

**IEEE 1981**  
**FRONTIERS OF COMPUTERS**  
**IN MEDICINE**

# **IEEE 1981 FRONTIERS OF COMPUTERS IN MEDICINE**

**Robin B. Lake, Editor**

Frontiers of Computers in Medicine

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# Welcome!

These national and international conferences are the highlight of our activities as a technical and professional society of the IEEE. This is our third conference year, and we hope it will contribute to the emergence of engineers in health care delivery. As the organizers and sponsor of this conference, we have made every possible effort to have it serve as a focal point for the latest developments involving technology in health care. Our technical committees, publication editors, and administrative committee members (AdCom) have planned for the past year to bring together the relevant research efforts, the clinical developments, and the industrial applications. Key people have been invited to organize the sessions and workshops. Many invited papers have been included to ensure the proper balance in the sessions. Tutorials and workshops have been included as part of the regular conference program. The full conference papers are published in these Conference Proceedings, and the abstracts of each paper were published in the August issue of our EMBS Transactions.

It is a gratifying moment to see our EMBS Society meet and work together at our own annual conference. May I welcome you to our conference, and I hope that your interaction with the other participants will lead to a more effective communication for our Society.

I would like to express my gratitude to those who have labored to bring this program together; and, in particular, to Bernard Cohen, Ph.D. for being the program chairman for the EMBS conference and Robin Lake, Ph.D. for being the program chairman for the CompMed conference.

Mort Schwartz, Ph.D.  
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## Introduction

COMPMED '81 is a new IEEE conference designed to focus on the interface between computer engineering/technology and biomedical computing/information science. The conference theme is Frontiers of Computers in Medicine. Just as computer technology has pushed forward its frontiers with faster, less expensive technology; higher capacity storage systems; new architectures; and advanced software techniques, so have the needs of biomedical computing challenged these very frontiers with areas of research and application that require ever faster and more complex systems. Yet, even routine applications of simple systems in the medical environment often present formidable problems.

The topics covered in COMPMED '81 represent but a few of biomedical computing's focal areas along its frontiers. As we attract a broader audience to COMPMED, we hope that future years will be better able to expand our scope to include three areas under-represented in this first conference: Computer Architecture; Networks & Distributed Systems; Multi-Microprocessor Systems; and Software Methodology.

That these Proceedings are bound together with the Proceedings of the Third Annual Conference of IEEE/EMBS reflects well the very close collaboration and support between COMPMED and EMBS. The Proceedings represents the efforts of many individuals who have volunteered their time to promote the exchange of professional ideas. The authors, who contributed papers of fine quality on short notice, are the real power behind COMPMED. The Program Committee and the sponsoring societies merely provide a framework for the expression of the authors' efforts.

Special thanks are due to the organizers on the several Committees who provided the extensive support necessary to quickly initiate a new conference. That EMBS has agreed to sponsor COMPMED '82 next year in Philadelphia indicates everyone's efforts are very well appreciated.

Robin B. Lake, Ph.D.  
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## MDX AND RELATED MEDICAL DECISION-MAKING SYSTEMS

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### Introduction

There are two key aspects to the MDX medical diagnosis system methodology: (1) The diagnostic knowledge is decomposed into a collection of hierarchically organized conceptual specialists; the scope of the specialists varies from more general to more particular as the hierarchy is traversed from top to bottom. (2) The problem solving proceeds in a top-down manner; the specialists in the hierarchy establish or reject their relevance to the case at hand, and those who have established their relevance pass the control to their successors, until all relevant specialists have added their explanation of abnormal data which the knowledge in their scope can explain.

The MDX system, which is wholly diagnostic in its knowledge, communicates with two auxiliary systems, PATREC and RADEX. PATREC is a data base assistant in the sense it acquires the patient data, organizes them, and answers the queries of MDX concerning the patient data. In all these activities PATREC uses various types of inferential knowledge embedded in an underlying conceptual model of the domain of medical data. RADEX is a radiology consultant to MDX, and it suggests or confirms diagnostic possibilities by reasoning based on its knowledge of imaging procedures and relevant anatomy. See (Mittal and Chandrasekaran, 1980) and (Chandrasekaran et al, 1980) for further details about these subsystems.

The current domain of MDX is a syndrome called cholestasis, which covers a large number of specific liver diseases. The pilot system was restricted to extra-hepatic diseases only; however, in the last several months, a large number of intrahepatic diseases have been added. The system is continually tested on a number of cases both from journals and hospital records. MDX is written in LISP, and is quite large: it runs in 120K words of DEC-20 memory, in addition to the memory requirements of the LISP system itself.

### Medical Knowledge Organization

For the past two years our research group (consisting of the author, F. Gomez, S. Mittal, and J. W. Smith, Jr.) has been investigating the issues of knowledge organization and representation, as well as the problem solving processes at work during medical diagnosis. In parallel with this investigation, we have also been building and extending a medical diagnosis system called MDX. This system currently handles substantially the entire domain of cholestatic diseases. The theoretical basis of our work is laid out in detail in Gomez and Chandrasekaran. Our work has been motivated by the following ideas:

(1) The central determinant of effective use of knowledge is how it is organized. Issues of representation should come after the organizational structure of knowledge that is needed for problem solving in a domain has been clarified.

(2) In a given domain of expertise, there are different types of problem solving that can go on. For instance, in the medical domain, the problem solving involved in diagnosis is a different type from that involved in reasoning about consequences of administering a therapy or a drug. To elaborate: Diagnosis is a process of classifying a case into a predefined diagnostic category, i.e., associating it with a node, as specifically, as possible, in a diagnostic hierarchy. On the other hand reasoning about a drug is a process of relating state changes at different levels of abstraction.

(3) For each type of problem solving that is identified in a domain of expertise, there exists a different knowledge structure, with the associated problem solving mechanism embedded in it. Thus this structure can be viewed as an active knowledge structure for problem solving of that type. Contrast this with the traditional view in which knowledge has an existence independent of the problem solvers that may use it.

It ought to be emphasized that in this view every piece of knowledge has implicitly associated with it information about how to use it, since the active knowledge structure invokes it at the appropriate context.

(4) In expert problem solving, the knowledge structure that is available for problem solving of a given type can in principle be decoupled from common sense knowledge. The role of common sense knowledge and learning processes is to construct and fill this and other knowledge structures. Thus the knowledge in this expert problem solving structures is in a highly compiled form. However, in a human, even one who is an acknowledged expert, these structures are often incomplete, and thus common sense knowledge and other domain-independent learning and knowledge acquisition processes are often invoked.

(5) The totality of reasoning and problem solving by a clinician is decomposable into a number of problem solving regimes as mentioned above. In the handling of an actual case, a physician is in the diagnostic mode only part of the time. Thus other structures are constantly invoked. There is considerable switching between different knowledge structures and the associated problem solving processes. A satisfactory account of this

overall process can only be given after the underlying conceptual structures and the problem solving regimes in them are identified. Our interest has so far been confined to the diagnostic structure.

(6) We have said that the diagnostic knowledge structure is an active knowledge structure. We can further view the diagnostic structure as a hierarchical organization of specialists. Specialists at the top levels have knowledge corresponding to the more general disease concepts (such as "liver", "heart," etc.), while specialists at lower levels have knowledge about concepts which are refinements of the top level concepts. Thus, the immediate subspecialists of the liver specialist may be "hepatitis", "cholestasis", etc., each of which will have further subspecialists. The tip nodes would correspond to the most specific disease states that are known or useful.

Note, however, that not all the concepts in the hierarchy are not necessarily disease concepts. Some of the nodes may be causes of diseases or any other concepts that are relevant in diagnostic reasoning. The creation of this structure is not automatic, but requires a careful epistemological analysis of the domain from the view point of the problem solving type that is involved.

(7) Much of the knowledge of the specialist can be represented in the form of production rules (first introduced in AI by Newell and Simon). In the case of the diagnostic structure, the production rules typically relate manifestations (symptoms, lab data etc.) to concepts in the structure itself (remember that there are conceptual specialists corresponding to these concepts). Thus we can view the right hand side of the production rules as giving information about which conceptual specialist may be of further relevance to the case at hand, i.e., may be able to provide further detailed analysis. Typically, these concepts will be subconcepts in which the production rule is located. Note that this process is similar to the real life situation of a GP turning over control to say a liver specialist, after first hypothesizing that the case at hand is likely a case falling under the specialist's competence. See Gomez and Chandrasekaran for a detailed account of how the problem solving by the specialists in the hierarchy is coordinated to produce a unified diagnosis.

A brief recapitulation may be in order. There are two key aspects to the methodology: (1) Diagnostic knowledge is decomposed into a collection of specialists, and (2) these specialists perform problem solving in certain specified ways (by appropriate transfer of control to other specialists) to achieve a unified diagnosis.

#### MDX and Its Problem Solving

A prototype diagnosis system called MDX has been built by our group and has been operational for some time. Details of system design and performance are available in Chandrasekaran et al and Mittal et al; here we content ourselves with a brief description of

its operation. As mentioned earlier, the current domain of MDX is cholestasis. The top level specialist in the system is GP (or internist), but all that it can do at this stage in the implementation is either to hypothesize cholestasis and transfer control to it, or to reject the case. The cholestasis hypothesis is generated by a collection of production rules which respond to the relevant lab data and physical signs and symptoms. When cholestasis gets control, its charge is to establish itself, and if successful, then further to refine it to account for all the abnormal manifestations. This establish-refine strategy is fairly general to the system as it currently exists.

Once cholestasis is established, a priority scheme is needed to call its subspecialists for further refinement. (MDX implements an essentially serial problem solving strategy. In Gomez' recent work, a parallel problem solving regime has been developed. See Gomez and Chandrasekaran for details.) This priority is provided by a collection of rules which suggest possible specialists on the basis of the patient data. These specialists are typically called to establish and refine themselves in turn. And when they succeed, they return those abnormal data which they can explain. The specialists which are established and the corresponding data are kept in an ACTIVE list. When the specialists in the top level ACTIVE list together can explain all the abnormalities in a non-overlapping way, the case is solved. (The above is a somewhat oversimplified description, and many subtleties such as multiple diseases etc. can arise. These are discussed in greater detail in Gomez and Chandrasekaran.)

Note that the specialists lower than cholestasis in the hierarchy may also have their own priority rules to select their subspecialists. The tip nodes, when called, match the data within their scope with their own confirmatory and disconfirmatory rules to establish or reject themselves. This information is passed up to the calling specialist. Each specialist thus organizes, by means of production rules, the priority by which it uses its subspecialists to arrive at an explanation of abnormal data within its scope. When the specialists explicitly suggested by the rules fail to explain the case, then an exhaustive interrogation of all subspecialists one level below will be made. Thus, the priority rules do not preclude the correct answer from being obtained eventually, even if they are mistaken in their hypotheses. On the average, however, these rules help to achieve a quick focussing on the most likely possibilities.

#### Associated Systems

The production rules in the specialists of MDX are purely diagnostic, that is they relate manifestations to disease or general diagnostic concepts. During the process of its reasoning, it calls upon other nondiagnostic knowledge structures for reasoning processes which are not directly diagnostic, but support diagnostic reasoning. These knowledge structures are also medical in nature, but are organized for the purposes of

intelligent data retrieval, or auxiliary activities such as radiological consultation. In our system, the retrieval of data from the patient data base is handled by a data base assistant called PATREC. This sort of division of labor has its counterpart in medical practice, where medically trained nurses are often charged with maintaining charts etc., and respond to queries by clinicians during the latter's diagnostic reasoning. The PATREC system is, like MDX, organized as a collection of conceptual specialists, but the problem solving involved is inferring answers to questions asked by MDX from specific data stored in the data base and its own domain knowledge. For example, if the data base has the datum, "The patient had biliary surgery," and MDX, during its problem solving, wishes to know if the patient has had abdominal surgery, the PATREC system will answer in the affirmative. In order to do this, PATREC needs medical knowledge (the anatomical relationship and how surgery data relate to it), but this knowledge is not diagnostic in nature. Human diagnosticians of course are not "pure" diagnosticians in this epistemological sense. When they need such data inferences, they briefly switch to other structures, and after completion of the necessary inference, they revert back to the diagnostic structure. Nevertheless it is interesting that the medical community organizes itself in the general spirit of our discussion.

Similarly, when MDX needs radiological consultation, it turns to an auxiliary specialist called RADEX. RADEX is capable of reasoning from stored low level descriptions of radiological images (currently restricted to liver-related image descriptions) to answer certain kinds of MDX queries, such as "Is there radiological evidence of tumor in the biliary duct?" Further details on RADEX are available in Chandrasekaran and Mittal.

Thus the principle of decomposition into specialists is applied in our system vertically as well as horizontally; vertically, in the sense that RADEX, PATREC and MDX is each decomposed into specialists of the same type --- all the specialists within MDX are diagnostic, within PATREC data retrieval etc.; horizontally in the sense that the overall medical knowledge required for solving the cases has been divided into specialist subsystems, each specializing in a different type of problem solving.

The MDX system (diagnostic as well as auxiliary systems) is being constantly expanded to include more diseases in its scope. The specialist approach permits a modular development. The current version of MDX, which can handle most of cholestasis, has been tested on a number of cases. The references at the end of the paper include a number of papers in which further discussion of the performance of MDX on several cases can be found.

#### Concluding Remarks

The reasoning processes of MDX and the associated systems clearly reflect only a subset of the distinct processes that together make up the problem solving of an expert clinician faced with a real life case. In order to model the totality of the processes,

not only do we need to identify other activities structures and the associated problem solving processes (e.g., therapy selection), but a whole collection of other structures that deal with social and ethical aspects of medical care. We make no pretense for that kind of completeness. In fact, precisely the decomposability into different structures and problem solving types that we have been talking about makes it possible that in the future AI systems can perform the role of consultants, which concentrate on purely medical aspects of a case. This may be supplemented by a physician with other knowledge structures of a nonmedical type.

We have so far not said anything at all about the knowledge structures and mechanisms that enable a physician to increase his or her expertise as a function of experience, e.g., exposure to and solution of difficult cases. Our belief is that questions of learning cannot be satisfactorily handled until we are clear about what it is that should result at the end of the learning process, i.e., what are the target structures that should be produced. Our work has concentrated on one type of target structure, viz., the diagnostic structure. Investigation of how this structure can be learned is an item in our agenda.

Most of our current activities are concentrated on the following problems:

- (1) Extension and testing of MDX over a larger domain.
- (2) Development of a language called CSRL--for Conceptual Structure Representation Language--which will enable high level specifications of the diagnostic specialists.
- (3) Implementation of the blackboards and the parallel problem solving regime as discussed in Gomez and Chandrasekaran.

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EMERGE is an interactive computer-based consultation program which serves as an "expert system" for emergency medicine department personnel treating patients with chest pain. EMERGE is written in the PASCAL language for operation on a microcomputer. EMERGE software can be used to create expert systems for any specialized areas whose knowledge can be expressed as sets of inference rules.

Expert or knowledge based systems are artificial intelligence (AI) terms for computer programs which model the reasoning process of experts. In contrast to conventional programs which use pre-specified control structures to follow sequences of operations, expert programs contain inference rules, which are triggered by patterns in the input data. These rule-based programs are called production systems and they can operate in any area of knowledge. Programs consist of knowledge rules, input data sets, and a control logic which examines the input and determines which rules to fire.

The concept that patterns in the input data can activate chains of inference rules is a powerful one for clinical decision making. Attention is immediately directed to the portions of the knowledge base relevant for the specific patient. Production systems have other advantages: reasoning which resembles human logical processes; conclusions which can be explained by citing the rules used; and knowledge which can be easily modified. MYCIN<sup>5</sup>, a computer based medical consultation system for selecting therapy for patients with bacterial infection, is the best known of the rule-based AI systems used in medicine.

The development of expert systems requires the acquisition of comprehensive knowledge in a narrow domain. The knowledge in EMERGE was abstracted from Criteria Maps developed for medical audit by the UCLA Experimental Medical Care Review Organization (EMCRO)<sup>1,3</sup>. Criteria Mapping is a retrospective medical review process designed to capture the decision making processes of physicians. A route through the map reviews the reasoning of the physician in treating the clinical condition of a particular patient. Criteria mapping is well-suited to emergency departments, where there must be rapid focus on critical decisions based on the presenting symptoms<sup>6</sup>. The production rules for EMERGE are the rules contained in a criteria map

## System Implementation

## Knowledge Base

The present knowledge base for EMERGE consists of 120 production rules derived from over 900 items in the criteria map developed by the department of Medicine at UCLA for management of chest pain in an emergency department. Figure 1 is a typical section of this criteria map. Criteria are organized in a branching logic format and are numbered to facilitate routing through the map. A positive response to an item leads to the next item to the right, and a negative response or missing information leads downward to the next vertical item. The criteria and their

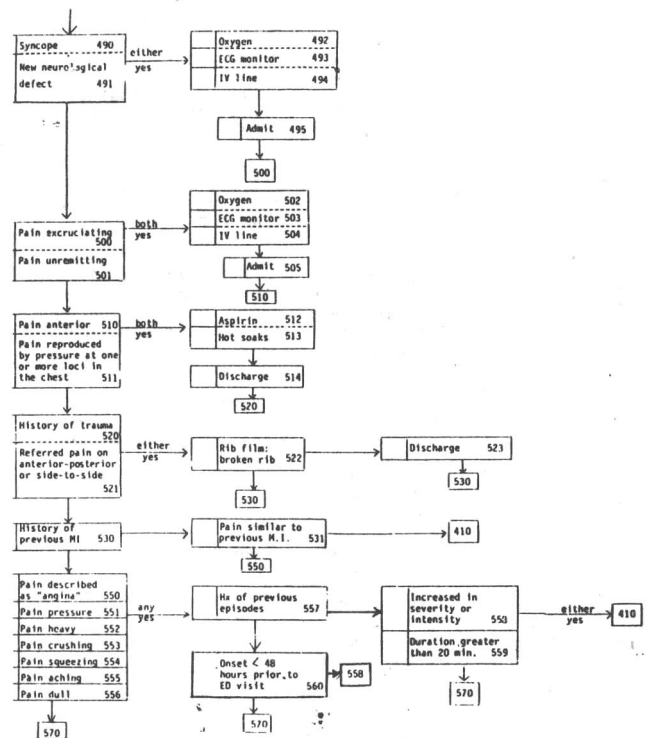


FIGURE 1: Items from Criteria Map for Chest Pain

linkages provide the knowledge contained in the production rules of EMERGE. For a patient who has fainted (syncope) or is in excruciating chest pain, but who has an ECG that is normal or unchanged from prior tracings, the consultation would quickly proceed to display rule 490 or rule 500. The final goal in the emergency room situation is the decision of whether or not to admit the patient. The criteria map directs this decision making process by eliciting required information and suggesting treatment advice along the way.

Figure 2 lists the production rules developed from the criteria items 490-530 of figure 1. Each item in the map translates into premise conditions or action items of production rules. Production rules are multiple premises and action items in the form of:

If A and B and C  
then D and E

A, B and C may be symptoms, signs or test finding and D and E may be treatment, diagnosis, or the decision to admit or discharge.

Figure 3 graphically illustrates the production rules 490 to 530 of figure 2, as part of a decision network. Premises and actions are represented as nodes, connected by paths. The left hand path emerging from a node indicates that all premises are satisfied while the right hand path indicates a

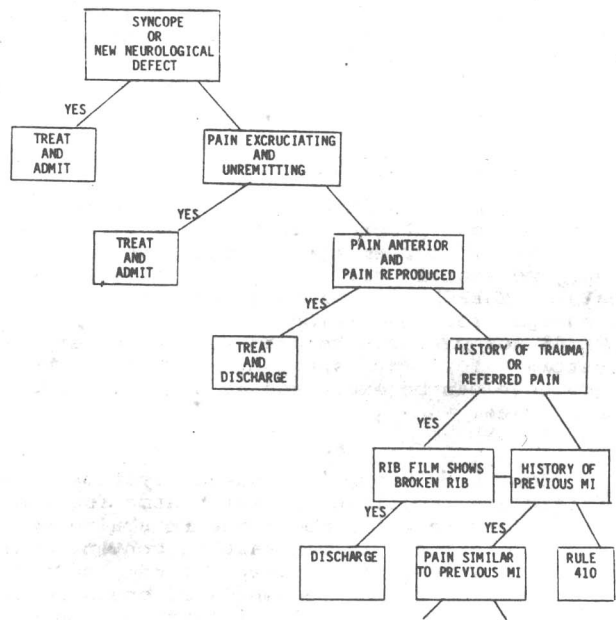


FIGURE 3: Inference-rule Network

negative response or lack of information. An EMERGE dialogue directly displays the English language text of the if-then production rules and no translation program is required.

- RULE 490  
IF [syncope OR new neurological defect] THEN [oxygen AND ECG monitor IV line AND admit]
- RULE 500  
IF [pain excruciating AND pain unremitting] THEN [oxygen AND ECG monitor AND IV line AND admit]
- RULE 510  
IF [pain anterior AND pain reproduced by pressure at one or more loci in chest]  
THEN [aspirin AND hot soaks AND discharge]
- RULE 520  
IF [history or trauma AND referred pain on anterior-posterior or side to side AND rib film shows broken rib]  
THEN [discharge]
- RULE 530  
IF [history or previous MI AND pain similar to previous MI]  
THEN [RULE 410\*]

Note that a loop is made here back to an earlier part of the criteria map.

FIGURE 2: Production Rules corresponding to Criteria Map of figure 1.

Sample Consultation

A consultation always begins with the system asking for demographic data on the patient. After this information has been entered, the user is asked whether clinical information is available. If the user answers negatively, the system will ask a series of questions to elicit clinical information. This dialogue is illustrated in figure 4. A series of initial questions to determine whether the patient is in shock, are answered negatively; the question on anterior pain is answered positively. The positive response triggers rule 510 of figure 2 and the system then asks the second condition of this premise rule: Is the pain reproduced at one or more locations? An affirmative response invokes the action items: aspirin, hot soaks and discharge.

Figure 5 illustrates a sample run for another patient, but the physician, after entering the demographic information, responds to the request for clinical information by entering: pain anterior. The system immediately selects production rule 510, and consultation then proceeds as in Figure 4 with the questioning dialogue bypassed.



## Control Structure

DO YOU WANT INSTRUCTIONS? PLEASE ANSWER YES OR NO  
? YES

THE SYSTEM WILL ASK YOU TO ENTER INFORMATION ON THE PATIENT. WHEN ENTERING INFORMATION, ENTER ONE ITEM PER LINE, I.E., AFTER ENTERING ONE ITEM OF INFORMATION, PRESS THE CARRIAGE RETURN. WAIT FOR A QUESTION MARK BEFORE ENTERING THE NEXT LINE. IF YOU HAVE NO FURTHER INFORMATION, ENTER END WHEN THE QUESTION MARK APPEARS. IF YOU HAVE NO INFORMATION, OR DO NOT KNOW WHAT TYPE OF INFORMATION IS DESIRED, THE COMPUTER WILL ASK YOU QUESTIONS.

PLEASE ENTER THE NAME OF THE PATIENT  
? Jane DOE  
WHAT IS THE AGE OF THE PATIENT?  
? 55  
WHAT IS THE SEX OF THE PATIENT? ENTER M OR F  
? F  
DO YOU HAVE ANY CLINICAL INFORMATION ON THIS PATIENT? PLEASE ANSWER YES OR NO  
? NO  
PLEASE ANSWER YES OR NO TO THE FOLLOWING:  
IS THERE AN ECG FINDING?  
? NO  
IS THERE ABNORMAL MENTAL STATE  
? NO  
IS THERE COLD, CLAMMY SKIN  
? NO  
IS THERE GREY, CYANOTIC SKIN  
? NO  
IS THERE WEAK PERIPHERAL PULSES  
? NO  
IS THERE CHEST X-RAY INTERPRETATION  
? NO  
IS THERE SYNCOPE  
? NO  
IS THERE NEW NEUROLOGICAL DEFECT  
? NO  
IS THERE PAIN ANTERIOR  
? YES  
IS THERE PAIN REPRODUCED AT ONE OR MORE LOCI IN CHEST  
? YES  
THE FOLLOWING TREATMENT SHOULD BE APPLIED ASPIRIN HOT SOAKS DISCHARGE

FIGURE 4: Sample Dialogue

PLEASE ENTER THE NAME OF THE PATIENT  
? JOHN DOE  
WHAT IS THE AGE OF THE PATIENT?  
? 37  
WHAT IS THE SEX OF THE PATIENT? ENTER M OR F  
? M  
DO YOU HAVE ANY CLINICAL INFORMATION ON THIS PATIENT? PLEASE ANSWER YES OR NO  
? YES  
PLEASE ENTER THE INFORMATION WHEN YOU HAVE NO MORE INFORMATION  
? PAIN ANTERIOR  
? END  
IS THERE PAIN REPRODUCED AT ONE OR MORE LOCI IN CHEST  
? YES  
THE FOLLOWING TREATMENT SHOULD BE APPLIED ASPIRIN DISCHARGE

FIGURE 5: Sample Dialogue

Figure 6 displays the structure which controls the sample dialogue of figure 4 and 5. The Recording Routine obtains basic patient data and then activates either the Questioning or Search Routine. If appropriate clinical information is not provided by the user, the right hand branch is taken and a series of questions are asked, as illustrated in the dialogue of figure 4. If clinical information is initially provided as in the dialogue of figure 5, control is passed to the Search Routine of the left hand branch.

The Search Routine finds a starting point in the network by comparing the input phrase with phrases in each production rule in turn. Matching of words are attempted only if their lengths are equal or differ by one character, and the ordering of words in a phrase is not significant. Both the comparing of words and the comparing of phrases utilize a combination of a modified form of fuzzy logic and a check for threshold values<sup>4</sup>. This approach allows a match of words when there are minor misspellings and a match of phrases even if all words do not match.

When the initial condition of the starting premise is verified, control is transmitted to the Verification Routine. Since a rule may contain multiple premises, remaining premises must be verified. The Verification Routine first checks to see if a match occurs with information already provided. If any premises still remain unverified, the user will be asked additional questions in an attempt to confirm them.

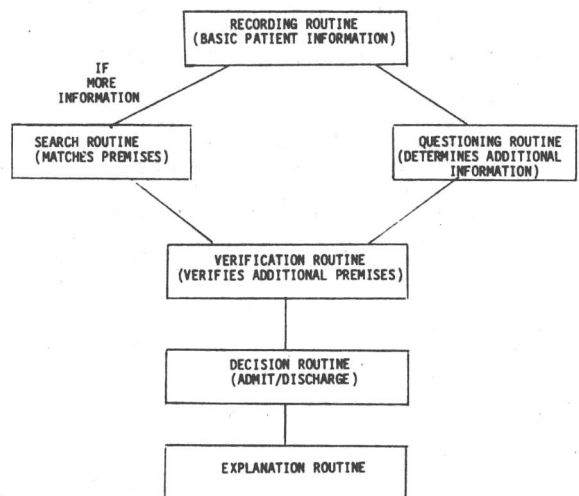


FIGURE 6: Program Modules

After all conditions of a premise have been verified, there are three types of action items which the Decision Routine may invoke: recommendation for standard treatment, transfer to another rule, and decision to admit or discharge. If the action is transfer to another rule, the premises of the new rule must be verified. This is done by the Decision Routine which calls itself recursively until the final admit/discharge decision is reached. The user can invoke the Explanation Routine to review the linkage of rules leading to the admit-discharge decision.

### Discussion

EMERGE illustrates the value of AI systems as aids to clinical decision making. Questions are only asked when the clinical findings of the patient indicate their relevance, in contrast to a fixed dialogue similar for all patients. By allowing multiple options and variations from conventional order, the characteristics of the patient lead EMERGE to request further investigation or to suggest relevant and appropriate treatment, concluding with the decision to admit or discharge the patient. A barrier to the design of expert systems like EMERGE, by computer scientists, is the difficulty in obtaining the medical expertise for the inference rules. The selection of a clinical area for which protocols or algorithms already exist partially solves this problem. Ultimately the validation and growth of a system will be dependent on use and feedback from physicians.

The EMERGE Consultation system was inspired by MYCIN. Implementation differs in several aspects, the most significant of which is choice of programming language. MYCIN rules are coded in INTERLISP, an interactive descendant of the LISP language. A translation program translates the rules into English for display to the user. LISP was developed by AI researchers to facilitate the processing of data organized in the form of LISTS. While LISP is very well suited to the representation and searching of production rules, it was designed to run on large machines. Implementation of LISP is system dependent and the language is not readily available to the medical community. Portability for use on small computers was made a critical specification for the EMERGE system.

PASCAL is a language with rich data structures and good string handling capability, making it a suitable candidate for implementing a system based on production rules. PASCAL allows recursive procedure calls to be implemented easily, which also is an important feature for AI programs. In addition programming is modular and permits development of structured programs which are easy to follow and to modify. PASCAL is standardized and readily available on small computers. EMERGE is written in PASCAL and can be used on any computer which has a PASCAL compiler, including micro-computers. The central memory requirement is less than 16,000 bytes and the disc requirement for the current knowledge base is approximately 300 kilobytes. The program is designed in modular fashion and can be readily integrated with data base systems or statistical packages.

The EMERGE system is available on floppy disc.

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# A HIERARCHICAL NONPARAMETRIC CLASSIFICATION STRATEGY FOR DISCRIMINATING EEG PATTERNS OF HIGHER CORTICAL FUNCTIONS (HCF)

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## Abstract

Brain Electrical potentials (BEP) recorded from the human scalp during cognitive engagement of the central nervous system (CNS) have been the subject of extensive research. To obtain significant results in this area it is important to establish a methodology which excludes as many external factors as possible. Also EEG classes obtained from higher cortical functions tend to be highly overlapping and therefore difficult to separate. In a search for a reliable pattern recognition system we have established a hierarchical nonparametric classification scheme. The 5 class classification problem was reconstructed as a four level dual binary tree scheme. Potential features were defined by parametrizing the EEG time series as a multivariate AR process. AR parameters were estimated from short (1-3 second) segmented epochs by the square root normalized maximum entropy method. A recursive nonparametric partitioning of the feature space by the Kolmogorov-Smirnov distance measure was used both for Bayes-efficient feature selection and classification. The method can be further applied to other dubious EEG recognition tasks and can be utilized in real-time applications.

## Introduction

Brain electrical potentials research is carried on in three distinct subareas: evoked response potentials (ERP); short-term analysis; and prolonged EEG recording as in patient monitoring or sleep scoring. This study deals with short-term EEG analysis related to higher cortical functions (HCF). Recent review papers by Gevins et al.<sup>1</sup>, Barlow<sup>2</sup> and Zetterberg<sup>3</sup> contain a fairly complete description of research results that have been obtained in this area. Some success in automatic classification of EEGs associated with higher cortical functions was recently reported by Yunk et al.<sup>4</sup> who examined a variety of parametric and nonparametric classification algorithms and laid out guidelines for future HCF and related recognition

problems. The possibility that this success and others reported in the literature<sup>5,6</sup> might be spurious has been raised by Gevins et al.<sup>7,8</sup>, who by carefully controlling demographic, behavioral, pharmacological and other extraneous variables, showed that certain spectral features of the EEG appeared to be uncorrelated with higher cortical activity.

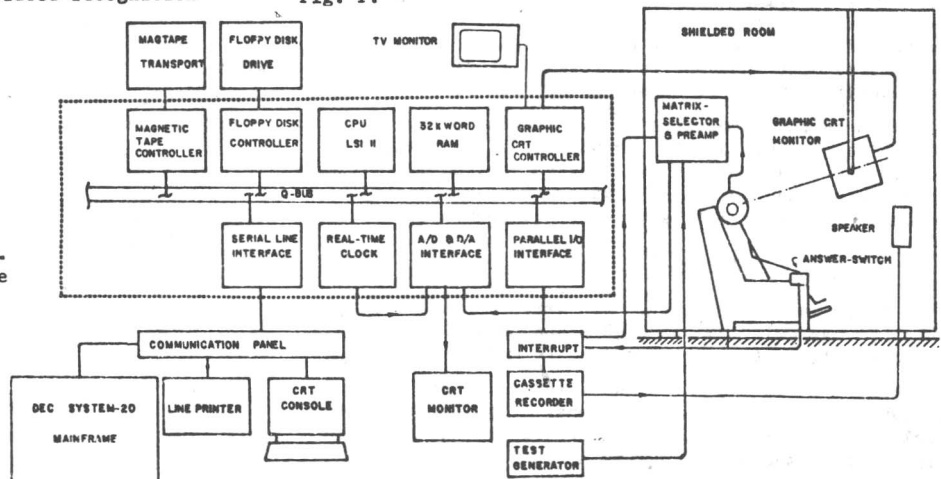
In recent years the fact that the EEG is probably not well modelled as a stationary process has been considered by a number of researchers and various segmentation methods have been proposed to permit modeling the EEG as a sequence of quasistationary waveforms<sup>9,10,11</sup>.

Compared to the reasonably well defined EEG classes observed in sleep staging, anaesthesia monitoring or metabolic disorders, the EEG classes associated with higher cortical functions tend to be highly overlapping and therefore difficult to separate. Intersubject variability is often substantially greater than the variability caused by changes in brain activity. For this reason sophisticated nonlinear, presumably nonparametric, layered classification strategies are called for. In our current study we have combined such nonparametric strategies with parametrization of the short-term EEG epochs by the maximum entropy method and with a unique experimental environment that implements the controls and general methodology suggested in<sup>1</sup>. In the following paragraphs we briefly discuss our experimental setup, data base, feature estimation, hierarchical design and classification results. An attempt is also made to analyze the selected feature set and to interpret its relation to the HCF classes.

## Experimental Setup - System and EEG Recording

A block diagram of the computerized data acquisition, graphics and analysis system is shown in Fig. 1.

Fig. 1 - Computerized EEG analysis system.



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The basic idea behind the system design is time-locking of the data acquisition cycle to the precise engagement time of the subject with a specific mental assignment. Tasks were designed as graphic images projected on a screen filling the subject's confined viewing field. A preliminary pilot study was used to determine optimal values for epoch time, repetition rate, and relaxation intervals. The experiment was based on a problem solving environment and subjects were supposed to respond (by pressing 1 of 4 buttons) to tasks projected for a preassigned time (typically 4-5 seconds). The complete session was controlled by the computer, and a "graphic feedback" was provided by the system to the subject as a response to an answer.

Approximately 250 distinct tasks from 5 different mental-task classes were used. They were grouped in sets of 12 tasks per class (5 classes were grouped into a "run" which was repeated 4 times). The classes were: Arithmetic (A), Cube folding (C), Short-term memory (S), Verbal (V) and visual fixation (F). The stimulus modality was the visual sensory system and efferent processes were unified by a unique answer pattern sequence. 8 referential channels to ipsilateral ears were utilized (M1-F3, M1-C3, M1-P3, M1-O1, M2-F4, M2-C4, M2-P4 and M2-O2). EEG potentials, bandlimited from 0.2 Hz to 30 Hz, were sampled at 200 samples/sec rate, digitized and stored on-line on magnetic tapes.

#### Subject Pool and Data Base

135 volunteer subjects (60% males) took part in the experiment. All were from a college population, were right-handed, nonsmoking, and did not use alcohol to excess. For each subject, variables such as coffee and medication intake, temperature and degree of wakefulness were recorded. The EEGs from the complete sample pool were screened by an interactive graphic artifact detection and elimination scheme. Extracerebral artifacts of various kinds were evaluated, identified and epochs were shortened or discarded completely if necessary to provide reasonably artifact-free traces for further analysis.

The artifact screening process resulted in a final set of 96 subjects (58 males and 38 females) and contained 24,000 epochs. Since task duration varied according to task engagement time, epoch length varied from 0.8 to 3.2 seconds. Poligraph traces from different classes were visually indistinguishable except for minor alpha-rhythm features observed in the visual fixation tasks.

#### Epoch Segmentation

Compressed spectral array (CSA) analysis of short overlapping segments of typical epochs indicated substantial nonstationarity in the longer epochs, and suggested that there might be significant class-dependent information contained in distinct segments. A number of segmentation techniques have appeared in the literature<sup>11,12</sup>. Because of epoch length constraints and for computational simplicity a simple ad hoc scheme was used whereby epochs of greater than 1.4 seconds were broken into 2-3 overlapping segments. Shorter epochs were left unsegmented.

#### The Dual Classifier

There is evidence that two basic rest EEG types, which we may call low and high alpha, exhibit fundamentally different responses to mental activities<sup>5</sup>. In order to reduce intersubject variability from this source we partitioned the subject pool into two alpha groups. Partitioning was based on alpha measurements made during a brief rest period

prior to the main session (single channel recording from Cz-O2 electrodes).

#### Feature Definition

Autoregressive modeling of EEG time series, extensively used in the past<sup>13,14</sup>, has been shown to be an adequate representation of these processes and to yield good data reduction over spectral domain features<sup>15</sup>. Very few studies have modeled the multichannel EEG as multivariate processes, mostly due to the substantial amount of computations needed<sup>6</sup>. Usually, AR and ARMA model parameters are estimated from long records or adaptively estimated by a Kalman Filter<sup>16</sup>. Our short records of variable length suggested the use of the Maximum Entropy Method (MEM), originally devised by J.P. Burg for spectral estimation<sup>17</sup>, and shown to be equivalent to a least-square estimation of AR parameters<sup>18</sup>. In fact, features were obtained from the prediction-error filter (the "whitening" filter) coefficients. A multichannel time series can be represented by a  $d$  by 1 column vector  $\underline{X}_n$ :

$$(1) \underline{X}_n = [\underline{X}_n^{(1)}, \underline{X}_n^{(2)}, \dots, \underline{X}_n^{(d)}]$$

We assume that  $\underline{X}_n$  is wide sense stationary. The  $d$  by  $d$  autocorrelation function of the lag parameter  $m$  is defined by:

$$(2) R_x(m) = E[(\underline{X}_n)(\underline{X}_{n-m})^T]$$

The correlation matrix of the vector process, for the case of a multichannel prediction-error filter of order  $M$ , is defined by the  $M+1$  by  $M+1$  block Toeplitz matrix as follows:

$$(3) R_x = \begin{bmatrix} R_x(0) & R_x(-1) & \dots & R_x(-M) \\ R_x(1) & R_x(0) & \dots & R_x(1-M) \\ \vdots & \vdots & \ddots & \vdots \\ R_x(M) & R_x(M-1) & \dots & R_x(0) \end{bmatrix}$$

The multichannel versions of the forward and backward prediction-error filters of order  $M$  are defined by the matrix-valued coefficients  $A_m^{(M)}$  and  $B_m^{(M)}$  respectively. Both  $A_0^{(M)} = B_0^{(M)} = I$  and  $m=0,1,2,\dots,M$ . The forward and backward prediction-errors are given by:

$$(4) \underline{e}_{f,n}^{(M)} = \sum_{m=0}^M A_m^{(M)} \underline{X}_{n-m}$$

$$(5) \underline{e}_{b,n}^{(M)} = \sum_{m=0}^M B_m^{(M)} \underline{X}_{n-m}$$

where  $\underline{e}_{f,n}^{(M)}$  and  $\underline{e}_{b,n}^{(M)}$  are  $d$  by 1 column vectors. The corresponding expressions for the prediction-error covariance matrices are:

$$(6) P_{f,M} = E[(\underline{e}_{f,n}^{(M)})(\underline{e}_{f,n}^{(M)})^T];$$

$$(7) P_{b,M} = E[(\underline{e}_{b,n}^{(M)})(\underline{e}_{b,n}^{(M)})^T]$$

Using (3-7) we may then write the multichannel version of the prediction-error filter equations:

$$(8) R_x \cdot \begin{bmatrix} I & B_M^{(M)} \\ A_1^{(M)} & B_{M-1}^{(M)} \\ \vdots & \vdots \\ A_M^{(M)} & I \end{bmatrix} = \begin{bmatrix} P_{f,M} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & P_{b,M} \end{bmatrix}$$