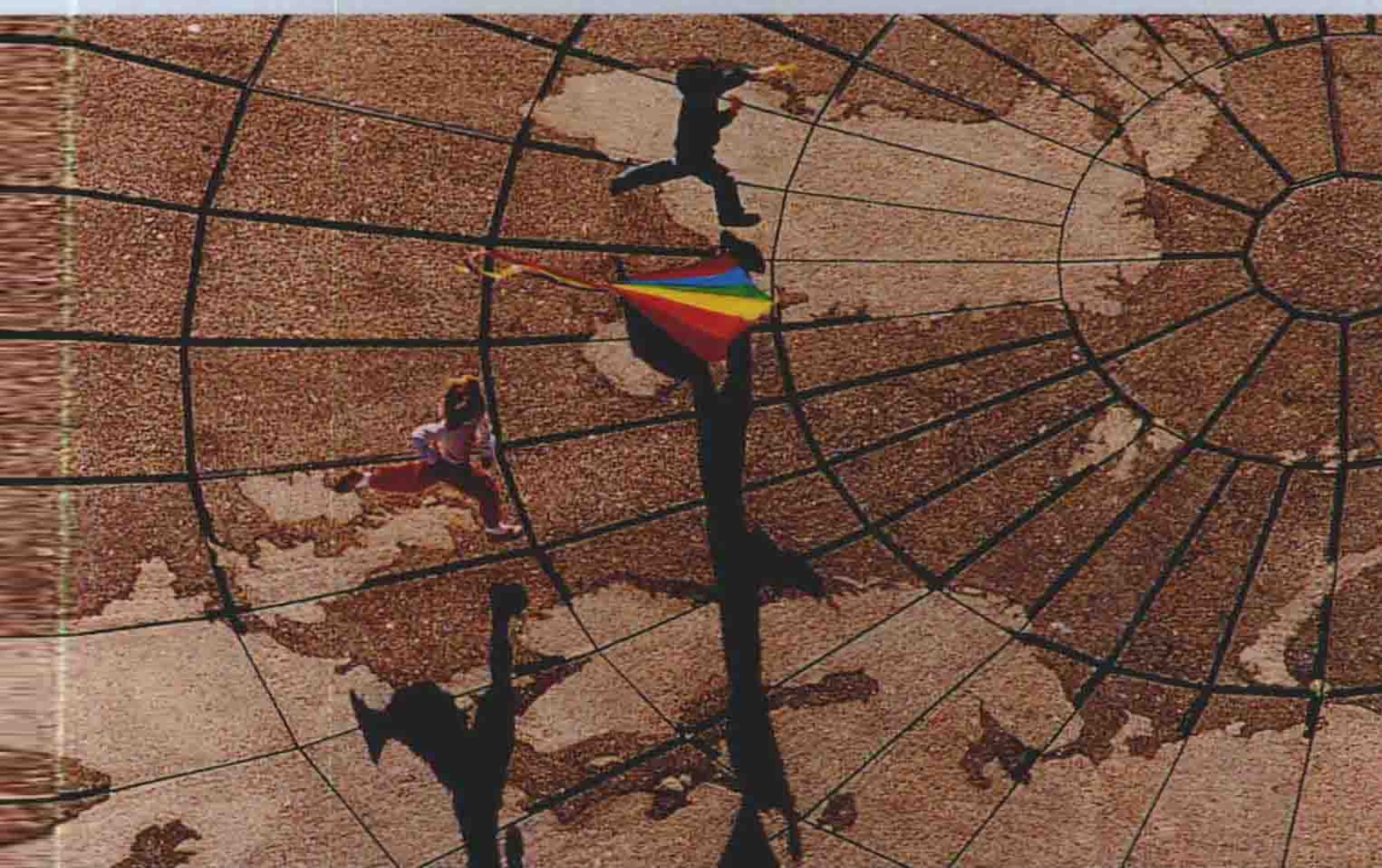
BRIEF CALCULUS

PRELIMINARY EDITION



FOR BUSINESS, SOCIAL SCIENCES, AND LIFE SCIENCES

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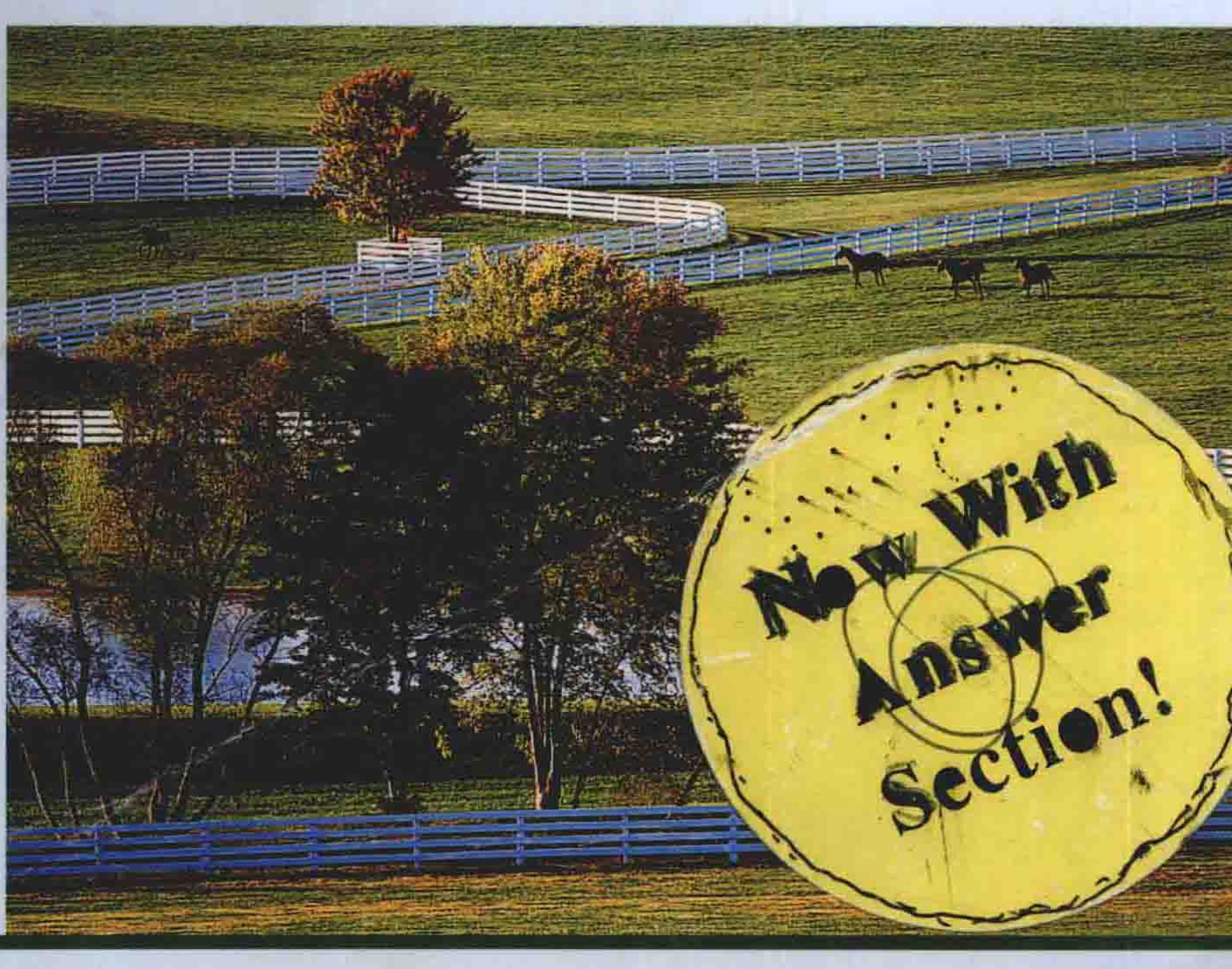

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PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures – thereby losing sight of both the mathematics and of its practical value. With the generous support of the National Science Foundation, our consortium set out to create a new calculus curriculum that would restore that insight. This book brings this new calculus curriculum to the applied calculus course.

Basic Principles

Two principles guided our efforts. The first is our prescription for restoring the mathematical content to calculus:

The Rule of Four: *Every topic should be presented geometrically, numerically, algebraically, and verbally.*

We continually encourage students to think about the geometrical and numerical meaning of what they are doing. It is not our intention to undermine the purely algebraic aspect of calculus, but rather to reinforce it by giving meaning to the symbols. In the homework problems dealing with applications, we continually ask students to explain verbally what their answers mean in practical terms.

The second principle, inspired by Archimedes, is our prescription for restoring practical understanding:

The Way of Archimedes: *Formal definitions and procedures evolve from the investigation of practical problems.*

Archimedes believed that insight into mathematical problems is gained by first considering them from a mechanical or physical point of view.[1] For the same reason, our text is problem driven. Whenever possible, we start with a practical problem and derive the general results from it. By practical problems we usually, but not always, mean real world applications. These two principles have led to a dramatically new curriculum – more so than a cursory glance at the table of contents might indicate.

Technology

We take advantage of computers and graphing calculators to help students learn to think mathematically. For example, using a graphing calculator to zoom in on functions is an excellent way of seeing local linearity. The

[1] ... I thought fit to write out for you and explain in detail ... the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge. From *The Method*, in *The Works of Archimedes* edited and translated by Sir Thomas L. Heath (Dover, NY).

ability to use technology effectively as a tool is important. Students are expected to use their own judgement to determine where technology is a useful tool.

However, the book does not require any specific software or technology. Test sites have used the materials with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice.

What Student Background is Expected?

This book is intended for students in business, the social sciences, and the life sciences. We have found the material to be thought-provoking for well-prepared students while still accessible to students with weak algebra backgrounds. Providing numerical and graphical approaches as well as the algebraic gives students several ways of mastering the material. This approach encourages students to persist, thereby lowering failure rates.

Content

We began work on this book by talking to faculty in business, economics, biology, and a wide range of other fields, as well as to many mathematicians who teach applied calculus. As a result of these discussions we included some new topics, and omitted some traditional topics whose inclusion we could not justify. In the process, we also changed the focus of certain topics. In order to meet individual needs or course requirements, topics can easily be added or deleted, or the order changed.

Chapter 1: Measuring Change

Chapter 1 introduces the concept of a function and the idea of change, including the distinction between total change and rate of change. Linear functions, exponential functions, and power functions are discussed. Although the functions are probably familiar, the graphical, numerical, and modeling approach to them is fresh. Our purpose is to acquaint the student with each function's individuality: the shape of its graph, characteristic properties, comparative growth rates, and general uses. We expect to give the student the skill to read graphs and think graphically, to read tables and think numerically, and to apply these skills, along with their algebraic skills, to modeling the real world. We introduce exponential functions at the earliest possible stage, since they are fundamental to the understanding of real-world processes. Further attention is given to using these functions to model real data through an understanding of regression analysis.

We encourage you to cover this chapter thoroughly, as the time spent on it will pay off when you get to the calculus.

Chapter 2: Rate of Change: The Derivative

Chapter 2 presents the key concept of the derivative according to the Rule of Four. The purpose of this chapter is to give the student a practical understanding of the meaning of the derivative and its interpretation as an instantaneous rate of change without complicating the discussion with differentiation rules. After finishing this chapter, a student will be able to find derivatives numerically (by taking arbitrarily fine difference quotients), visualize derivatives graphically as the slope of the graph, and interpret the meaning of first and second derivatives in various applications. The student will also understand the concept of marginality and recognize the derivative as a function in its own right.

Chapter 3: Accumulated Change: The Definite Integral

Chapter 3 presents the key concept of the definite integral, along the same lines as Chapter 2. Chapter 3 (and Section 5.7 on antiderivatives) can be delayed until after Chapter 5 without difficulty.

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums, and to bring out the connection between the derivative and the definite integral in the Fundamental

Theorem of Calculus. We use the same method as in Chapter 2, introducing the fundamental concept in depth without going into technique. The motivating problem is computing the total distance traveled from the velocity function. The student will finish the chapter with a good grasp of the definite integral as a limit of Riemann sums, with the ability to compute it numerically, and with an understanding of how to interpret the definite integral in various contexts.

Chapter 4: A Library of Functions

Chapter 4 extends the library of functions begun in Chapter 1. Exponential functions with base e, logarithmic functions, polynomials, periodic functions, logistic functions, and surge functions are all introduced as families of functions used to model real world phenomena. The emphasis is on understanding the behavior of the functions and the effect of parameters on this behavior. Further attention is given to constructing new functions from old – how to shift, flip and stretch the graph of any basic function to give the graph of a new related function.

Chapter 5: Short-Cuts to Differentiation

Chapter 5 presents the symbolic approach to differentiation. The title is intended to remind the student that the basic methods of differentiation are not to be regarded as the definition of the derivative. The derivatives of all the basic functions are introduced as well as the rules for differentiating combinations of functions. Antiderivatives are introduced in Section 5.7 The student will finish this chapter with basic proficiency in differentiation and an understanding of why the various rules are true.

Chapter 6: Applications

Chapter 6 presents applications of the derivative and the definite integral. Our aim in this chapter is to enable the student to use calculus in solving problems, rather than to learn a catalogue of application templates. It is not meant to be comprehensive, and you do not need to cover all the sections. The student will finish this chapter with the experience of having successfully tackled a few problems that required sustained thought over more than one session.

Chapter 7: Functions of Many Variables

Chapter 7 introduces functions of two variables from several points of view, using contour diagrams, formulas, and tables. It gives students the skills to read contour diagrams and think graphically, to read tables and think numerically, and to apply these skills, along with their algebraic skills, to modeling the real world. The idea of the partial derivative is introduced from graphical, numerical, and analytical viewpoints. Partial derivatives are then applied to optimization problems, ending with a discussion of Lagrange multipliers. Students will finish this chapter with a solid understanding of functions of two variables.

Appendices

There are two appendices: one on roots and accuracy and one on compound interest.

What is the Relationship Between This Book and the Calculus Books by the Same Consortium?

Much of this book is based on Chapters 1–8 of the text *Calculus* (first edition, 1994) and Chapters 11, 13, and 14 of *Multivariable Calculus* (first edition, 1997), both by the same consortium. However, the content of this book was thought out from scratch with substantial input from faculty in business, social, and life sciences. For example, symbolic antidifferentiation plays a much smaller role in this text; the emphasis is instead on when and how to use a definite integral. Since a firm understanding of graphs and tabular data is especially important for this audience, this book emphasizes the graphical and numerical aspects to an even greater extent than in the original texts.

What is the Relationship Between This Book and the Two-Semester Applied Calculus Book by the Same Consortium?

This book is designed for a one-semester applied calculus course. The book contains material from the text *Applied Calculus* (Preliminary Edition, 1996). We have limited the amount of material included in this text to ensure that students have time to develop a solid understanding of the key ideas. We have increased the emphasis on the concept of change and the distinction between rate of change and accumulated change. In addition, we have divided the first chapter of the *Applied Calculus* book into two chapters in this book (Chapters 1 and 4). This book starts by emphasizing conceptual ideas in the first three chapters: functions and change, the rate of change, and accumulated change.

Supplementary Materials

- **Instructor's Manual containing** containing teaching tips, calculator programs, and some overhead transparency masters.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Answer Manual** with brief answers to all odd-numbered problems.
- **Student's Solution Manual** with complete solutions to half the odd-numbered problems.
- **Student Workbook** with study guides and supplementary materials.

Acknowledgements

First and foremost, we want to express our appreciation to the National Science Foundation for their faith in our ability to produce a revitalized calculus curriculum and, in particular, to Louise Raphael, John Kenelly, John Bradley, Bill Haver, and James Lightbourne. We also want to thank the members of our Advisory Board, Benita Albert, Lida Barrett, Bob Davis, Lovenia DeConge-Watson, John Dossey, Ron Douglas, Don Lewis, Seymour Parter, John Prados, and Steve Rodi for their ongoing guidance and advice.

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To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.
- There are few examples in the text that are exactly like the homework problems, so homework problems can't be done by searching for similar–looking "worked out" examples. Success with the homework will come by grappling with the ideas of calculus.
- Many of the problems in the book are open-ended. This means that there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common sense ideas that are not stated in the problem explicitly but which you know from everyday life.
- This book assumes that you have access to a calculator or computer that can graph functions, find (approximate) roots of equations, and compute integrals numerically. There are many situations where you may not be able to find an exact solution to a problem, but can use a calculator or computer to get a reasonable approximation. An answer obtained this way is usually just as useful as an exact one. However, the problem does not always state that a calculator is required, so use your own judgement.

  If you mistrust technology, listen to this student, who started out the same way:

  > Using computers is strange, but surprisingly beneficial, and in my opinion is what leads to success in this class. I have difficulty visualizing graphs in my head, and this has always led to my downfall in calculus. With the assistance of the computers, that stress was no longer a factor, and I was able to concentrate on the concepts behind the shapes of the graphs, and since these became gradually more clear, I got increasingly better at picturing what the graphs should look like. It's the old story of not being able to get a job without previous experience, but not being able to get experience without a job. Relying on the computer to help me avoid graphing, I was tricked into focusing on what the graphs meant instead of how to make them look right, and what graphs symbolize is the fundamental basis of this class. By being able to see what I was trying to describe and learn from, I could understand a lot more about the concepts, because I could change the conditions and see the results. For the first time, I was able to see how everything works together . . . .

  That was a student at the University of Arizona who took calculus in Fall 1990, the first time we used some of the material in this text. She was terrified of calculus, got a C on her first test, but finished with an A for the course.
- This book attempts to give equal weight to three methods for describing functions: graphical (a picture), numerical (a table of values) and algebraic (a formula). Sometimes it's easier to translate a problem given in one form into another. For example, you might replace the graph of a parabola with its equation, or plot a table of values to see its behavior. It is important to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the most important accomplishments of the millennium – calculus –

as well as a real sense of the power of mathematics in the age of technology.

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CHAPTER ONE

MEASURING CHANGE

Calculus is the study of change. We investigate the total change in a quantity and the average rate of change of one quantity with respect to another. The idea of changing one quantity to investigate the resulting change in another quantity leads us naturally to the concept of a function. Functions are truly fundamental to mathematics, and the study of calculus begins with the study of functions.

This chapter will lay the foundation for studying calculus by surveying the behavior of linear functions, exponential functions, and power functions. We will explore ways of handling the graphs, tables, and formulas that represent these functions. We will consider the different ways in which these functions change, and we will investigate some applications and how to use these functions in modeling data.

1.1 HOW DO WE MEASURE CHANGE?

Change is all around us. The temperature outside, the population of your town, the price of a stock, the size of a cancerous tumor, or the velocity of a baseball are all examples of quantities that are changing. Calculus is the study of change.

The Height of a Child

Kari was born in May of 1982, and her height (in inches) each year on her birthday is given in Table 1.1.

TABLE 1.1 *The growth of a child*

Age (yrs.)	Birth	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Height (in.)	19	28	33	36	39	42	44	47	49	51	54	56	59	61	63

Kari is growing, and so her height is changing. What is the change in her height during the first four years of her life? We see that

$$\begin{array}{l}\text{Total change in height}\\ \text{between birth and age 4}\end{array} = 39 \text{ inches} - 19 \text{ inches} = 20 \text{ inches.}$$

What is the change in her height between age 4 and age 14?

$$\begin{array}{l}\text{Total change in height}\\ \text{between age 4 and age 14}\end{array} = 63 \text{ inches} - 39 \text{ inches} = 24 \text{ inches.}$$

Total Change and Rate of Change

Was Kari growing faster during the first four years of her life or the following ten years? We saw above that she grew 20 inches during the first 4 year period and she grew 24 inches during the following 10 years. However, the numbers 20 and 24 are not very helpful in answering the question of when she was growing fastest. To answer this question, we need a *rate of change*. While the change in height is measured in inches, the rate of change is measured in inches *per year*.

Since Kari grew 20 inches during her first 4 years, the average rate of change during this period is 20 inches divided by 4 years, or 5 inches per year. We see that

$$\begin{array}{l}\text{Average rate of change}\\ \text{between age 0 and age 4}\end{array} = \frac{\text{change in height}}{\text{change in age}} = \frac{39-19}{4-0} = \frac{20}{4} = 5 \text{ in/yr.}$$

$$\begin{array}{l}\text{Average rate of change}\\ \text{between age 4 and age 14}\end{array} = \frac{\text{change in height}}{\text{change in age}} = \frac{63-39}{14-4} = \frac{24}{10} = 2.4 \text{ in/yr.}$$

Kari grew at an average rate of 5 inches per year between birth and age 4, and at an average rate of 2.4 inches per year between age 4 and age 14. Kari was growing faster during the first 4 years of her life.

$$\begin{array}{l}\text{The }\textbf{change}\text{ or }\textbf{total change}\\ \text{in a quantity between}\\ \text{time } a \text{ and time } b\end{array} = \begin{array}{l}\text{the value of}\\ \text{the quantity}\\ \text{at time } b\end{array} - \begin{array}{l}\text{the value of}\\ \text{the quantity}\\ \text{at time } a\end{array}$$

$$\begin{array}{l}\text{The }\textbf{average rate of change}\\ \text{of a quantity between}\\ \text{time } a \text{ and time } b\end{array} = \frac{\text{the change in quantity}}{\text{the change in time}}$$

Public Debt of the United States

Table 1.2 gives the public debt, D, of the United States for the years 1980 to 1993.[1] The total change in the public debt during this 13-year period was $4351.2 - 907.7 = 3443.5$ billion dollars. If we want to know the rate at which the public debt has been increasing, in billions of dollars per year, we use the average rate of change.

$$\begin{aligned}\text{Average rate of change of the public debt between 1980 and 1993} &= \frac{\text{the change in the public debt}}{\text{the change in time}}\\ &= \frac{4351.2 - 907.7}{1993 - 1980}\\ &= \frac{3443.5 \text{ billion dollars}}{13 \text{ years}}\\ &= 264.88 \text{ billion dollars per year.}\end{aligned}$$

Notice that the units for the average rate of change are units of public debt over units of time, or billions of dollars per year. Between 1980 and 1993, the public debt of the United States increased at an average rate of 264.88 billion dollars per year. (This represents an increase of $725,700,000, over 700 million dollars, every day!)

The Δ Notation

To find the change in the public debt, D, we subtracted one value of D from another. The notation ΔD stands for the change in D and is a difference of two values of D. Likewise, Δt stands for the change in t and is a difference of two values of t.

$\Delta D =$ Change in D. The units of ΔD are the same as the units of D.

$$\text{Average rate of change of } D = \frac{\text{change in } D}{\text{change in } t} = \frac{\Delta D}{\Delta t}.$$ The units of $\frac{\Delta D}{\Delta t}$ are the D units over the t units.

TABLE 1.2 *Public Debt of the United States*

Year	Debt (billions of dollars)	Year	Debt (billions of dollars)
1980	907.7	1987	2350.3
1981	997.9	1988	2602.3
1982	1142.0	1989	2857.4
1983	1377.2	1990	3233.3
1984	1572.3	1991	3665.3
1985	1823.1	1992	4064.6
1986	2125.3	1993	4351.2

[1] *The World Almanac 1995.*

Example 1 Find the average rate of change of the US public debt between 1980 and 1985, and between 1985 and 1993.

Solution Between 1980 and 1985

$$\text{Average rate of change} = \frac{\Delta D}{\Delta t} = \frac{1823.1 - 907.7}{1985 - 1980} = \frac{915.4}{5} = 183.1 \text{ billion dollars per year.}$$

Between 1985 and 1993

$$\text{Average rate of change} = \frac{\Delta D}{\Delta t} = \frac{4351.2 - 1823.1}{1993 - 1985} = \frac{2528.1}{8} = 316.0 \text{ billion dollars per year.}$$

Farmland in the United States

Example 2 Figure 1.1 shows a graph of the number of farms (in millions) in the United States between 1940 and 1993.[2] Estimate the average rate at which the number of farms is changing between 1950 and 1970. Interpret your answer.

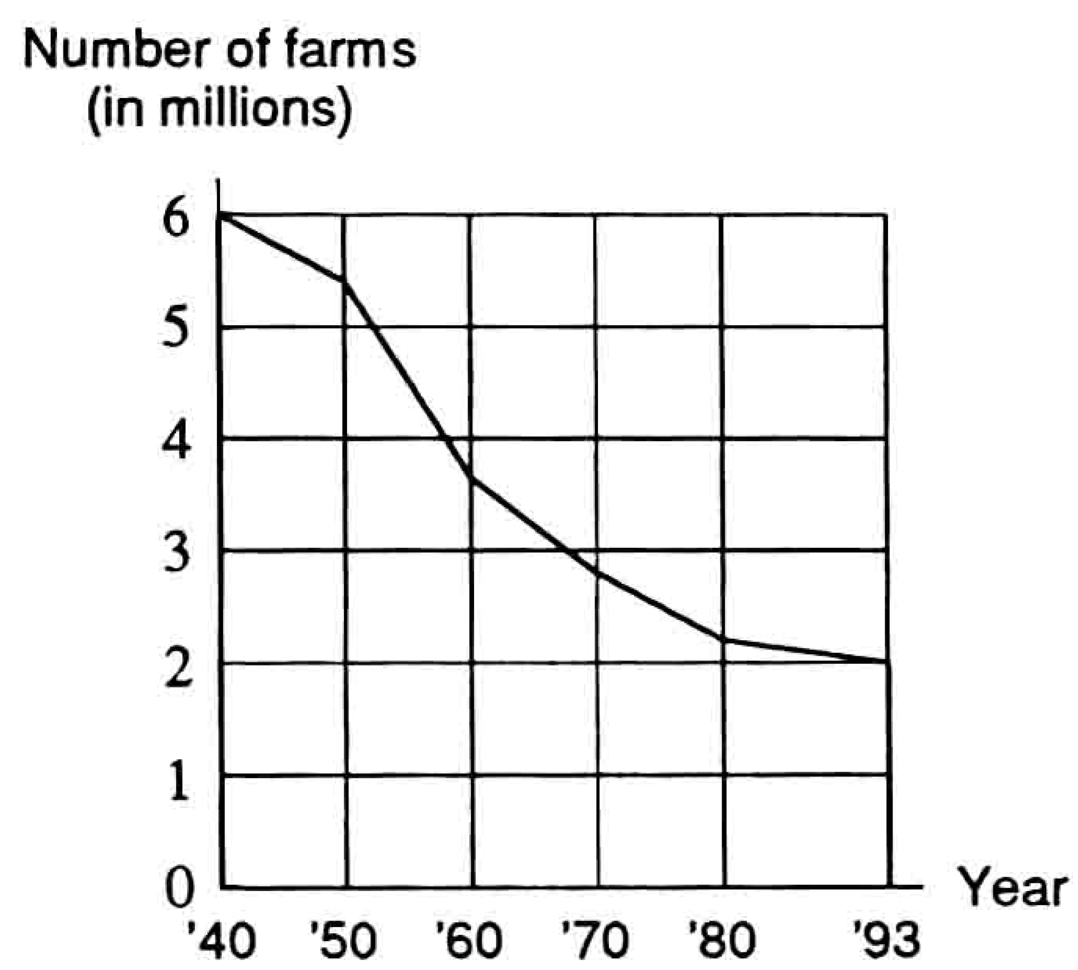

Figure 1.1: Number of farms in the US

Solution We see from Figure 1.1 that the number of farms in the US is approximately 5.4 million in 1950 and approximately 2.8 million in 1970. We have

$$\text{Average rate of change} = \frac{2.8 - 5.4}{1970 - 1950} = -0.13 \text{ million farms per year.}$$

The average rate of change is negative since the number of farms is decreasing. We see that during this 20 year period, the number of farms in the US went down at an average rate of 0.13 million farms per year, or an average decrease of 130,000 farms per year.

PCB's and Pelicans

We have looked at the change in a child's height, the change in the public debt, and the change in the number of farms. All of these quantities are changing over time. In this example, we look at a quantity that is changing for a reason other than the passage of time.

[2]*The World Almanac 1995.*

High levels of PCB (polychlorinated biphenyl, an industrial pollutant) in the environment affect many animal populations. In Table 1.3 we look at the effect of PCB on the thickness of pelican eggs. The concentration of PCB in the eggshell is given in parts per million, and the thickness of the shell is given in millimeters. We see in Table 1.3 that as the concentration of PCB goes up, the thickness of the eggshell goes down, which is bad for pelicans.[3]

TABLE 1.3

Concentration (in ppm)	87	147	204	289	356	452
Thickness (in mm)	0.44	0.39	0.28	0.23	0.22	0.14

Example 3 Find the average rate of change in the thickness of the shell as the PCB concentration changes from 87 ppm to 452 ppm. Give units with your answer. What does the fact that your answer is negative tell you about PCB and pelican eggs?

Solution Since we are looking for the average rate of change of thickness, with respect to change in PCB concentration rather than change in time, we have

$$\begin{aligned}\text{Average rate of change of thickness} &= \frac{\text{change in the thickness}}{\text{change in the PCB level}}\\ &= \frac{0.14-0.44}{452-87}\\ &= \frac{-0.30}{365}\\ &= -0.00082\frac{\text{mm}}{\text{ppm}}\end{aligned}$$

The units are thickness units (mm) over PCB concentration units (ppm), or millimeters over parts per million. The average rate of change is negative because the thickness of the eggshell goes *down* as the PCB concentration goes up. We see that the thickness of pelican eggs goes down by an average of 0.00082 mm for every additional part per million of PCB in the eggshell.

Distance and Velocity

Consider the motion of a grapefruit thrown up in the air. The height of the grapefruit above ground is changing: the grapefruit will go up, turn around, fall down, and then "splat". (See Figure 1.2.) The height of the grapefruit is increasing and then decreasing. Table 1.4 gives the height, y, of the grapefruit above ground t seconds after it is thrown.

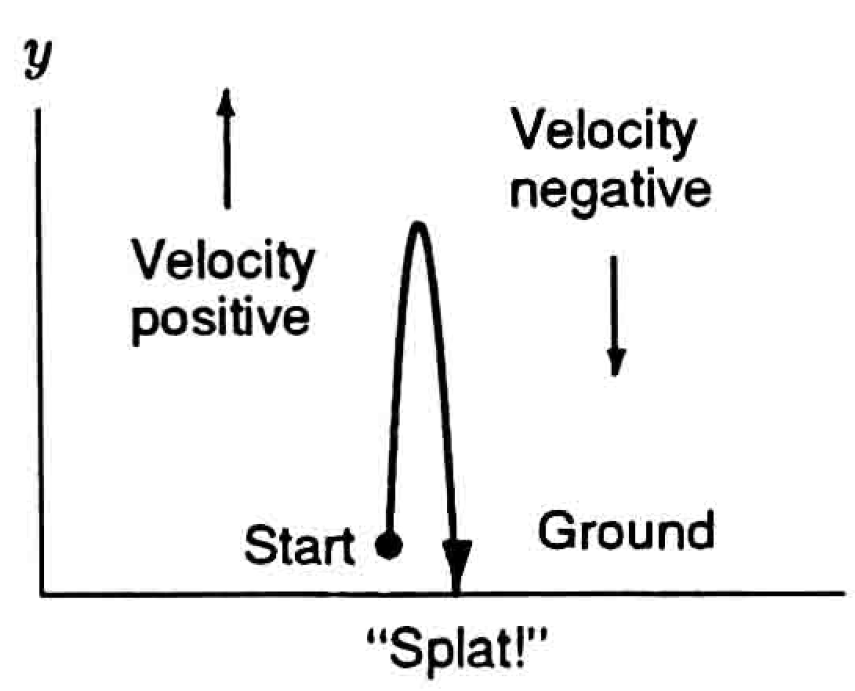

Figure 1.2: The grapefruit's path

TABLE 1.4 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

[3]Risebrough, R.W., "Effects of environmental pollutants upon animals other than man." *Proceedings of the 6th Berkeley Symposium on Mathematics and Statistics, VI*, (Berkeley: University of California Press, 1972) 443-463.