

Arch W. Naylor
George R. Sell

Applied Mathematical Sciences 40

Linear Operator Theory in Engineering and Science

工程与科学中的线性算子理论

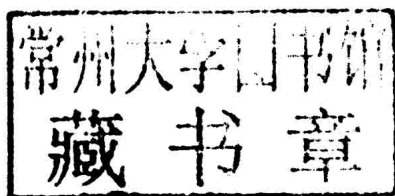
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Arch W. Naylor
George R. Sell

Linear Operator Theory in Engineering and Science

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Arch W. Naylor
University of Michigan
Department of Electrical
and Computer Engineering
Ann Arbor, MI 48104
USA

George R. Sell
University of Minnesota
Institute for Mathematics
and its Applications
514 Vincent Hall
206 Church Street, S.E.
Minneapolis, MN 55455
USA

Editors

J. E. Marsden
Department of
Mathematics
University of California
Berkeley, CA 94720
USA

L. Sirovich
Division of
Applied Mathematics
Brown University
Providence, RI 02912
USA

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Advisors

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**To
Andrée
and
Geraldine**

Preface

The goal of this book is to present the basic facts of functional analysis in a form suitable for engineers, scientists, and applied mathematicians. Although the Definition–Theorem–Proof format of mathematics is used, careful attention is given to motivation of the material covered and many illustrative examples are presented.

The text can be used by students with various levels of preparation. However, the typical student is probably a first-year graduate student in engineering, one of the formal sciences, or mathematics. It is also possible to use this book as a text for a senior-level course. In order to facilitate students with varying backgrounds, a number of appendices covering useful mathematical topics have been included. Moreover, there has also been an attempt to make the pace in the beginning more gradual than that of later chapters.

The first five chapters are concerned with the “geometry” of normed linear spaces. The basic approach is to “disassemble” this geometric structure first, study the pieces, then reassemble and study the whole geometry. The pieces that result from this disassembly are set-theoretic, topological, and algebraic structures. Hence, Chapter 2 covers the appropriate set theory; Chapter 3 treats topological structure, in particular, metric spaces; and Chapter 4 handles algebraic structure, in particular, linear spaces. The reassembly takes place in Chapter 5 where normed linear spaces are studied. The main topic of this chapter is the geometry of Hilbert spaces.

The authors have found that the material covered in these first five chapters can be presented in a one-semester beginning graduate course. Indeed, the authors have done so a number of times in engineering

and mathematics departments at a number of universities in the United States, Europe, and South America. Needless to say, the mode of presentation depends upon the audience. For certain audiences, motivation and examples are emphasized while proofs are only highlighted. For others, the converse is the case. An attempt has been made to make the book suitable for both modes of presentation. Moreover, there is material in the large collection of exercises appropriate for each type of audience.

Chapters 6 and 7 take the geometric structure developed in the first five chapters and apply it to the geometric analysis of linear operators. Chapter 6 covers the Spectral Theorem (the eigenvalue-eigenvector representation) for compact operators. Chapter 7 extends this material to certain discontinuous operators, in particular it treats those operators with compact resolvents. These two chapters also contain many illustrative examples.

Many chapters are divided into parts (Part A, Part B, and so forth). Part A contains basic introductory concepts. The subsequent parts of each chapter develop additional concepts and special topics. Thus, if a relatively quick introduction is desired, Part A can be covered first and material from the rest of the chapter can be added as needed.

For the person who is interested in getting to the spectral theory of linear operators as soon as possible it is recommended that he cover Part A of Chapters 3 and 4, Sections 1-8, 12-24 of Chapter 5, and then Chapters 6 and 7.

There is an important problem concerning integration theory. Although integration theory is not needed to understand the basic material covered, there are certain examples that do make reference to the Lebesgue integral and probability spaces. This problem can be handled in at least two ways. First, it can be more or less ignored. That is, the student can be told that there is such a thing as a Lebesgue integral and what its relation to the, presumably familiar, Riemann integral is. Probability spaces can be "glossed" over in the same way. The other way to approach the problem is to use the appendices. Appendix D gives an introduction to Lebesgue integration theory, and Appendix E presents the basic facts about probability spaces.

Each chapter is denoted by a numeral; that is, Chapter 3. The tenth section of the third chapter is denoted Section 3.10. However within Chapter 3, the 3 may be dropped and Section 10 used instead of Section 3.10. Theorem 5.5.4 (or Definition 5.5.4, Lemma 5.5.4, Corollary 5.5.4) refers to the fourth theorem in Section 5 of Chapter 5.

The notation "■" is used to denote the end of proofs and examples. This allows the proof or examples to be skimmed on first reading.

The authors would like to thank a number of people who have aided in the development of this book. First, there are the students at various universities who have taken courses from one or the other of us based upon manuscript versions. Their suggestions have been invaluable. Next, we would like to thank colleagues who have aided us in various ways: H. Antosiewicz, M. Damborg, K. Irani, G. Kallianpur, W. Kaplan, W. Littman, W. Miller, R. Perret, W. Porter, T. Pitcher, P. Rejto, Y. Sibuya, H. van Nauta Lemke, and H. Weinberger. We

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Ann Arbor
Minneapolis
1971

Arch W. Naylor
George R. Sell

Preface to the Second Edition

We are very pleased that the new edition is being published and we are grateful to Springer-Verlag for doing this. The number of inquiries that we received each year made us believe that a new edition would be welcomed. We hope we were right, and we hope that it will be of use to our colleagues and their students.

We further hope, probably unrealistically, that we have corrected all errors of the first edition.

Ann Arbor
Minneapolis
1982

Arch W. Naylor
George R. Sell

Contents

<i>Preface</i>	vii
Chapter 1 Introduction	1
1. Black Boxes	2
2. Structure of the Plane	4
3. Mathematical Modeling	5
4. The Axiomatic Method. The Process of Abstraction	6
5. Proofs of Theorems	7
Chapter 2 Set-Theoretic Structure	11
1. Introduction	12
2. Basic Set Operations	14
3. Cartesian Products	17
4. Sets of Numbers	18
5. Equivalence Relations and Partitions	19
6. Functions	22
7. Inverses	29
8. Systems Types	38
Chapter 3 Topological Structure	43
1. Introduction	44
Part A Introduction to Metric Spaces	45
2. Metric Spaces: Definition	45
3. Examples of Metric Spaces	47
4. Subspaces and Product Spaces	56
5. Continuous Functions	61
6. Convergent Sequences	69
7. A Connection Between Continuity and Convergence	74
Part B Some Deeper Metric Space Concepts	77
8. Local Neighborhoods	77
9. Open Sets	82
10. More on Open Sets	92
11. Examples of Homeomorphic Metric Spaces	97

12. Closed Sets and the Closure Operation	101
13. Completeness	112
14. Completion of Metric Spaces	120
15. Contraction Mapping	125
16. Total Boundedness and Approximations	134
17. Compactness	141
 Chapter 4 Algebraic Structure	 159
1. Introduction	160
Part A Introduction to Linear Spaces	161
2. Linear Spaces and Linear Subspaces	161
3. Linear Transformations	165
4. Inverse Transformations	171
5. Isomorphisms	173
6. Linear Independence and Dependence	176
7. Hamel Bases and Dimension	183
8. The Use of Matrices to Represent Linear Transformations	188
9. Equivalent Linear Transformations	192
Part B Further Topics	196
10. Direct Sums and Sums	196
11. Projections	201
12. Linear Functionals and the Algebraic Conjugate of a Linear Space	204
13. Transpose of a Linear Transformation	208
 Chapter 5 Combined Topological and Algebraic Structure	 213
1. Introduction	214
Part A Banach Spaces	215
2. Definitions	215
3. Examples of Normal Linear Spaces	218
4. Sequences and Series	224
5. Linear Subspaces	229

6. Continuous Linear Transformations	234
7. Inverses and Continuous Inverses	243
8. Operator Topologies	247
9. Equivalence of Normed Linear Spaces	257
10. Finite-Dimensional Spaces	264
11. Normed Conjugate Space and Conjugate Operator	270
<i>Part B</i> Hilbert Spaces	272
12. Inner Product and Hilbert Spaces	272
13. Examples	278
14. Orthogonality	282
15. Orthogonal Complements and the Projection Theorem	292
16. Orthogonal Projections	300
17. Orthogonal Sets and Bases: Generalized Fourier Series	305
18. Examples of Orthonormal Bases	322
19. Unitary Operators and Equivalent Inner Product Spaces	331
20. Sums and Direct Sums of Hilbert Spaces	340
21. Continuous Linear Functionals	344
<i>Part C</i> Special Operators	352
22. The Adjoint Operator	352
23. Normal and Self-Adjoint Operators	367
24. Compact Operators	379
25. Foundations of Quantum Mechanics	388
<i>Chapter 6</i> Analysis of Linear Operators (Compact Case)	395
1. Introduction	396
<i>Part A</i> An Illustrative Example	397
2. Geometric Analysis of Operators	397
3. Geometric Analysis. The Eigenvalue-Eigenvector Problem	399
4. A Finite-Dimensional Problem	401

<i>Part B</i>	The Spectrum	411
5.	The Spectrum of Linear Transformations	411
6.	Examples of Spectra	414
7.	Properties of the Spectrum	431
<i>Part C</i>	Spectral Analysis	439
8.	Resolutions of the Identity	439
9.	Weighted Sums of Projections	449
10.	Spectral Properties of Compact, Normal, and Self-Adjoint Operators	449
11.	The Spectral Theorem	459
12.	Functions of Operators (Operational Calculus)	468
13.	Applications of the Spectral Theorem	470
14.	Nonnormal Operators	476
Chapter 7	Analysis of Unbounded Operators	485
1.	Introduction	486
2.	Green's Functions	488
3.	Symmetric Operators	493
4.	Examples of Symmetric Operators	495
5.	Sturm-Liouville Operators	498
6.	Gårding's Inequality	505
7.	Elliptic Partial Differential Operators	510
8.	The Dirichlet Problem	516
9.	The Heat Equation and Wave Equation	523
10.	Self-Adjoint Operators	527
11.	The Cayley Transform	533
12.	Quantum Mechanics, Revisited	539
13.	Heisenberg Uncertainty Principle	541
14.	The Harmonic Oscillator	543
<i>Appendix A</i>	The Hölder, Schwartz, and Minkowski Inequalities	548
<i>Appendix B</i>	Cardinality	552

<i>Appendix C</i>	Zorn's Lemma	556
<i>Appendix D</i>	Integration and Measure Theory	558
1.	Introduction	558
2.	The Riemann Integral	559
3.	A Problem with the Riemann Integral	564
4.	The Space C_0	564
5.	Null Sets	566
6.	Convergence Almost Everywhere	569
7.	The Lebesgue Integral	572
8.	Limit Theorems	576
9.	Miscellany	581
10.	Other Definitions of the Integral	586
11.	The Lebesgue Spaces, L_p	589
12.	Dense Subspaces of L_p , $1 \leq p < \infty$	591
13.	Differentiation	593
14.	The Radon-Nikodym Theorem	596
15.	Fubini Theorem	598
<i>Appendix E</i>	Probability Spaces and Stochastic Processes	599
1.	Probability Spaces	599
2.	Random Variables and Probability Distributions	600
3.	Expectation	602
4.	Stochastic Independence	603
5.	Conditional Expectation Operator	604
6.	Stochastic Processes	607
	<i>Index of Symbols</i>	615
	<i>Index</i>	617

