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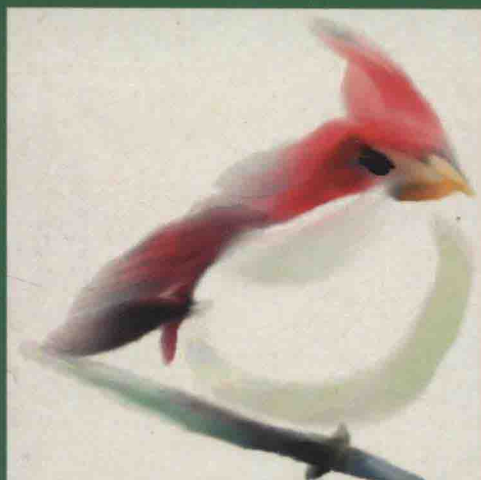
Modern Machine Learning Approaches



Masashi Sugiyama

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Masashi Sugiyama
University of Tokyo
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Foreword

How can agents learn from experience without an omniscient teacher explicitly telling them what to do? Reinforcement learning is the area within machine learning that investigates how an agent can learn an optimal behavior by correlating generic reward signals with its past actions. The discipline draws upon and connects key ideas from behavioral psychology, economics, control theory, operations research, and other disparate fields to model the learning process. In reinforcement learning, the environment is typically modeled as a Markov decision process that provides immediate reward and state information to the agent. However, the agent does not have access to the transition structure of the environment and needs to learn how to choose appropriate actions to maximize its overall reward over time.

This book by Prof. Masashi Sugiyama covers the range of reinforcement learning algorithms from a fresh, modern perspective. With a focus on the statistical properties of estimating parameters for reinforcement learning, the book relates a number of different approaches across the gamut of learning scenarios. The algorithms are divided into model-free approaches that do not explicitly model the dynamics of the environment, and model-based approaches that construct descriptive process models for the environment. Within each of these categories, there are policy iteration algorithms which estimate value functions, and policy search algorithms which directly manipulate policy parameters.

For each of these different reinforcement learning scenarios, the book meticulously lays out the associated optimization problems. A careful analysis is given for each of these cases, with an emphasis on understanding the statistical properties of the resulting estimators and learned parameters. Each chapter contains illustrative examples of applications of these algorithms, with quantitative comparisons between the different techniques. These examples are drawn from a variety of practical problems, including robot motion control and Asian brush painting.

In summary, the book provides a thought provoking statistical treatment of reinforcement learning algorithms, reflecting the author's work and sustained research in this area. It is a contemporary and welcome addition to the rapidly growing machine learning literature. Both beginner students and experienced researchers will find it to be an important source for understanding the latest reinforcement learning techniques.

Daniel D. Lee
GRASP Laboratory
School of Engineering and Applied Science
University of Pennsylvania, Philadelphia, PA, USA

Preface

In the coming *big data* era, *statistics* and *machine learning* are becoming indispensable tools for data mining. Depending on the type of data analysis, machine learning methods are categorized into three groups:

- **Supervised learning:** Given input-output paired data, the objective of supervised learning is to analyze the input-output relation behind the data. Typical tasks of supervised learning include *regression* (predicting the real value), *classification* (predicting the category), and *ranking* (predicting the order). Supervised learning is the most common data analysis and has been extensively studied in the statistics community for long time. A recent trend of supervised learning research in the machine learning community is to utilize side information in addition to the input-output paired data to further improve the prediction accuracy. For example, *semi-supervised learning* utilizes additional input-only data, *transfer learning* borrows data from other similar learning tasks, and *multi-task learning* solves multiple related learning tasks simultaneously.
- **Unsupervised learning:** Given input-only data, the objective of unsupervised learning is to find something useful in the data. Due to this ambiguous definition, unsupervised learning research tends to be more ad hoc than supervised learning. Nevertheless, unsupervised learning is regarded as one of the most important tools in data mining because of its automatic and inexpensive nature. Typical tasks of unsupervised learning include *clustering* (grouping the data based on their similarity), *density estimation* (estimating the probability distribution behind the data), *anomaly detection* (removing outliers from the data), *data visualization* (reducing the dimensionality of the data to 1–3 dimensions), and *blind source separation* (extracting the original source signals from their mixtures). Also, unsupervised learning methods are sometimes used as data pre-processing tools in supervised learning.
- **Reinforcement learning:** Supervised learning is a sound approach, but collecting input-output paired data is often too expensive. Unsupervised learning is inexpensive to perform, but it tends to be ad hoc. Reinforcement learning is placed between supervised learning and unsupervised learning — no explicit supervision (output data) is provided, but we still want to learn the input-output relation behind the data. Instead of output data, reinforcement learning utilizes *rewards*, which

evaluate the validity of predicted outputs. Giving implicit supervision such as rewards is usually much easier and less costly than giving explicit supervision, and therefore reinforcement learning can be a vital approach in modern data analysis. Various supervised and unsupervised learning techniques are also utilized in the framework of reinforcement learning.

This book is devoted to introducing fundamental concepts and practical algorithms of statistical reinforcement learning from the modern machine learning viewpoint. Various illustrative examples, mainly in robotics, are also provided to help understand the intuition and usefulness of reinforcement learning techniques. Target readers are graduate-level students in computer science and applied statistics as well as researchers and engineers in related fields. Basic knowledge of probability and statistics, linear algebra, and elementary calculus is assumed.

Machine learning is a rapidly developing area of science, and the author hopes that this book helps the reader grasp various exciting topics in reinforcement learning and stimulate readers' interest in machine learning. Please visit our website at: http://www.ms.k.u-tokyo.ac.jp.

Masashi Sugiyama
University of Tokyo, Japan

Author

Masashi Sugiyama was born in Osaka, Japan, in 1974. He received Bachelor, Master, and Doctor of Engineering degrees in Computer Science from All Tokyo Institute of Technology, Japan in 1997, 1999, and 2001, respectively. In 2001, he was appointed Assistant Professor in the same institute, and he was promoted to Associate Professor in 2003. He moved to the University of Tokyo as Professor in 2014.

He received an Alexander von Humboldt Foundation Research Fellowship and researched at Fraunhofer Institute, Berlin, Germany, from 2003 to 2004. In 2006, he received a European Commission Program Erasmus Mundus Scholarship and researched at the University of Edinburgh, Scotland. He received the Faculty Award from IBM in 2007 for his contribution to machine learning under non-stationarity, the Nagao Special Researcher Award from the Information Processing Society of Japan in 2011 and the Young Scientists' Prize from the Commendation for Science and Technology by the Minister of Education, Culture, Sports, Science and Technology for his contribution to the density-ratio paradigm of machine learning.

His research interests include theories and algorithms of machine learning and data mining, and a wide range of applications such as signal processing, image processing, and robot control. He published *Density Ratio Estimation in Machine Learning* (Cambridge University Press, 2012) and *Machine Learning in Non-Stationary Environments: Introduction to Covariate Shift Adaptation* (MIT Press, 2012).

The author thanks his collaborators, Hirotaka Hachiya, Sethu Vijayakumar, Jan Peters, Jun Morimoto, Zhao Tingting, Ning Xie, Voot Tangkaratt, Tetsuro Morimura, and Norikazu Sugimoto, for exciting and creative discussions. He acknowledges support from MEXT KAKENHI 17700142, 18300057, 20680007, 23120004, 23300069, 25700022, and 26280054, the Okawa Foundation, EU Erasmus Mundus Fellowship, AOARD, SCAT, the JST PRESTO program, and the FIRST program.

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Part I

Introduction