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Structural Vibration

A Uniform Accurate Solution for Laminated Beams, Plates and Shells with General Boundary Conditions

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Preface

In practical applications that range from outer space to the deep oceans, engineering structures such as aircraft, rockets, automobiles, turbines, architectures, vessels, and submarines often work in complex environments and can be subjected to various dynamic loads, which can lead to the vibratory behaviors of the structures. In all these applications, the engineering structures may fail and collapse because of material fatigue resulting from vibrations. Many calamitous incidents have shown the destructive nature of vibrations. For instance, the main span of the famous Tacoma Narrows Bridge suffered severe forced resonance and collapsed in 1940 due to the fact that the wind provided an external periodic frequency that matched one of the natural structural frequencies of the bridge. Furthermore, noise generated by vibrations always causes annoyance, discomfort, and loss of efficiency to human beings. Therefore, it is of particular importance to understand the structural vibrations and reduce them through proper design to ensure a reliable, safe, and lasting structural performance. An important step in the vibration design of an engineering structure is the evaluation of its vibration modal characteristics, such as natural frequencies and mode shapes. This modal information plays a key role in the design and vibration suppression of the structure when subjected to dynamics excitations. In engineering applications, a variety of possible boundary restraining cases may be encountered for a structure. In recent decades, the ability of predicting the vibration characteristics of structures with general boundary conditions is of prime interest to engineers and designers and is the mutual concern of researchers in this field as well.

Beams, plates, and shells are basic structural elements of most engineering structures and machines. A thorough understanding of their vibration characteristics is of great significance for engineers to predict the vibrations of the whole structures and design suitable structures with low vibration and noise radiation characteristics. There exists many books, papers, and research reports on the vibration analysis of beams, plates, and shells. In 1969, Prof. A.W. Leissa published the excellent monograph *Vibration of Plates*, in which theoretical and experimental results of approximately 500 research papers and reports were presented. And in 1973, he organized and summarized approximately 1,000 references in the field of shell vibrations and published another famous monograph entitled *Vibration of Shells*. New survey shows that the literature on the vibrations of beams, plates, and shells has expanded rapidly since then. Based on the Google Scholar search tool, the numbers of article related to the following keywords from 1973 up to 2014 are: 315,000 items for "vibration & beam," 416,000 items for "vibration & plates," and 101,000 items for "vibration & shell." This clearly reveals the importance of the vibration analysis of beams, plates, and shells.

Undeniably, significant advances in the vibration analysis of beams, plates, and shells have been achieved over the past four decades. Many accurate and efficient computational methods have also been developed, such as the Ritz method, differential quadrature method (DQM), Galerkin method, wave propagation approach, multiquadric radial basis function method (MRBFM), meshless method, finite element method (FEM), discrete singular convolution approach (DSC), etc. Furthermore, a large variety of classical and modern theories have been proposed by researchers, such as the classical structure theories (CSTs), the first-order shear deformation theories (FSDTs), and the higher order shear deformation theories (HSDTs).

However, after the review of the literature in this subject, it appears that most of the books deal with a technique that is only suitable for a particular type of classical boundary conditions (i.e., simply supported supports, clamped boundaries, free edges, shear-diaphragm restrains and their combinations), which typically requires constant modifications of the solution procedures and corresponding computation codes to adapt to different boundary cases. This will result in very tedious calculations and be easily inundated with various boundary conditions in practical applications since the boundary conditions of a beam, plate, or shell may not always be classical in nature, a variety of possible boundary restraining cases, including classical boundary conditions, elastic restraints, and their combinations may be encountered. In addition, with the development of new industries and modern processes, laminated beams, plates, and shells composed of composite laminas are extensively used in many fields of modern engineering practices such as space vehicles, civil constructions, and deep-sea engineering equipments to satisfy special functional requirements due to their outstanding bending rigidity, high strength-weight and stiffness-weight ratios, excellent vibration characteristics, and good fatigue properties. The vibration results of laminated beams, plates, and shells are far from complete. It is necessary and of great significance to develop a unified, efficient, and accurate method which is capable of universally dealing with laminated beams, plates, and shells with general boundary conditions. Furthermore, a systematic, comprehensive, and up-to-date monograph which contains vibration results of isotropic and laminated beams, plates, and shells with various lamination schemes and general boundary conditions would be highly desirable and useful for the senior undergraduate and postgraduate students, teachers, engineers, and individual researchers in this field.

In view of these apparent voids, the present monograph presents an endeavor to complement the vibration analysis of laminated beams, plates, and shells. The title, *Structural Vibration: A Uniform Accurate Solution for Laminated Beams, Plates and Shells with General Boundary Conditions*, illustrates the main aim of this book, namely:

(1) To develop an accurate semi-analytical method which is capable of dealing with the vibrations of laminated beams, plates, and shells with arbitrary lamination schemes and general boundary conditions including classical boundaries, elastic supports and their combinations, aiming to provide a unified and reasonable accurate alternative to other analytical and numerical techniques.
(2) To provide a summary of known results of laminated beams, plates, and shells with various lamination schemes and general boundary conditions, which may serve as benchmark solutions for the future research in this field.

The book is organized into eight chapters. Fundamental equations of laminated shells in the framework of classical shell theory and shear deformation shell theory are derived in detail, including the kinematic relations, stress–strain relations and stress resultants, energy functions, governing equations, and boundary conditions. The corresponding fundamental equations of laminated beams and plates are specialized from the shell ones. Following the fundamental equations, a unified modified Fourier series method is developed. Then both strong and weak form solution procedures are realized and established by combining the fundamental equations and the modified Fourier series method. Finally, numerous vibration results are presented for isotropic, orthotropic, and laminated beams, plates, and shells with various geometry and material parameters, different lamination schemes and different boundary conditions including the classical boundaries, elastic ones, and their combinations. Summarizing, the work is arranged as follows:

The theories of linear vibration of laminated beams, plates, and shells are well established. In this regard, Chap. 1 introduces the fundamental equations of laminated beams, plates, and shells in the framework of classical shell theory and the first-order shear deformation shell theory without proofs.

Chapter 2 presents a modified Fourier series method which is capable of dealing with vibrations of laminated beams, plates, and shells with general boundary conditions. In the modified Fourier series method, each displacement of a laminated beam, plate, or shell, regardless of boundary conditions, is invariantly expressed as a new form of trigonometric series expansions in which several supplementary terms are introduced to ensure and accelerate the convergence of the series expansion. Then one can seek the solutions either in strong form solution procedure or the weak form one. These two solution procedures are fully illustrated in this chapter.

Chapters 3–8 deal with laminated beams, plates, and cylindrical, conical, spherical and shallow shells, respectively. In each chapter, corresponding fundamental equations in the framework of classical and shear deformation theories for the general dynamic analysis are developed first, which can be useful for potential readers. Following the fundamental equations, numerous free vibration results are presented for various configurations including different boundary conditions, laminated sequences, and geometry and material properties.

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Contents

Chapter 1
Fundamental Equations of Laminated Beams, Plates and Shells

Beams, plates and shells are named according to their size or/and shape features. Shells have all the features of plates except an additional one-curvature (Leissa 1969, 1973). Therefore, the plates, on the other hand, can be viewed as special cases of shells having no curvature. Beams are one-dimensional counterparts of plates (straight beams) or shells (curved beams) with one dimension relatively greater in comparison to the other two dimensions. This chapter introduces the fundamental equations (including kinematic relations, stress-strain relations and stress resultants, energy functions, governing equations and boundary conditions) of laminated shells in the framework of the classical shell theory (CST) and the shear deformation shell theory (SDST) without proofs due to the fact that they have been well established. The corresponding equations of laminated beams and plates are specialized from the shell ones.

1.1 Three-Dimensional Elasticity Theory in Curvilinear Coordinates

Consider a three-dimensional (3D) shell segment with total thickness h as shown in Fig. 1.1, a 3D orthogonal coordinate system (α, β and z) located on the middle surface is used to describe the geometry dimensions and deformations of the shell, in which co-ordinates along the meridional, circumferential and normal directions are represented by α, β and z, respectively. R_α and R_β are the mean radii of curvature in the α and β directions on the middle surface ($z = 0$). U, V and W separately indicate the displacement variations of the shell in the α, β and z directions. The strain-displacement relations of the three-dimensional theory of elasticity in orthogonal curvilinear coordinate system are (Leissa 1973; Soedel 2004; Carrera et al. 2011):

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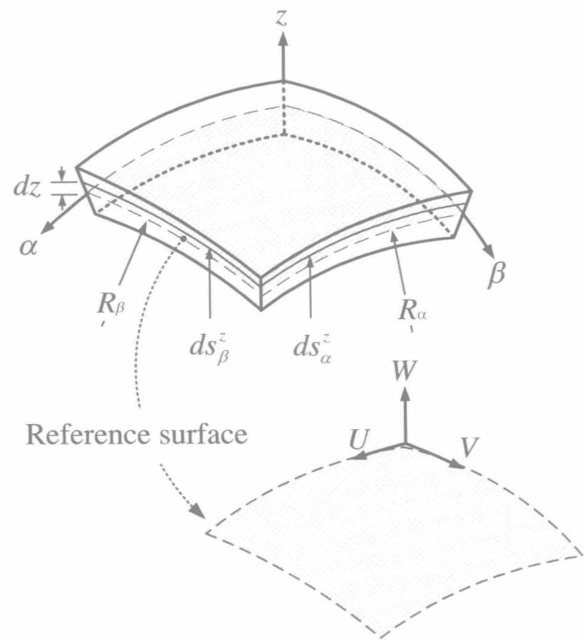


Fig. 1.1 Notations in shell coordinate system (α, β and z)

$$
\begin{aligned}
\varepsilon_\alpha &= \frac{1}{(1+z/R_\alpha)}\left(\frac{1}{A}\frac{\partial U}{\partial \alpha}+\frac{V}{AB}\frac{\partial A}{\partial \beta}+\frac{W}{R_\alpha}\right)\\
\varepsilon_\beta &= \frac{1}{(1+z/R_\beta)}\left(\frac{U}{AB}\frac{\partial B}{\partial \alpha}+\frac{1}{B}\frac{\partial V}{\partial \beta}+\frac{W}{R_\beta}\right)\\
\varepsilon_z &= \frac{\partial W}{\partial z}\\
\gamma_{\alpha\beta} &= \frac{A(1+z/R_\alpha)}{B(1+z/R_\beta)}\frac{\partial}{\partial \beta}\left[\frac{U}{A(1+z/R_\alpha)}\right]\\
&\quad+\frac{B(1+z/R_\beta)}{A(1+z/R_\alpha)}\frac{\partial}{\partial \alpha}\left[\frac{V}{B(1+z/R_\beta)}\right]\\
\gamma_{\alpha z} &= \frac{1}{A(1+z/R_\alpha)}\frac{\partial W}{\partial \alpha}+A(1+z/R_\alpha)\frac{\partial}{\partial z}\left[\frac{U}{A(1+z/R_\alpha)}\right]\\
\gamma_{\beta z} &= \frac{1}{B(1+z/R_\beta)}\frac{\partial W}{\partial \beta}+B(1+z/R_\beta)\frac{\partial}{\partial z}\left[\frac{V}{B(1+z/R_\beta)}\right]
\end{aligned}
\tag{1.1a–f}
$$

where the quantities A and B are the Lamé parameters of the shell. They are determined by the shell characteristics and the selected orthogonal coordinate system. The detail definitions of them are given in Sect. 1.4. The lengths in the α and β directions of the shell segment at distance dz from the shell middle surface are (see Fig. 1.1):

$$ds_\alpha^z = A\left(1 + \frac{z}{R_\alpha}\right)d\alpha \quad ds_\beta^z = B\left(1 + \frac{z}{R_\beta}\right)d\beta \tag{1.2}$$

The above equations contain the fundamental strain-displacement relations of a 3D body in curvilinear coordinate system. They are specialized to those of CST and FSDT by introducing several assumptions and simplifications.

1.2 Fundamental Equations of Thin Laminated Shells

According to Eq. (1.1), it can be seen that the 3D strain-displacement equations of a shell are rather complicated when written in curvilinear coordinate system. Typically, researchers simplify the 3D shell equations into the 2D ones by making certain assumptions to eliminate the coordinate in the thickness direction. Based on different assumptions and simplifications, various sub-category classical theories of thin shells were developed, such as the Reissner-Naghdi's linear shell theory, Donner-Mushtari's theory, Flügge's theory, Sanders' theory and Goldenveizer-Novozhilov's theory, etc. In this book, we focus on shells composed of arbitrary numbers of composite layers which are bonded together rigidly. When the total thickness of a laminated shell is less than 0.05 of the wavelength of the deformation mode or radius of curvature, the classical theories of thin shells originally developed for single-layered isotropic shells can be readily extended to the laminated ones. Leissa (1973) showed that most thin shell theories yield similar results. In this section, the fundamental equations of the Reissner-Naghdi's linear shell theory are extended to thin laminated shells due to that it offers the simplest, the most accurate and consistent equations for laminated thin shells (Qatu 2004).

1.2.1 Kinematic Relations

In the classical theory of thin shells, the four assumptions made by Love (1944) are universally accepted to be valid for a *first approximation shell theory* (Rao 2007):

1. *The thickness of the shell is small compared with the other dimensions.*
2. *Strains and displacements are sufficiently small so that the quantities of second- and higher-order magnitude in the strain-displacement relations may be neglected in comparison with the first-order terms.*
3. *The transverse normal stress is small compared with the other normal stress components and may be neglected.*
4. *Normals to the undeformed middle surface remain straight and normal to the deformed middle surface and suffer no extension.*

The first assumption defines that the shell is thin enough so that the deepness terms z/R_α and z/R_β can be neglected compared to unity in the strain-displacement relations (i.e., $z/R_\alpha \ll 1$ and $z/R_\beta \ll 1$). The second assumption ensures that the differential equations will be linear. The fourth assumption is also known as Kirchhoff's hypothesis. This assumption leads to zero transverse shear strains and zero transverse normal strain ($\gamma_{\alpha z} = 0$, $\gamma_{\beta z} = 0$ and $\varepsilon_z = 0$). Taking these assumptions into consideration, the 3D strain-displacement relations of shells in orthogonal curvilinear coordinate system can be reduced to those of 2D classical thin shells as:

$$\begin{aligned} \varepsilon_\alpha &= \frac{1}{A}\frac{\partial U}{\partial \alpha} + \frac{V}{AB}\frac{\partial A}{\partial \beta} + \frac{W}{R_\alpha} \\ \varepsilon_\beta &= \frac{U}{AB}\frac{\partial B}{\partial \alpha} + \frac{1}{B}\frac{\partial V}{\partial \beta} + \frac{W}{R_\beta} \\ \gamma_{\alpha\beta} &= \frac{A}{B}\frac{\partial}{\partial \beta}\left[\frac{U}{A}\right] + \frac{B}{A}\frac{\partial}{\partial \alpha}\left[\frac{V}{B}\right] \end{aligned} \tag{1.3a–c}$$

According to the Kirchhoff hypothesis, the displacement variations in the α, β and z directions are restricted to the following linear relationships (Leissa 1973):

$$\begin{aligned} U(\alpha,\beta,z) &= u(\alpha,\beta) + z\phi_\alpha(\alpha,\beta) \\ V(\alpha,\beta,z) &= v(\alpha,\beta) + z\phi_\beta(\alpha,\beta) \\ W(\alpha,\beta,z) &= w(\alpha,\beta) \end{aligned} \tag{1.4}$$

where u, v and w are the displacement components on the middle surface in the α, β and z directions. ϕ_α, ϕ_β represent the rotations of transverse normal respect to β- and α-axes, respectively. They are determined by substituting Eq. (1.4) into Eq. (1.1e, f) and letting $\gamma_{\alpha z} = 0$, $\gamma_{\beta z} = 0$, i.e.:

$$\phi_\alpha = \frac{u}{R_\alpha} - \frac{1}{A}\frac{\partial w}{\partial \alpha} \quad \phi_\beta = \frac{v}{R_\beta} - \frac{1}{B}\frac{\partial w}{\partial \beta} \tag{1.5}$$

Substituting Eqs. (1.4) and (1.5) into Eq. (1.3), the strain-displacement relations of thin shells can be rewritten as:

$$\begin{aligned} \varepsilon_\alpha &= \varepsilon_\alpha^0 + z\chi_\alpha \\ \varepsilon_\beta &= \varepsilon_\beta^0 + z\chi_\beta \\ \gamma_{\alpha\beta} &= \gamma_{\alpha\beta}^0 + z\chi_{\alpha\beta} \end{aligned} \tag{1.6}$$

where ε_α^0, ε_β^0 and $\gamma_{\alpha\beta}^0$ denote the normal and shear strains in the middle surface. χ_α, χ_β and $\chi_{\alpha\beta}$ are the corresponding curvature and twist changes. They are written in terms of shell displacements u, v and w as:

$$\begin{aligned}
\varepsilon_\alpha^0 &= \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{v}{AB}\frac{\partial A}{\partial \beta} + \frac{w}{R_\alpha} \\
\varepsilon_\beta^0 &= \frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{u}{AB}\frac{\partial B}{\partial \alpha} + \frac{w}{R_\beta} \\
\gamma_{\alpha\beta}^0 &= \frac{1}{A}\frac{\partial v}{\partial \alpha} - \frac{u}{AB}\frac{\partial A}{\partial \beta} + \frac{1}{B}\frac{\partial u}{\partial \beta} - \frac{v}{AB}\frac{\partial B}{\partial \alpha} \\
\chi_\alpha &= \frac{1}{A}\frac{\partial \phi_\alpha}{\partial \alpha} + \frac{\phi_\beta}{AB}\frac{\partial A}{\partial \beta} \\
\chi_\beta &= \frac{1}{B}\frac{\partial \phi_\beta}{\partial \beta} + \frac{\phi_\alpha}{AB}\frac{\partial B}{\partial \alpha} \\
\chi_{\alpha\beta} &= \frac{1}{A}\frac{\partial \phi_\beta}{\partial \alpha} - \frac{\phi_\alpha}{AB}\frac{\partial A}{\partial \beta} + \frac{1}{B}\frac{\partial \phi_\alpha}{\partial \beta} - \frac{\phi_\beta}{AB}\frac{\partial B}{\partial \alpha}
\end{aligned} \tag{1.7}$$

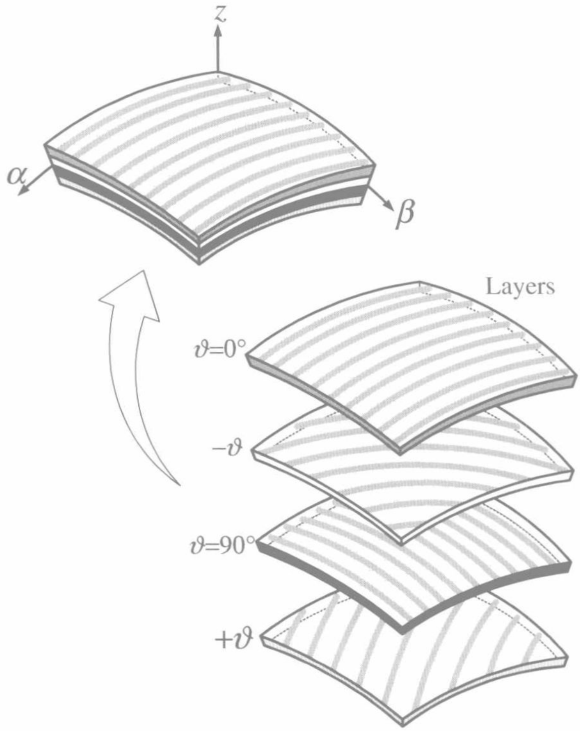
Equation (1.7) constitutes the strain-displacement relations of a thin shell in curvilinear coordinates.

1.2.2 Stress-Strain Relations and Stress Resultants

With the development of new industries and modern processes, composite materials are extensively used in many fields of modern engineering practices such as aircraft and spacecraft, civil constructions and deep-ocean engineering to satisfy special functional requirements due to their outstanding bending rigidity, high strength-weight and stiffness-weight ratios, excellent vibration characteristics and good fatigue properties. For instance, more than 20 % of the A380's airframe is composite materials.

Typically, composite materials are made of reinforcement material distributed in matrix material. There commonly exist three types of composite materials (Reddy 2003; Ye 2003): (1) fiber composites, in which the reinforcements are in the form of fibers. The fibers can be continuous or discontinuous, unidirectional, bidirectional, woven or randomly distributed; (2) particle composites, which are composed of macro size particles of reinforcement in a matrix of another, such as concrete; (3) laminated composites, which consist of layers of various materials, including composites of the first two types. As for many other kinds of composite structures, beams, plates and shells composed of arbitrary numbers of unidirectional fiber reinforced layers with different fiber orientations (see Fig. 1.2) are most frequently used in the engineering applications and are the mutual concern of researchers in this field as well. In such cases, by appropriately orientating the fibers in each lamina of the structure, desired strength and stiffness parameters can be achieved. As a consequence, this book is devoted to the vibration analysis of laminated beams, plates and shells made of this type of laminated composite.

Fig. 1.2 A laminated shell made up of composite layers with different fiber orientations



In this section we primarily study the stress-strain relations of a unidirectional fiber reinforced layer, which is the basic building block of a composite laminated structure. A unidirectional fiber reinforced layer can be treated as an orthotropic material whose material symmetry planes are parallel and transverse to the fiber direction (Reddy 2003). See Fig. 1.3, suppose the laminated shell is constructed by N unidirectional fiber-reinforced layers which are bonded together rigidly. The principal coordinates of the composite material in the kth layer are denoted by 1, 2 and 3, in which the coordinate axes 1 and 2 are taken to be parallel and transverse to the fiber orientation. The 3 axis is parallel to the normal direction of the shell. The angle between the material axis 1 (or 2) and the α axis (or β) is denoted by ϑ^k and Z_{k+1} and Z_k are the distances from the top surface and the bottom surface of the layer to the referenced middle surface, respectively. Thus, according to generalized Hooke's law, the corresponding stress-strain relations in the kth layer of the laminated shell can be written as:

$$\begin{Bmatrix} \sigma_\alpha \\ \sigma_\beta \\ \tau_{\alpha\beta} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}^k} & \overline{Q_{12}^k} & \overline{Q_{16}^k} \\ \overline{Q_{12}^k} & \overline{Q_{22}^k} & \overline{Q_{26}^k} \\ \overline{Q_{16}^k} & \overline{Q_{26}^k} & \overline{Q_{66}^k} \end{bmatrix} \begin{Bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \gamma_{\alpha\beta} \end{Bmatrix}_k \tag{1.8}$$