ROUTLEDGE ADVANCES IN RISK MANAGEMENT

Volatility Surface and Term Structure

High-profit options trading strategies

Shifei Zhou, Hao Wang, Kin Keung Lai and Jerome Yen

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Preface

Options trading became popular from 26 June, 1973 when the Chicago Board Options Exchange (CBOE) standardized and integrated options contract transactions. Options expand the range of investment choices and help investors explore different investment channels. Generally, options can provide investors with good opportunities to obtain a higher income. There are many branches of research on options. The risk-hedge functionality of options is also welcomed by investors. Proper understanding and manipulation of the Greek risk indicators for options can help investors measure and manage risk.

This book proposes different financial models based on options prices to predict the underlying asset price and proposes designs for risk hedging strategies. The authors review the literature and improve traditional volatility models. Theoretical innovation is made for making these models suitable for real markets. Risk management and hedging strategies are designed and introduced based on different criteria. These strategies can provide practical guidance for real options trading based on results from theoretical models.

Half the chapters in this book focus on volatility models and the application of these models to market forecasting. The other half is oriented towards risk management and option trading strategies design.

Chapter 2 describes the use of an implied volatilities term structure-based Heston model to forecast the underlying asset price. The parameters of the Heston model are estimated by the least squares method. The term structure is calculated and applied to the Heston model as the long-run mean level. Finally, we simulate price distribution of the underlying assets on the basis of the Heston and constant elasticity of variance (CEV) models.

Chapter 3, motivated by the disadvantages of the traditional Heston model, proposes an adaptive correlation coefficient scheme to estimate Heston's parameters. To precisely estimate this correlation, the Heston model is trained by the least squares method on historical data every day when the underlying price is simulated based on the implied volatility term structure.

Chapter 4 describes the use of basic theories of option pricing and the method of simulation to clarify the significant role that options can play in risk management. The characteristics and hedging effects of options are analyzed by combining options with the underlying asset. This does not only widen the method of controlling risk but also promotes the development of options.

Chapter 5 describes the proposed way to manage risk in equity investments by applying VaR (Value at Risk) and CVaR (Conditional Value at Risk) as standard criteria. A model based on these criteria is built for the design of optimal strategies. Through empirical tests, we find that a good profit can be made when the prediction of the trend of the underlying asset price is highly efficient and precise. The loss is also effectively controlled even when the prediction is bad and inaccurate.

Chapter 6 studies the traditional local volatility model and proposes a novel one with a mean-reversion process. The more the local volatility departs from its mean level, the higher the rate at which local volatility reverts to the mean. Then, a Bi-cubic B-spline surface-fitting scheme is used to recover the local volatility surface. The Monte Carlo simulation is adopted to estimate the underlying asset price trend. Finally, empirical tests show our mean-reversion local volatility model has good predictive power compared with the traditional local volatility model.

In Chapter 7, we propose a regression-based dynamic correlation between the volatility and the underlying asset price which is estimated by three different regression models: simple, polynomial, and auto-regression. The prediction performance of these models is compared in empirical tests.

Chapter 8 describes options risk management by buying or selling the underlying assets to hedge potential risk, to a certain degree. We propose a self-risk management method to control risk. The combination of different types of options is designated self-risk management. The underlying asset price is predicted and used to back-test our self-risk management method.

In Chapter 9, we propose a novel call–put spread-based model for forecasting the price of the underlying asset. Curves of implied volatility of call and put options are calculated separately. The distance between the call and put implied volatility curves contains important market information. We use this distance to predict the underlying asset level and obtain a good result.

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1 Introduction

Stock options trading is a new way of stock trading developed in the 1970s, commonly used in the United States since the early 1990s. A stock option is a right to buy or sell a certain stock at a fixed price within a certain period of time. The buyer is not given the stock itself but the right to buy it at an agreed price.

Execution of an option in options trading is a process where the buyer decides whether to buy or sell the underlying asset at the price the seller and the buyer have agreed. The seller can only passively accept the compliance obligations. Once the buyer asks to execute the option contract, the seller must fulfill their obligations and settle the position specified in the option contract. Hence, the rights and obligations between the seller and the buyer are not equal. The underlying assets of options include commodities, stocks, stock index futures contracts, bonds, and foreign exchange.

The underlying asset of stock index options is the spot index. For example, in the case of the currently popular European option, buyers and sellers directly settle the option contract by cash for stock index options when these options expire. For real option trading, the buyer of a stock index option can only execute the buy right when the option generates the floating profit (earnings greater than the transaction fees) and foregoes the right when there is a loss (including the case where earnings are less than the transaction fee). Hence, there is less risk for the option buyer.

Stock index options sellers face the risk of loss only when they have to passively execute an option contract. However, as long as the premium that sellers collect from buyers can cover the losses, they can hedge the risk of losses. Based on volatility calculated from the stock index and stock index futures contracts, options sellers are able to control the risk of losses to a ninety-five percent confidence level by collecting a conservative premium rate of ten to fifteen percent.

Compared with commodity options, the execution of stock index options and stock index futures are two independent delivery processes. The underlying asset of stock index futures contracts is the spot price index. The stock index is usually composed of a basket of stocks. Commonly, the cash mode is used for settlement of stock index options. That is, the gains and losses of stock index investors are settled by cash at the maturity date of futures contracts.

The execution of stock index options is similar to stock index futures, which can be divided into two modes, US and European. Since the underlying asset of stock index options is also the spot stock index, stock index options are converted into stock index futures on the execution date (US mode) and are exercised with futures, such as the small Chicago Mercantile Exchange (CME) Standard & Poor's (S&P) 500 index futures and options. The European option is directly executed with cash at maturity, such as the CBOE S&P 500 Index options. However, for European options, the market price usually contains intrinsic value and time value. The intrinsic value is the expected value between the underlying asset price and the strike price at maturity. Hence, in this book, we make use of the market option price and do not discuss the intrinsic and time value of option prices in detail. The option price is also referred to as the market option price by default.

Options trading based on futures can provide a hedging function for the futures trader. To format a multi-level, especially low-risk high-yield portfolio in the futures market, you need to take advantage of options. To reduce the trading risk of futures investors, to expand the scope of market participants, and to improve market stability and liquidity also requires options. In addition, options can provide hedging tools for addressing risk in contractual agriculture and to protect farmers' revenue. The US government encouraged farmers to successfully combine their government subsidy with the options market so as to transmit the huge risks of the agricultural market to the futures market. This policy not only reduces the government's fiscal expenditure but also stabilizes agricultural production and effectively protects farmers' benefits.

In addition to hedging, stock index options also contain certain market information from participants. During the options trading process, buyers and sellers provide bid and ask prices for options. These prices contain views of buyers and sellers about what the underlying asset price is expected to be at the maturity date. Black and Scholes (1973) proposed that implied volatility, calculated from options prices, reflects market information. If the implied volatility of a put option is large, it means market participants are panicky about the future market trend. Conversely, if the implied volatility of a call option is large, this indicates that the market will go up in the near future. Therefore, the study of option implied volatility is significant and beneficial.

1.1 Implied volatility

The implied volatility of an option contract is volatility of price of the underlying asset which is implied in the price of the option. Implied volatility varies with different strikes and time to maturities. For a given time to maturity, the implied volatility varies with strikes; for a given strike, the implied volatility varies with different maturities. To price European options, Black and Scholes (1973) proposed that the price of underlying assets satisfies the following:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where μ is the mean value of historical price, σ is the constant variance of the underlying asset's price, and W represents a standard Wiener process. From Ito's

Lemma, the logarithmic of the underlying price should follow the formula:

$$d \ln S = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW$$

Therefore, the logarithmic of the underlying price follows a normal distribution. However, when applying this theory into practical market options, the normal distribution expresses the phenomenon of a fat tail at both sides of the distribution. Black and Scholes proposed this model based on the following assumptions: (1) no dividend before the option maturity; (2) no arbitrage; (3) a constant risk-free interest rate; (4) no transaction cost or taxes; (5) divisible securities; (6) continuous trading; and (7) constant volatility. For a given time during the trading day, if the market releases the option price, the option volatility can be calculated by inverting the Black–Scholes formula.

On the one hand, if the time to maturity is fixed, volatility smile is defined as a curve that the implied volatility changes with different strikes. In a long-observed pattern, the volatility smile looks like a smile. The implied volatility of an at-the-money option is smaller than that of in- or out-of-the-money options. When the implied volatility of an out-of-the-money put option is larger than that of an out-of-the-money call option, this curve is called implied volatility skew. Zhang and Xiang (2008) defined the concept of "moneyness" as the logarithm of the strike price over the forward price, normalized by standard deviation of expected asset return as follows:

$$\xi \equiv \frac{\ln(K/F_0)}{\overline{\sigma}\sqrt{\tau}}$$

where $\overline{\sigma}$ is the historical volatility of the underlying asset price; τ is the time to maturity; K is strike price; F is the forward index level. Then, implied volatility smirk is defined as employing the moneyness as an independent variable. Implied volatility changes according to moneyness.

On the other hand, if implied volatilities with different strikes and a given maturity are combined together under a certain weighted scheme, then the implied volatility term structure can be defined as a curve that implied volatilities change with different maturities. When the implied volatilities of call options and put options are calculated, we obtain a call implied volatility curve and a put implied volatility curve. The spread between call and put curve is called call–put term structure spread. This spread contains certain market information.

When implied volatility of put term structure is larger than that of call term structure, market participants worry about the market on the maturity date. If these two curves cross, there are two conditions. First, when implied volatility of the put curve is larger than call curve before the cross point and smaller after the cross point, it means the market trend may reverse in a short term and go up. Second, when implied volatility of call curve is larger than the put curve before the cross point and smaller after the cross point, it means the market trend may reverse in a short term and go down. Finally, when the implied volatility of call term structure is larger than that of put term structure, the view from investors is that the market still goes up. By using this functionality of term structure, we can employ the term structure to predict the underlying asset price efficiently, which is discussed in greater detail in Chapters 2 and 9.

1.2 Local volatility model

Local volatility also performs well in forecasting the underlying asset price. Local volatility is an instantaneous volatility that is a function of time t and underlying asset price S_t. Typically, Dupire (1994) presented a deterministic equation to calculate the local volatility from option price based on the assumption that all call options with different strikes and maturities should be priced in a consistent manner.

$$\sigma(K,T)=\sqrt{2\left(\frac{\frac{\partial C}{\partial T}+rK\frac{\partial C}{\partial K}}{K^2\frac{\partial^2 C}{\partial K^2}}\right)}$$

Nevertheless, this deterministic function suffers two weaknesses. First, because local volatility is a function of both strike and time to maturity and it is possible that not all strikes are available at each time to maturity, the number of local volatilities is finite and is usually not enough for further calculation and applications. As a result, researchers are inclined to use interpolation to obtain a series data of local volatility for further calculations. In this way, the algorithm of interpolation becomes very important because a weak algorithm results in problem of inadequate precision. Second, there is the intrinsic problem of the Dupire's equation. The indeterminancy of the equation may cause local volatility to be extremely large or very small. In Chapter 6, we propose a mean-reversion process to overcome these faults and improve the model. As local volatility is a function of time and underlying asset price, it has the ability of predicting the underlying asset price if we can construct the local volatility surface.

1.3 Stochastic volatility model

In the stochastic volatility model, volatility is considered as a stochastic process. The stochastic volatility model assumes that the underlying asset price follows a geometric Brownian process. In the Black–Scholes model, volatility is assumed to be constant over the time to maturity. However, this can explain the phenomenon that volatility smile and skew vary with different strikes. The stochastic volatility model can solve this problem. Typically, Heston (1993) proposed a model which considers the underlying asset price process and the volatility process as random processes. Moreover, these two processes have a constant correlation.

The underlying level process of Heston model is composed of two terms, the price drift term and the volatility with random motion term. The volatility process is also composed of two terms, a volatility drift with mean reversion functionality and a volatility of volatility with random motion. The model is formulated as follows:

$$dS_t = \mu(t)S_t dt + \sqrt{V_t}S_t dW_1$$
$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_2$$

where θ is the long-run mean level of volatility, κ is the speed of instant volatility returning to long-run mean level, and σ is volatility of volatility. These three parameters satisfy the condition of $2\kappa_t\theta > \sigma^2$ and ensure the process of V_t is strictly positive. Furthermore, W_1 and W_2 are two standard Wiener processes and have a correlation of ρ.

The Heston model has been widely applied in equities, gold, and foreign exchange markets. Furthermore, many extended models based on the Heston model have been proposed. Christoffersen *et al.* (2006) proposed a two-factor stochastic volatility model based on the Heston model to control the level and slope of the volatility smirk. Andersen *et al.* (2002) and Chernov and Ghysels (2000) employed an Efficient Method of Moments approach to estimate structural parameters of the Heston model. Bates (2000) used an iterative two iterations procedure to measure the structural parameters and spot volatilities.

As the Heston model simulates the relationship between implied volatility and the underlying asset price, this model also has the ability of forecasting the underlying asset price. In Chapter 3, we use the Heston model to predict the Hang Seng Index (HSI) by considering the correlation of the underlying asset level and volatility process as dynamic variables. The underlying asset price is simulated by the analytic solution of geometric Brownian motion.

2 A novel model-free term structure for stock prediction

Implied volatility term structure contains market views. This chapter first calculates model-free implied volatility term structure and then applies it as the long-run mean level of Heston model. Since the Heston model assumes both underlying asset price process and volatility process as stochastic processes, the geometric Brownian motion is used to forecast the underlying asset price.

2.1 Introduction

2.1.1 Background

Volatility term structure represents that the implied volatility varies with different times to maturities. While analyzing implied volatility term structure, the key point is to figure out how the implied volatility is calculated from options market data. There are mainly two types of implied volatility valuation methods: the model-based implied volatility and the model-free implied volatility.

The most widely used model-based implied volatility valuation method is the Black–Scholes (BS) model. Researchers reverse the BS model and obtain a deterministic volatility function. However, this method suffers a number of constraints. Heston (1993) proposed a stochastic volatility model which assumes volatility is a stochastic process. Secondly, in the case of model-free volatility, Carr and Madan (1998), Demeterfi *et al.* (1999), Britten-Jones and Neuberger (2000) and Carr and Wu (2009) have presented a volatility expectation based on variance swap contracts. Britten-Jones and Neuberger (2000) further proposed an integrated volatility defined as the integral of call option price and put option price on all strikes at a given expiry date.

2.1.2 Motivation

Extant literature has showed that the BS model has a few shortcomings. First, for a given maturity, options with different strikes have different implied volatilities. This is the reason why the BS model can not explain the volatility smile curve. Second, it assumes implied volatility at a given maturity to be constant, which is unsuitable for predicting the underlying asset price trend. Hence, in this chapter, we prefer the model-free integrated implied volatility model but use a discrete form of the integral and consider the process risk-neutral.

Furthermore, the Heston model considers the process of change of the underlying asset's price as Geometrical Brownian movement with volatility as a stochastic process. We apply the Heston model, as well as the constant elasticity variance (CEV) model, to establish a deterministic relationship between the underlying price and volatility.

2.2 Volatility model

The Heston model is more complicated than the CEV model. It takes the mean-reversion term into consideration and the assumption is that the two processes of the underlying price and volatility are stochastic processes with a constant correlation to each other. The original Heston model is defined as follows:

$$\begin{aligned} dS_t &= \mu(t)S_t dt + \sqrt{V_t} S_t dW_1 \\ dV_t &= \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_2 \\ \rho &= \langle W_1, W_2 \rangle \end{aligned} \tag{2.1}$$

where θ is the long-run mean level of volatility, κ is the speed of instant volatility returning to long-run mean level, and σ is volatility of volatility. These three parameters satisfy the condition of $2\kappa_t\theta > \sigma^2$ and ensure the process of V_t to be strictly positive. Furthermore, W_1 and W_2 are two standard Wiener processes having correlation ρ.

2.3 Model-free term structure

2.3.1 Model-free implied volatility

Taylor *et al.* (2010) defined a model-free implied volatility which is an expected implied volatility obtained by integrating different option strikes by option prices. They compared this model-free implied volatility with at-the-money implied volatility and realized volatility in respect of volatility information content. They found that the model-free implied volatility outperforms the other volatility model and is more informative. Similarly, Carr and Wu (2009) defined a risk-neutral integrated volatility, marked as $E_t^Q[IV_{t,T}]$, over a given period $[t, T]$ through options with different strike prices.

$$E_t^Q[IV_{t,T}] = \frac{2}{T-t}\int_0^{\infty} \frac{Q_t(K,T)}{P_t(T)K^2} dK \tag{2.2}$$

where $P_t(T)$ is the value of a bond at time t, $Q_t(K, T)$ is the call option price at time t with strike price K, and maturity T when strike price K is greater than the underlying asset price or the put option price at time t with strike price K and maturity T when strike price K is less than the underlying asset price.

However, this is a continuum formula for model-free implied volatility. As we focus on the Hong Kong options market, we use a discrete version of Equation (2.2), as follows:

$$\sigma_t^2 = \frac{2}{T-t}\sum_{i}^{N}\frac{\Delta K_i}{K_i^2}e^{r(T-t)}Q_t(K,T) - \frac{1}{T-t}\left[\frac{F}{K_0}-1\right]^2 \tag{2.3}$$

where N is the number of options for a given maturity T at time t, F is the forward price. According to the Hang Seng Index (HSI), forward price is defined as the forward index level of the HSI, calculated as follows:

$$F = K + e^{r(T-t)} \times (C_K - P_K) \tag{2.4}$$

where K is the strike price of out-of-the-money call or put options. C_K is the out-of-the-money call option price. P_K is the out-of-the-money put option price.

Since the model-free implied volatility is calculated on a given maturity with different strikes, we can draw an implied volatility term structure with different times to maturities, though for a given expiry, implied volatility is constant through the option lifetime. This is not really true in real markets. Therefore, we propose a cubic function to fit the implied volatility term structure. The spirit of the cubic function is similar to the least squares method.

We define the cubic function $\phi(x)$ as follows; the left hand side of the function represents the fitted implied volatility:

$$\phi(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = \sum_{i=0}^{m} a_i x^i \tag{2.5}$$

where $m = 3$ is the largest power of the variable, x represents time to maturity (measured in years). a denotes the corresponding parameters. Our aim is to find an optimal parameter vector a such that the difference between fitted implied volatility and real implied volatility is minimal. We define the difference δ_j to be the result of real data minus fitted data.

$$\delta_j = y_j - \phi_m(x_j) \tag{2.6}$$

where $j = 1, \ldots, n$, n means the total number of real implied volatility, y_j is the real implied volatility, and $\phi_m(x_j)$ is the estimated implied volatility at jth data point. It is hard to get an optimal vector only by comparing this difference because the difference may be small when the real data fluctuate dramatically through a constant mean value. Hence, we use a squared error method to compare the difference.

$$F(a_0, a_1, \ldots, a_m) = \sum_{j=1}^{n}\delta_j^2 = \sum_{j=1}^{n}[y_j - \varphi_m(x_j)]^2 \tag{2.7}$$