Game Theory

with Engineering Applications

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Game Theory
with Engineering Applications

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To the loving memory of my parents.

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