Arch W. Naylor
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Linear Operator Theory in Engineering and Science

工程与科学中的线性算子理论

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Linear Operator Theory in Engineering and Science

With 120 Figures

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To
Andrée
and
Geraldine

Preface

The goal of this book is to present the basic facts of functional analysis in a form suitable for engineers, scientists, and applied mathematicians. Although the Definition–Theorem–Proof format of mathematics is used, careful attention is given to motivation of the material covered and many illustrative examples are presented.

The text can be used by students with various levels of preparation. However, the typical student is probably a first-year graduate student in engineering, one of the formal sciences, or mathematics. It is also possible to use this book as a text for a senior-level course. In order to facilitate students with varying backgrounds, a number of appendices covering useful mathematical topics have been included. Moreover, there has also been an attempt to make the pace in the beginning more gradual than that of later chapters.

The first five chapters are concerned with the "geometry" of normed linear spaces. The basic approach is to "disassemble" this geometric structure first, study the pieces, then reassemble and study the whole geometry. The pieces that result from this disassembly are set-theoretic, topological, and algebraic structures. Hence, Chapter 2 covers the appropriate set theory; Chapter 3 treats topological structure, in particular, metric spaces; and Chapter 4 handles algebraic structure, in particular, linear spaces. The reassembly takes place in Chapter 5 where normed linear spaces are studied. The main topic of this chapter is the geometry of Hilbert spaces.

The authors have found that the material covered in these first five chapters can be presented in a one-semester beginning graduate course. Indeed, the authors have done so a number of times in engineering and mathematics departments at a number of universities in the United States, Europe, and South America. Needless to say, the mode of presentation depends upon the audience. For certain audiences, motivation and examples are emphasized while proofs are only highlighted. For others, the converse is the case. An attempt has been made to make the book suitable for both modes of presentation. Moreover, there is material in the large collection of exercises appropriate for each type of audience.

Chapters 6 and 7 take the geometric structure developed in the first five chapters and apply it to the geometric analysis of linear operators. Chapter 6 covers the Spectral Theorem (the eigenvalue-eigenvector representation) for compact operators. Chapter 7 extends this material to certain discontinuous operators, in particular it treats those operators with compact resolvents. These two chapters also contain many illustrative examples.

Many chapters are divided into parts (Part A, Part B, and so forth). Part A contains basic introductory concepts. The subsequent parts of each chapter develop additional concepts and special topics. Thus, if a relatively quick introduction is desired, Part A can be covered first and material from the rest of the chapter can be added as needed.

For the person who is interested in getting to the spectral theory of linear operators as soon as possible it is recommended that he cover Part A of Chapters 3 and 4, Sections 1-8, 12-24 of Chapter 5, and then Chapters 6 and 7.

There is an important problem concerning integration theory. Although integration theory is not needed to understand the basic material covered, there are certain examples that do make reference to the Lebesgue integral and probability spaces. This problem can be handled in at least two ways. First, it can be more or less ignored. That is, the student can be told that there is such a thing as a Lebesgue integral and what its relation to the, presumably familiar, Riemann integral is. Probability spaces can be "glossed" over in the same way. The other way to approach the problem is to use the appendices. Appendix D gives an introduction to Lebesgue integration theory, and Appendix E presents the basic facts about probability spaces.

Each chapter is denoted by a numeral; that is, Chapter 3. The tenth section of the third chapter is denoted Section 3.10. However within Chapter 3, the 3 may be dropped and Section 10 used instead of Section 3.10. Theorem 5.5.4 (or Definition 5.5.4, Lemma 5.5.4, Corollary 5.5.4) refers to the fourth theorem in Section 5 of Chapter 5.

The notation "▮" is used to denote the end of proofs and examples. This allows the proof or examples to be skimmed on first reading.

The authors would like to thank a number of people who have aided in the development of this book. First, there are the students at various universities who have taken courses from one or the other of us based upon manuscript versions. Their suggestions have been invaluable. Next, we would like to thank colleagues who have aided us in various ways: H. Antosiewicz, M. Damborg, K. Irani, G. Kallianpur, W. Kaplan, W. Littman, W. Miller, R. Perret, W. Porter, T. Pitcher, P. Rejto, Y. Sibuya, H. van Nauta Lemke, and H. Weinberger. We especially want to thank F. Beutler for the many suggestions that arose out of his classroom use of the manuscript. Finally, we would like to thank the many secretaries at various universities who have helped in the preparation of the manuscript. In particular, we would like to thank the secretarial staffs of the Department of Electrical Engineering at the University of Michigan and the School of Mathematics at the University of Minnesota.

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Preface to the Second Edition

We are very pleased that the new edition is being published and we are grateful to Springer-Verlag for doing this. The number of inquiries that we received each year made us believe that a new edition would be welcomed. We hope we were right, and we hope that it will be of use to our colleagues and their students.

We further hope, probably unrealistically, that we have corrected all errors of the first edition.

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