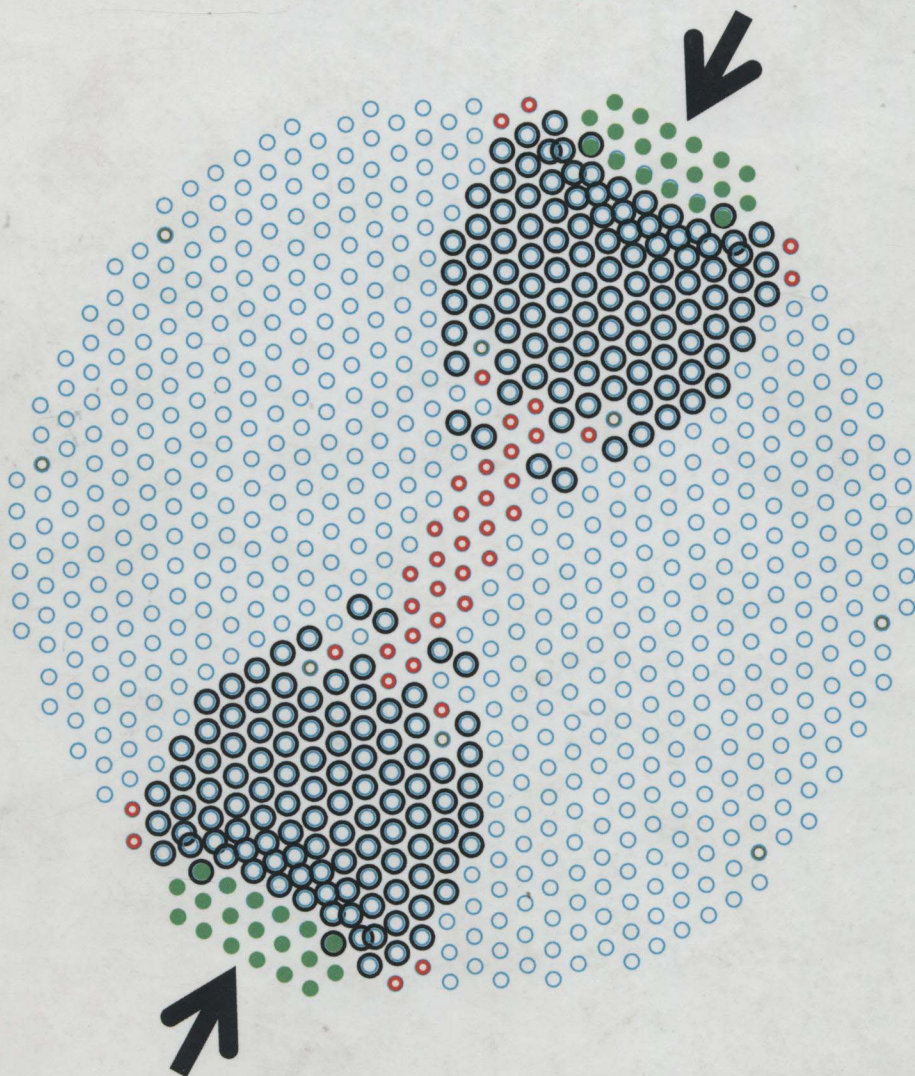


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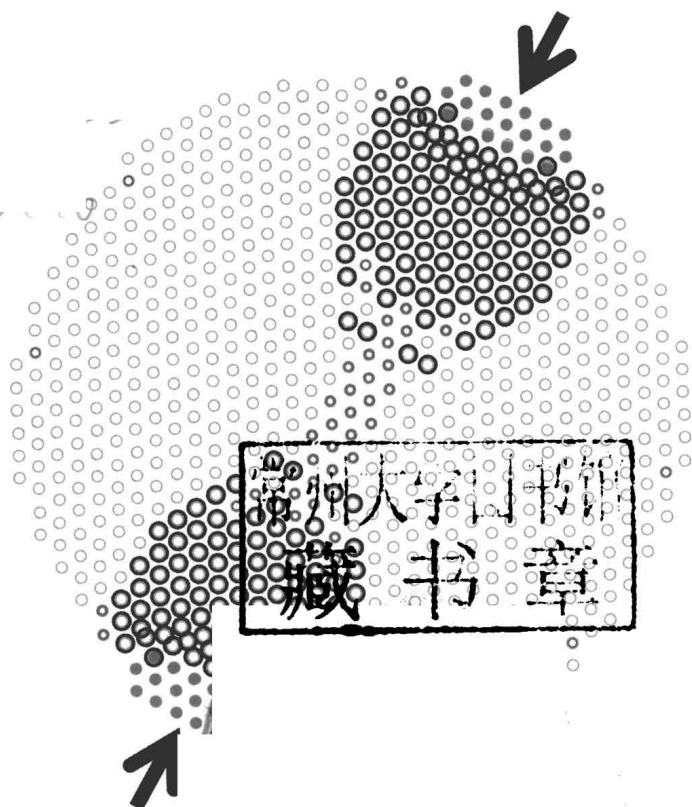
Computational Solid Mechanics Without Stress and Strain

Walter Herbert Gerstle



INTRODUCTION TO PRACTICAL PERIDYNAMICS

Computational Solid Mechanics Without Stress and Strain



Walter Herbert Gerstle

University of New Mexico, USA

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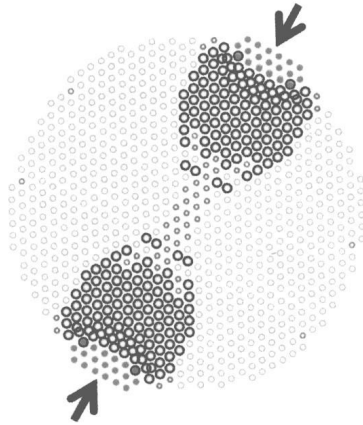
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Vol. 1 Introduction to Practical Peridynamics:

Computational Solid Mechanics Without Stress and Strain

by *Walter Herbert Gerstle*

Dedication

In memory of my father and mother
Kurt and Eva Gerstle

To my wife and son
Irene and David Gerstle

And, of course, to all of my students

Preface

This book proposes a departure from business-as-usual in the computational simulation of solids. Solid mechanics has, until now, usually been framed in terms of stress and strain. Indeed, the modern concept of “material” arises from, and is almost inseparable from, the concepts of strain and stress, which Augustin-Louis Cauchy invented in the early 1820s. The Navier-Cauchy linear partial differential equations of elasticity, as well as the Navier-Stokes equations of fluid mechanics, served as the archetype from which the more general discipline of continuum mechanics arose in the 1950s, primarily through the publications of Truesdell and Noll. Concurrently, to model cracking, which continuum mechanics fails adequately to address, Griffith, Irwin and many others invented the discipline of fracture mechanics.

With the advent of computers in the 1950s, structural engineers sought methods to solve realistic problems in structural mechanics, and the finite element method provided the necessary bridge that allowed engineers to solve the Navier-Cauchy equations of linear elasticity on a computer. The finite element method was wildly successful, and it continues to be very important in analyzing and designing countless modern technologies.

Following the success of the finite element method in solving problems of linear elasticity, engineers also began to solve nonlinear problems including such behaviors as plasticity, damage, creep, and fracture. However, a number of difficulties emerged; principally engineers found that the theory of continuum mechanics was inadequate to solve fracture problems, which are discontinuous at their core.

Most researchers, however, were reluctant to deviate from the theory of continuum mechanics, so they patched up the theory, with contrivances such as singularity elements, the crack band model, discrete fracture propagation models, and localization limiters. Researchers for the most part continued to toe the continuum mechanics line.

I have taught structural analysis, mechanics of materials, advanced mechanics of materials, finite elements, and fracture mechanics during the thirty years of my academic career. In recent years, computers have become more and more powerful. I, and many others, have become increasingly uncomfortable teaching engineering practices involving assumptions that ignore the power of computers – and which are therefore perhaps unlikely to survive the current computer revolution. We no longer teach once-important engineering practices like the moment distribution method and the conjugate beam method. We now teach matrix methods in every engineering curriculum. Engineering education continues to change in response to improvements in computer technology.

Digital computers are not capable of directly representing the basic ideas of continuum mechanics (like continuous analytic fields of strain and stress). Consequently, continuum mechanics theories must be discretized, in the form of a finite element (or other discrete) model, to be represented by a computer.

The situation is currently very peculiar. Why do we take a discrete physical model, and then through mathematical gymnastics (like taking limits) turn it into a set of partial differential equations, only to then later, through yet more mathematical manipulation, turn the problem back into a set of algebraic equations that can only then be represented and solved by a computer?

Why not just start from the outset, in describing our physical model, with a discrete algebraic computational model, directly solvable on the computer?

This book strives for the simplest possible computational model for solid bodies that both yields practical engineering results and that also connects in a meaningful way with the human mind.

I have written this book both as a treatise and as a textbook for a graduate-level class in computational solid mechanics, and therefore it

includes chapters that introduce the classical theory of elasticity, continuum mechanics, fracture mechanics, peridynamics, solid modeling, plasticity, and damage mechanics. However, I introduce all of these theories with a critical eye, because our ultimate goal is to develop a theory that is at the same time sufficiently true to physical reality, sufficiently comfortable for the human mind, and sufficiently computer-friendly. While important physical behaviors must be included, our model both demands and provides no more precision than is warranted by the solid materials that we seek to represent. While this book focuses on solids, the methods described herein are quite general, and one can extend these methods to many other physical phenomena (like fluids, thermo-mechanics, electromagnetics, and soils).

Fundamental inspiration for this book comes from my father, Kurt H. Gerstle, of the University of Colorado, graduate advisor Anthony R. Ingraffea, of Cornell University, and Stewart Silling, the inventor of peridynamics, of Sandia National Laboratories. Professor Shaofan Li, of University of California, Berkeley, invited me to write this book. I appreciate the encouragement and guidance provided by my colleague and friend Timothy Ross.

Thanks go to my graduate students who have helped to develop the state based peridynamic lattice method described in this book. They are listed in the sequence that they worked with me: Nicolas Sau, Eduardo Aguilera, Navid Sahkavand, Kiran Tuniki, Asifur Rahman, Hossein Honarvar, Raybeau Richardson, Aziz Asadollahi, Seth McVey, and Shreya Vemuganti.

I also thank the University of New Mexico for allowing me the time to write this book during a sabbatical leave, and Susan Atlas, director of the Center of Advanced Research Computing at UNM, for collaborating with me in the development of the parallel particle simulation code, pdQ, and for hosting me during the writing of this book.

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June 2015

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