FLUID DYNAMICS of Oil Production

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PREFACE

SUBSURFACE FLUID DYNAMICS

At the present time, mathematical modelling is frequently applied to the mechanics of continuous media and in particular to subsurface fluid dynamics. The latter deals with such important theoretical and practical issues as water flow through dams, soil salinization, the spread of pollution by groundwater flows, oil production, groundwater flow into artesian wells and many others. The similarity of the physical processes involved in these phenomena means that their models also have many similarities, although the model equations all have their own special characteristics. It is in fact these special characteristics that make it very difficult to validate the models and solve the equations.

Filtration is defined as fluid flow through a porous medium. A medium is regarded as porous if it contains a large number of voids which are small by comparison with the typical dimensions of the medium. Porosity is defined quantitatively by the ratio of pore volume to bulk volume: $m = V_{por}/V_{total}$. Mathematical flow models are based on the law of conservation, the mechanics of continuous media, their effects, and other accepted equations. Primary equations include the continuity equation (taking porosity into account), the heat balance equation and equations of state. The main assumption of the flow theory is the replacement of Euler's or Navier-Stokes equations of motion with Darcy's Law.

The simplest two-phase flow model is the well-known Buckley-Leverett (BL) model (Chapter 1, Section 1.1), which assumes the equality of phase pressures, and therefore does not allow for the effect of capillary forces on fluid flow. The difficulties which arise in solving its equations (the potential ambiguity of the solution) are resolved by making the flow process mathematically ideal by assuming that the final function contains a point of inflexion. Convective processes are central to the BL model. To take additional effects into account, the mathematical flow model needs to be adjusted in various ways.

The introduction of capillary forces produces the Muskat-Leverett (ML) model (Chapter 1, Sections 1.1−1.4) which uses a Laplace equation to allow for these forces. Unidimensional transformation of the model produces a non-linear degenerate second-order differential equation. The solution of this equation has no point of inflexion, and the high-gradient region is confined to a limited area, which is perfectly justified in physical terms. Another advantage of this equation is that although it is parabolic, the model retains an important and physically natural property, in that it allows perturbations to propagate within a defined range of velocities (provided that its functional parameters have been correctly selected) (Chapter 2, Section 2.1).

The flow model is further refined (and therefore further complicated) if we allow for the interaction of velocity and temperature in oil-bearing formations, which means that an energy equation needs to be added to the model. Models of non-isothermal two-phase flow were studied by O.B. Bocharov, V.N. Monakhov, R. Yuing (MLT-model) [2; 15; 16; 44], E.B. Chekalyuk [140] and others. O.B. Bocharov and V.N. Monakhov [16] proposed and investigated an even more generalized MLT-model, which included variable (temperature-dependent) residual saturation.

Other generalized flow models include non-linear, multiphase and multicomponent flow models and others.

In our book, we concentrate on the effect of temperature on fluid flow processes as applied to modelling water-oil displacement and the production of fluid. The inclusion of non-isothermal flow makes it possible to approximate the real conditions, making the physical and therefore the mathematical model less abstract, and provides some corrections to the accepted hydrodynamic methods of calculating oil production.

Studies have shown that oil recovery factors can be significantly increased only by changing the physical and physico-chemical properties of the displaced phase, withthermal recovery being increasingly favoured. The importance of thermal recovery methods is largely due to the fact that they use easily available media—water and air. Another major advantage over most other methods (e.g. physico-chemical) is the potential for increasing recovery in a variety of physico-geological oil field conditions. Thermal recovery methods are based on the fact that the viscosity of oil decreases considerably when it is heated, so that their primary application is in high-viscosity oil fields. At the same time, thermal recovery involves virtually all known oil displacement mechanisms, together with a variety of phase transitions, so that it offers promise even in the case of low-viscosity oil fields which have long been operated under water injection. It should be noted that the injection of water at a temperature lower than formation temperature (e.g. sea water or injection during winter) reduces oil recovery. In particular, it may lead to wax precipitation directly in the porous medium.

It is well known that in water wet rocks capillary forces can play a very important role in the process of oil displacement. If a low-permeability section is surrounded by high–permeability rock, the water will flow around the oil contained in the section. If water wet formations are flooded, the oil can be frequently recovered only by the use of capillary forces. The existence of this mechanism has been confirmed both experimentally and by analysing fields consisting of inhomogeneous water wet rocks. Capillary saturation may also have a decisive influence on the mechanism of oil recovery in stratified beds. Therefore, we need to know how the non-isothermal process of oil displacement by capillary forces will affect the recovery of oil from such heterogeneous formations.

All these phenomena require thorough study, and the Muskat-Leverett thermal flow model provides an effective tool.

Many problems formulated using these models can be studied in a given sequence, forming a specific process cycle, such as for instance steam treatment (Chapter 2, Section 2.1), which may be described in a simplified form as consisting of the following steps (the corresponding mathematical statements are shown in brackets):

1. Steam (superheated water) injection at a specified temperature and flow rate (non-isothermal two-phase flow with convective forces predominating);
2. Soaking for a specified time without water injection (thermocapillary saturation)
3. Steam or water injection (possibly, at a different temperature and flow rate) (non-isothermal two-phase flow with convective forces predominating).

Therefore, if we know how to model these steps we can use them to study more complicated processes and make multivariate optimizing calculations.

For all the above models, we need to find specific solutions, including self-similar (analytical) solutions, and this problem is dealt with in Sections 2.1–2.5 of Chapter 2.

NUMERICAL MODELLING OF OIL PRODUCTION PROCESSES

The most common oil-field development systems are based on symmetrical well patterns. This means that rather than studying a whole field, we can study a single development unit, which usually consists of two wells. For example, for a five-spot water flood, the basic element is a rectangle with no flow boundaries, containing an injection and a production well in opposite corners.

Since calculations of the development of basic elements of symmetrical well patterns can be reduced to calculations of linear flow (for an in-line pattern) or a plane-radial flow (for an areal pattern), this simplifies flow model equations, making them one−dimensional.

The formulation of the initial and boundary conditions for the basic elements is also simplified: the production rate, pressure or saturation are specified for each well. Consolidated figures are then calculated for the production unit as a whole, followed by the calculation of the 2D process of two-phase flow in the basic element−this program can be attached to more detailed multi-parameter ID programs, providing them with coefficients allowing for the fact that the processes are not unidimensional. However, the 2D basic element calculation is important not only because it supplements the ID programs but also as a stand-alone petroleum engineer's tool, in which the multiple parameters of the ID models can be incorporated, provided sufficient computing power is available. In addition, as field development proceeds, well patterns and well operation become asymmetrical, and this can only be allowed for by 2D calculations, performed by a program which calculates the process of oil production in a 2D basic element without assuming that the boundary conditions are symmetrical.

The calculation program produces oil saturation and pressure fields within the basic element and calculates the oil recovery factor and water cut as a function of the injected pore volumes of water. The information may be presented in graphical form and then printed out as data files for use in further analysis of the oil production process and/or printed out as numerical files.

The contents of the book. If we include submodels and combined models, Chapter 1 contains the description of over 30 different mathematical models of oil formations, provides analyses of a number of some generally accepted flow models and offers new models of some physical effects not covered elsewhere. In designing these models, we have attempted to achieve a good numerical implementation without increasing the number of their key parameters. As a rule, the proposed model design changes are accompanied by small "slippage" terms introduced into the equation by analogy with the computing "slippage" in finite-difference equations. It should be noted that other authors have also introduced some of the filtration model changes proposed in Chapter 1, but did not analyse the resultant models sufficiently.

S.N. Antontsev and V.N. Monakhov [4] proposed a general oil formation flow model containing a range of functional parameters. By making a careful selection, some of the models proposed in Chapter 1 can be derived from them.

Mathematical models can be subdivided into three main classes:

1. Single-phase Darcy models and contact models (Section 1.1);
2. Two-phase models (e.g. the Muskat-Leverett model – Sections 1.1–1.4);
3. Combined models (e.g. of two inhomogeneous liquids – Section 1.7).

In addition to the conventional Darcy and Muskat-Leverett models and the Muskat-Leverett thermal model (in the form proposed by O.B. Bocharov and V.N. Monakhov [15; 16]), Chapter 1 describes the Navier-Stokes and Zhukovsky models (Section 1.1), used by the authors to optimize oil production control and production forecasts.

The book also contains some unconventional models, such as the models describing the process of "foaming" in oil formations (Section 1.6), the combination of reservoir flow with liquid flow in wells (Section 1.5) and others.

Of the new and modified models (e.g. the reduced-pressure ML and MLT models) Chapter 1 discusses only the models developed by V.N. Monakhov and studied by him and his colleagues and students, S.N. Antontsev, O.B. Bocharov, A.A. Papin, R. Yuing, E.M. Turganbayev, V. N. Starovoitov, N.V. Khusnutdinov, A.E. Osokin, and others [4; 15; 16; 18; 20; 32; 44; 61; 69; 75; 91; 94; 101; 124; 134].

Chapter 2 presents a theoretical and numerical analysis of one-dimensional and self-similar (analytical) thermal two-phase flow patterns, while its Section 2.1 provides additional information based on the ordinary differential equation theory, which is also of independent interest.

The core of the chapter is formed by Sections 2.2 and 2.3, which present the results of V.N. Monakhov, O.B. Bocharov, A.E. Osokin, and T.V. Kantayeva's work [20; 69; 92]. These include the theorem of existence of self-similar (analytical) solutions of the MLT model for constant and variable residual saturation, the identification of a restricted range of velocities of propagation of perturbations, and the computer implementations of the numerical algorithms proposed by the authors and their substantiation.

Section 2.4 contains a theoretical analysis of the analytical solutions (B.T. Zhumagulov, V.N. Monakhov [58].

The existence and uniqueness of self-similar (analytical) solutions of the model of two-phase flow of non-linear-viscous liquids is demonstrated in 2.5 (E.G. Galkina, A.A. Papin [32]). Section 2.6 establishes the convergence of Rothe-type methods in a one-dimensional MLT model (A. E. Osokin [100]).

Section 2.7 is devoted to the substantiation of a new method of integrating ML and BL model solutions and to their numerical implementation (I.G. Telegin, [129]; B.T. Zhumagulov, Sh.S. Smagulov, V.N. Monakhov, N.V. Zubov [61]).

The existence and uniqueness of "im Kleinen" (small-scale) solutions of the first boundary-value problem, based on the initial data for the two interpenetrating viscous liquids flow model is demonstrated in Section 2.8 (A. A. Papin [101]).

Chapter 3 deals with numerical modelling of two-dimensional subsurface hydrodynamics processes with reference to Muskat-Leverett isothermal and temperature models as well as Navier-Stokes and Zhukovsky models.

In this chapter, Section 3.1 demonstrates the convergence and stability of effective finite–difference schemes [38] for Navier-Stokes velocity vs. pressure finite difference equations, while Section 3.2 uses velocity vs. flux function and the method of virtual regions to provide numerical calculations of reservoir flows in multiply connected regions [45] and geometrically complex regions (Section 3.3) [45]. In Section 3.4, similar numerical methods are applied to the Zhukovsky model. [49].

Section 3.5 provides a solution to a key problem of subsurface hydrodynamics—that of determining formation pressure from measured well pressure values [49]. We have performed a numerical calculation of formation heating, which forms one of the stages of steam treatment, based on the classical thermal convection model (Section 3.6) [63]. Section 3.9 [49] provides a numerical solution of water-oil displacement from inhomogeneous oil formations, based on the ML model, while Section 3.7 and Section 3.8 present the mathematical substantiation of the finite-difference equations used in Section 3.8 for more general models [18, 44]. And finally, Section 3.10 offers a hydrodynamic analysis of the results of numerical calculations of subsurface hydrodynamics problems based on different formation models [57].

Sections 3.1–3.6 and 3.9 present the results obtained by B.T. Zhumagulov and his colleagues, Sh.S. Smagulov, N.T. Danayev, B.G. Kuznetsov, G.T. Balakayeva, N.T. Temirbekov, K.Zh. Baigelov, K.M. Baimirov, K.B. Esikeyev [38; 47-52; 56; 59].

Results obtained jointly by V.N. Monakhov with R. Ewing [44], O. B. Bocharov [18] and B.T. Zhumagulov [57] appear in Sections 3.7, 3.8 and 3.10.

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In 1996, B.T. Zhumagulov, N.V. Zubov, V.N. Monakhov, and Sh.S. Smagulov published a book entitled *New Computer Technologies in Oil Production* (Almaty, Gylym). It described a computer-aided oil and gas field development analysis system developed jointly by a team of Russian and Kazakh scientists, led by the authors.

Subsequently, the research continued independently at the Lavrentyev Institute of Hydrodynamics, where it was led by V.N. Monakhov, at the Kazakhstan Engineering Academy and the Al-Farabi Kazakh State University, where it was conducted by B.T. Zhumagulov, Sh.S. Smagulov and their colleagues.

These studies form the basis of this book, which presents both the authors' own and collaborative work, except for Sections 2.3, 2.5–2.8 of Chapter 2, which include the work of V.N. Monakhov's students, A. A. Panin, A. E. Osokin, T. V. Kantayeva, I.G. Telegin and E. G. Galkina.

There is no doubt of the important contribution to the content and the scientific value of the book made by our co-authors, Sh. S. Smagulov, S.N. Antontsev, N.T. Danayev, O.B. Bocharov, N.V. Khusnutdinov, B.G. Kuznetsov, and N.M. Temirbekov, to all of whom we wish to express our sincere gratitude. To our editor-in-chief, R.I. Nigmatulin, and our reviewer, Sh.S. Smagulov, our profound gratitude for the many years of fruitful support, which has in many ways determined the style and the conceptual framework of the book.

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CONTENTS

CHAPTER 1

Fluid Flow Models

1.1 INTRODUCTION

In this chapter we analyze a number of well-known mathematical models of homogeneous and non-homogeneous fluid flow in porous media, and propose some new models. As the existing models [89; 105; 143] are based on specific conceptions of these processes, the inclusion of each new effect requires a revision of their underlying assumptions, as well as a revision of the model.

The fact that there are many forms of Darcy's Law means that we need to ask ourselves how to select the form which will best describe each specific situation. While the work on this question has progressed in recent decades, it has involved virtually no review of the fundamentals of conventional models. Frequently, experimental data processed to fit the conventional models have been unstable (not easily reproducible), while published experimental results did not, as a rule, provide sufficient information to fit them to other models. Some effects are simply impossible to describe in terms of the existing models.

Basic mathematical analysis of the various forms of flow models may prove extremely useful for the modelling of phenomena. At the same time, new physical factors need to be taken into account, that is, the minor effects which stabilize the numerical calculations (i.e. the physical "slippage terms" in the equations). For instance, transition to linear models often leads to a loss of divergence in equations, and when it comes to numerical calculations, does not simplify the initial nonlinear model. Equally, striving to achieve a mathematically satisfactory model can lead to a lack of conformity with the physics of the phenomenon, as is the case with the divergent form of Darcy's Law for inhomogeneous media, proposed by Sheidegger [143].

At present, the widespread use of computers has led to the establishment of a well-defined "process flow diagram" for solving specific problems in the mechanics of continuous media, including multiphase fluid flow. The work flow progresses from the problem under consideration to a mathematical model, from there to a numerical algorithm, the implementation software and finally to the analysis of the results. While the individual components of the process are not isolated but interconnected, linking both forwards and backwards, the most important factor for success is likely to be the choice of an appropriate mathematical model.

There are several principal requirements applicable to phenomenological flow models:

1. Experiment reproducibility. The ability to define all parameters experimentally, without needing to involve additional "theories", and good reproducibility of the experiments.
2. A clear distinction between the underlying hypotheses, and a clear definition of the limits of their applicability, both in qualitative terms (what kinds of physical effects they can describe) and quantitatively.
3. Ability to incorporate simpler models into higher-level models, so that new physical factors can be taken into account.
4. Mathematical feasibility and correctness.

Needless to say, these are not rigid requirements and could even be seen as programmes of study of the models. Moreover, the features of phenomenological models can be determined in laboratory conditions, using higher-level models containing independently determined parameters. For instance, the Navier-Stokes model could be used to determine phase permeabilities in two-phase flow models and to check various properties (e.g. saturation). Below we comment on several examples of multiphase fluid flow. There is no point in calculating total oil recovery using models which specify the total flow rate for injection wells, and the flow rate of only the displaced phase for production wells. If the phases are incompressible, then the answer lies in correctly stating the well conditions.

With these examples, we hope to have provided some insight into the difficulties of choosing an appropriate model with which to describe the physical process of fluid flow in porous media as it occurs in reality.

1.2 SINGLE-PHASE AND TWO-PHASE FLUID FLOW MODELS

1.2.1 Darcy's Model and Contact Models

1.2.1.1 The Properties of Porous Media

The main property of a porous material, its porosity (effective or dynamic), is described by the ratio $m = V_n / V_0$, where V_n is the interconnected pore volume and V_0 is the bulk volume.