{\sigma\rho}(x_2))(\hat{w}_{rs}(x_1)+\hat{\hat{w}}_{rs}(x_1))\Big)a^*_{s_i}C_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle|C_{\eta_f}a_{s_f}(\bar{u}_\sigma(x_2)u_s(x_1)+\bar{u}_\sigma(x_2)\bar{v}_s(x_1)$$

$$+v_\sigma(x_2)u_s(x_1)-\bar{v}_s(x_1)v_\sigma(x_2))(\hat{w}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1)+\hat{\hat{w}}_{rs}(x_1)\hat{w}_{\sigma\rho}(x_2)$$

$$+\hat{\hat{w}}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1)+\hat{\hat{w}}_{\sigma\rho}(x_2)\hat{\hat{w}}_{rs}(x_1))a^*_{s_i}C^*_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle|a_{s_f}(\bar{u}_\sigma(x_2)u_s(x_1)+\bar{u}_\sigma(x_2)\bar{v}_s(x_1)+v_\sigma(x_2)u_s(x_1)$$

$$-\bar{v}_s(x_1)v_\sigma(x_2))a^*_{s_i}C_{\eta_f}(\underline{\hat{w}_{\sigma\rho}(x_2)\,\hat{w}_{rs}(x_1)}+\hat{\hat{w}}_{rs}(x_1)\hat{w}_{\sigma\rho}(x_2)$$

$$+\hat{w}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1)+\hat{\bar{w}}_{\sigma\rho}(x_2)\hat{\bar{w}}_{rs}(x_1))C^*_{\eta_i}|\rangle_0$$

$$= +\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle a_{s_t}(\bar{u}_\alpha(x$$

$$\cdot C_{\eta_f}(\hat{\hat{w}}_{rs}(x_1)\hat{w}_{\sigma\rho}(x_2)+\hat{\hat{v}}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1))C^*_{\eta_i}|\rangle_0$$

$$\underset{\text{正能}}{=\!=} -\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle|a_{s_f}\bar{u}_\sigma(x_2)u_s(x_1)a^*_{s_i}C_{\eta_f}(\hat{\hat{w}}_{\sigma\rho}(x_2)\bar{w}_{rs}(x_1)$$

$$\pm\hat{\hat{w}}_{rs}(x_1)\hat{\hat{w}}_{\sigma\rho}(x_2))C^*_{\eta_i}|\rangle_0$$

讨论几条公式

$$\{a_{s_f},\bar{u}_\sigma(x_2)\}=\{a_{s_f},\sum_f a^*_s\bar{\varphi}_{s\sigma}(x_2)\}=\bar{\varphi}_{s_f\sigma}(x_2)$$

$$\{u_s(x_1),a^*_{s_i}\}=\{\sum_s a_s\varphi_{s_s}(x_1),a^*_{s_i}\}=\varphi_{s_is}(x_1)$$

$$[C_{\eta_f},\hat{w}_{\sigma\rho}(x_2)]=[C_{\eta_f},\sum_\eta\frac{\hat{e}_{\sigma\rho}(\eta)}{\sqrt{2w}}e^{-ikx_2}C^*_\eta]$$

$$=\frac{\hat{e}_{\sigma\rho}(\eta_f)}{\sqrt{2w_f}}e^{-ik_fx_2}$$

$$[\hat{w}_{rs}(x_1)C^*_{\eta_i}]=[\sum_\eta\frac{\hat{a}_{rs}(\eta)}{\sqrt{2w}}e^{ikx}C_\eta.C^*_{\eta_i}]=\frac{\hat{e}_{rs}(\eta_i)}{\sqrt{2w_i}}e^{ik_ix_1}$$

$$N=-\frac{1}{2}S_{f,\rho r}(x_2-x_1)_0\langle|\{a_{s_f}\bar{u}_\sigma(x_2)\}u_s(x_1)a^*_{s_i}C_{\eta_f}(\hat{w}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1)$$

$$+\hat{\hat{w}}_{rs}(x_1)\hat{w}_{\sigma\rho}(x_2)C^*_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2} S_{f\cdot\rho r}(x_2-x_1)\bar{\varphi}_{s_f\sigma}(x_2)\,{}_0\langle|\{u_s(x_1)a^*_{s_i}\}C_{\eta_f}(\hat{\hat{w}}_{\sigma\rho}(x_2)\hat{w}_{rs}(x_1)$$

$$+\hat{\hat{w}}_{rs}(x_1)\hat{w}_{\sigma\rho}(x_2))C^*_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2} S_{f\cdot\rho r}(x_2-x_1)\bar{\varphi}_{s_f\sigma}(x_2)\varphi_{g_i s}(x_1)\,{}_0\langle|\Big([C_{\eta_f},\hat{\hat{w}}_{\sigma\rho}(x_i)]+\hat{w}_{rs}(x_1)$$

$$+[C_{\eta_f}\hat{\hat{w}}_{rs}(x_1)]\hat{w}_{\sigma\rho}(x_2)\Big)C^*_{\eta_i}|\rangle_0$$

$$= -\frac{1}{2} S_{f\cdot\rho r}(x_2-x_1)\bar{\varphi}_{g_f\sigma}(x_2)\varphi_{g_i s}(x_1)\,{}_i\langle|\frac{\hat{e}_{\sigma\rho}(\eta_f)}{\sqrt{2w_f}}e^{-ik_f x_2}\times$$

$$\times[\hat{w}_{rs}(x_1),C^*_{\eta_i}]+\frac{\hat{e}_{rs}(\eta_f)}{\sqrt{2w_f}}e^{-ik_f x_1}[\hat{w}_{\sigma\rho}(x_2)C^*_{\eta_i}]|\rangle_0$$

$$= -\frac{1}{2} S_{f\cdot\rho r}(x_2-x_1)\bar{\varphi}_{s_f\sigma}(x_1)\,{}_0\langle|\frac{\hat{e}_{\sigma\rho}(\eta_f)e^{-ik_f x_2}}{\sqrt{2w_f}}\frac{\hat{e}_{rs}(\eta_i)}{\sqrt{2w_i}}e^{ik_i x_1}$$

$$+\frac{\hat{e}_{rs}(\eta_f)}{\sqrt{2w_f}}e^{-ik_f x_1}\frac{\hat{e}_{\sigma\rho}(\eta_i)}{\sqrt{2w_i}}e^{ik_i x_2}|\rangle_0$$

$$= \bar{\varphi}_{g_f}(x_2)\frac{\hat{e}(\eta_f)}{\sqrt{2w_f}}e^{-ik_f x_2}(-\frac{1}{2})S_f(x_2-x_1)\frac{\hat{e}(\eta_i)}{\sqrt{2w_i}}e^{ik_i x_1}\varphi_{g_i}(x_1)$$

$$+\bar{\varphi}_{s_f}(x_2)\frac{\hat{e}(\eta_i)}{\sqrt{2w_i}}e^{ik_i x_2}(-\frac{1}{2}S_f(x_2-x_1)\frac{\hat{e}(\eta_f)}{\sqrt{2w}}e^{-ik_f x_1}\varphi_{g_i}(x_1)$$

◇ (8·98) — (8·100)

$$N = {}_0\langle|a_{g_4}a_{g_3}N(\bar{\varphi}_\sigma(x_2)\hat{A}_{\sigma\rho}(x_2)\varphi_\rho(x_2)\bar{\varphi}_r(x_1)\hat{A}_{rs}(x_1)\varphi_s(x_1))\cdot$$

$$\cdot a^*_{g_2}a^*_{g_1}|\rangle_0$$

$$= \hat{A}_{\sigma\rho}(x_2)\hat{A}_{rs}(x_1)\,{}_0\langle|a_{g_4}a_{g_3}N(\bar{\varphi}_\sigma(x_2)\varphi_\rho(x_1)\bar{\varphi}_r(x_1)\varphi_s(x_1))a^*_{g_2}a^*_{g_1}|\rangle_0$$

其中

$$\hat{A}_{\sigma\rho}(x_2)\hat{A}_{rs}(x_1) = (\gamma_\mu)_{\sigma\rho}A_\mu(x_2)(\gamma_\nu)_{rs}A_\nu(x_1)$$

$$= (\gamma_\mu)_{\sigma\rho}(\gamma_\nu)_{rs}\frac{1}{2}\delta_{\mu\nu}D_f(x_2-x_1) = (\gamma_\mu)_{\sigma\rho}(\gamma_\mu)_{rs}\frac{1}{2}D_f(x_2-x_1)$$

$$\text{“}N\text{”} = {}_0\langle|a_{g_4}a_{g_3}N(\bar{\varphi}_\sigma(x_2)\varphi_\rho(x_2)\bar{\varphi}_r(x_1)\varphi_s(x_1))a_{s_2}a^*_{s_1}|\rangle_0$$

$$= {}_0\langle|a_{g_4}a_{g_3}N\Big((\bar{u}_\sigma(x_2)+v_\sigma(x_2))(u_\rho(x_2)+\bar{v}_\rho(x_2))(\bar{u}_r(x_1)$$

$$+v_r(x_1))(u_s(x_1)+\bar{v}_s(x_1))\Big)\cdot a^*_{g_2}a^*_{g_1}|\rangle_0$$

其中 ，取消灭反粒子

$$= \langle_0 | a_{\rho_4} a_{\rho_3} N(\bar{u}_\alpha(x_2) u_\rho(x_2) \bar{u}_\gamma(x_1) u_s(x_1)) a^*_{\rho_2} a^*_{\rho_1} | \rangle_0$$

$$= -\langle_0 | a_{\rho_4} a_{\rho_3} \bar{u}_\alpha(x_2) \bar{u}_1(x_1) u_\rho(x_2) u_s(x_1) a^*_{\rho_2} a^*_{\rho_1} | \rangle_0$$

$$= -\langle_0 | a_{\rho_4} \left[\{a_{\rho_3}, \bar{u}_\alpha(x_2)\} - \bar{u}_\alpha(x_2) a_{\rho_3} \right] \bar{u}_\gamma(x_1) u_\rho(x_2) u_s(x_1) a^*_{\rho_2} a^*_{\rho_1} | \rangle_0$$

$$= -\bar{\varphi}_{\rho_3\alpha}(x_2) \langle_0 | \{a_{\rho_4} \bar{u}_\gamma(x_1)\} u_\rho(x_2) u_s(x_1) a^*_{\rho_2} a^*_{\rho_1} | \rangle_0$$

$$+ \langle_0 | \{a_{\rho_4} \bar{u}_\alpha(x_2)\} a_{\rho_3} \bar{u}_\gamma(x_1) u_\rho(x_2) u_s(x_1) a^*_{\rho_2} a^*_{\rho_1} | \rangle_0$$

$$= -\bar{\varphi}_{\rho\alpha}(x_2) \bar{\varphi}_{\rho_4\gamma}(x_1) \langle_0 | u_\rho(x_2) \left[\{u_s(x_1) a^*_{\rho_2}\} - a^*_{\rho_2} u_s(x_1) \right] a^*_{\rho_1} | \rangle_0$$

$$+ \bar{\varphi}_{\rho_4\alpha}(x_2) \langle_0 | \{a_{\rho_2} \bar{u}_\gamma(x_1)\} u_\rho(x_2) \left[\{u_s(x_1) a^*_{\rho_2}\} - a^*_{\rho_2} u_2(x_1) \right] a^*_{\rho_1} | \rangle_0$$

$$= -\bar{\varphi}_{\rho_3\alpha}(x_2) \bar{\varphi}_{\rho_4}\gamma(x_1) \varphi_{\rho_2}(x_1) \langle_0 | \{u_\rho(x_2), a^*_{\rho_1}\} | \rangle_0$$

$$+ \bar{\varphi}_{\rho_3\alpha}(x_2) \bar{\varphi}_{\rho_4\gamma}(x_1) \langle_0 | \{u_\rho(x_2) a^*_{\rho_2}\} \{u_s(x_1), a^*_{s_1}\} | \rangle_0$$

$$+ \bar{\varphi}_{\rho_4\alpha}(x_2) \bar{\varphi}_{\rho_3\gamma}(x_1) \varphi_{\rho_2 s}(x_1) \langle_0 | \{u_\rho(x_2), a^*_{\rho_1}\} | \rangle_0$$

$$- \bar{\varphi}_{\rho_4\alpha}(x_2) \bar{\varphi}_{\rho_3\gamma}(x_1) \langle_0 | \{u_\rho(x_2) a^*_{\rho_2}\} \{u_s(x_1) a^*_{\rho_1}\} | \rangle_0$$

$$= -\bar{\varphi}_{\rho_3\alpha}(x_2) \varphi_{\rho_1\rho}(x_2) \bar{\varphi}_{\rho\gamma}(x_1) \varphi_{\rho_2 s}(x_1)$$

$$+ \bar{\varphi}_{\rho_3\alpha}(x_2) \varphi_{\rho_2\rho}(x_2) \bar{\varphi}_{\rho_4\gamma}(x_1) \bar{\varphi}_{\rho_1 s}(x_1)$$

$$+ \varphi_{\rho_4\alpha}(x_2) \varphi_{\rho_1\rho}(x_2) \bar{\varphi}_{\rho_3\gamma}(x_1) \varphi_{\rho_2 s}(x_1)$$

乘起来

$$N = -\bar{\varphi}_{\rho_3}(x_2) \gamma_\mu \varphi_{\rho_1}(x_2) \frac{1}{2} D_f(x_2 - x_1) \bar{\varphi}_{\rho_4}(x_1) \gamma_\mu \varphi_{\rho_2}(x_1)$$

$$+ \varphi_{\rho_3}(x_2) \gamma_\mu \varphi_{\rho_2}(x_2) \frac{1}{2} D_f(x_2 - x_1) \bar{\varphi}_{\rho_4}(x_1) \gamma_\mu \varphi_{\rho_1}(x_1)$$

$$+ \varphi_{\rho_4}(x_2) \gamma_\mu \varphi_{\rho_1}(x_2) \frac{1}{2} D_f(x_2 - x_1) \bar{\varphi}_{\rho_3}(x_1) \gamma_\mu \varphi_{\rho_2}(x_1)$$

$$- \bar{\varphi}_{\rho_4}(x_2) \gamma_\mu \varphi_{\rho_2}(x_2) \frac{1}{2} D_f(x_2 - x_1) \bar{\varphi}_{\rho_3}(x_1) \gamma_\mu \varphi_{\rho_1}(x_1)$$

由于

$$\frac{1}{2} D_f(x - y) = \theta(x_0 - y_0) [W_1(x) \bar{W}_1(y)] + \theta(y_0 - x_0) [W_1(y) \bar{W}_1(x)]$$

$$\frac{1}{2} D_f(y - x) = \theta(y_0 - x_0) [W_1(y) \bar{W}_1(x)] + \theta(x_0 - y_0) [W_1(x) \bar{W}_1(y)]$$