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实用符号动力学与混沌

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序　　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任

中国科学院院士，北京大学教授

王恩哥

2010 年 5 月于燕园

Preface for the Second Edition

The authors are very happy to see the second revised edition of this monograph appearing in a joint effort of the Peking University Press and the World Scientific Publishing Co., Inc., Singapore. The printing of this book in Beijing greatly increases the availability of the book to readers within China.

The hard work of revising the text and figures was mainly done by Dr. Wei-mou Zheng. The revisions concern mainly the application of symbolic dynamics to ordinary differential equations via constructing two-dimensional symbolic dynamics of the corresponding Poincare maps for the ODEs. I would like to emphasize once more that the way of getting into two-dimensional maps and ODEs was paved by Dr. Zheng almost single-handed since the early 1990s. This approach significantly extends the qualitative study of ODEs by numerical means under the guidance of topology, as symbolic dynamics is topological in nature. However, many difficult problems remain unsolved regarding the relation of symbolic dynamics to knot theory and formal language theory. These problems are only touched briefly in the last chapters with the hope to inspire further studies. Criticism and feedback from the readers are mostly welcome.

Special thanks go to Ms. Xiao-hong Chen and Mr. Zhao-yuan Yin from the Peking University Press who have been very patient and helpful during the yearly long process of preparing the second edition.

Bai-lin Hao
1 August 2014, Beijing

Preface

Symbolic dynamics is a coarse-grained description of dynamics. It has been a long-studied chapter of the mathematical theory of dynamical systems, but its abstract formulation has kept away many practitioners of physical sciences and engineering from appreciating its simplicity, beauty, and power. At the same time, symbolic dynamics provides almost the only rigorous way to understand global systematics of periodic and, especially, chaotic motion in dynamical systems. In a sense, everyone who enters the field of chaotic dynamics should begin with the study of symbolic dynamics. However, this has not been an easy job for non-mathematicians to accomplish. On one hand, the method of symbolic dynamics has been developed to such an extend that it may well become a practical tool in studying chaotic dynamics, both on computers and in laboratories. On the other hand, most of the existing literature on symbolic dynamics is mathematics-oriented. This book is an attempt at partially filling up this apparent gap by emphasizing the applied aspects of symbolic dynamics without pretending to mathematical rigor.

No previous knowledge of dynamical systems theory is required in order to read this book. The mathematics used does not exceed basic calculus taught at engineering schools. Starting from simple one-dimensional maps, we go through circle maps and two-dimensional maps to arrive at numerical study of some ordinary differential equations under the guidance of symbolic dynamics. Instead of numbered formal definitions and proofs, the reader will find many examples and figures which embody the idea and method of symbolic dynamics. We have also included two kinds of computer programs in the book. A few short BASIC programs, implementing one or another procedure just described in the text, may help the reader to understand the method thoroughly. These programs may be considered part of the text or be skipped at first reading. Some more sophisticated programs written in C language are listed in the Appendix. These may be easily modified to treat systems not studied in the book and are aimed at the research need of some readers.

The book is organized as follows.

Chapter 1 is a brief introduction to the general idea of symbolic dynamics.

Chapter 2 studies symbolic dynamics of unimodal, one-hump map of the interval, the simplest yet very rich dynamical system. Recent development of the applied direction of symbolic dynamics, what we call *Applied Symbolic Dynamics*[1], has drawn much inspiration from the unimodal map.

Chapter 3 studies one-dimensional maps of the interval with multiple critical points or points of discontinuity. These maps occur naturally in many applications and they are needed in understanding ordinary differential equations with dissipation, i.e., those equations that allow the existence of strange attractors.

Chapter 4 is devoted to symbolic dynamics of circle maps as the simplest model of physical systems with competing frequencies. The symbolic dynamics of circle maps possesses some specific features absent in interval maps. The knowledge is of much help in the study of periodically forced systems described by ordinary differential equations.

Chapter 5 develops symbolic dynamics of two-dimensional maps. A mostly analytical study will be carried out on some piecewise linear 2D maps, providing hints and clues to deal with more general maps. Being a major progress in the last decade, the symbolic dynamics of 2D maps are essential for the symbolic dynamics study of differential equations as the usual Poincaré maps are two-dimensional.

Chapter 6 focuses on numerical study of ordinary differential equations under the guidance of symbolic dynamics. This mostly "experimental", at present time, approach is capable to provide some global understanding of the system that cannot be reached neither by purely analytical means nor by numerical methods alone. For example, we are able to list and locate all short periodic orbits, stable as well as unstable ones, in fairly large region of the parameter space for the Lorenz model and for the periodically forced Brusselator.

Chapter 7 provides the complete solution of a counting problem which is related to but goes beyond symbolic dynamics, namely, the number of periods

[1] The name was suggested by Ian Percival during his visit to Beijing in 1988.

in continuous maps of the interval with multiple critical points. It also contains partial result for maps with discontinuity.

Symbolic sequences fit naturally into the framework of formal languages. The well-established machinery of formal language theory is of great help in the classification of orbital types and their complexity in 1D maps. Many new results have been obtained in the last few years. Since there has just appeared a nice book on this topic (Xie [B1996]), we confine ourselves to a brief summary of this line of study in Chapter 8.

Chapter 9 discusses the relation of symbolic dynamics with another topological approach to dynamical systems, namely, the study of periodic orbits as knots and links. Although knots and links are objects in three-dimensional space, many problems may be posed using 1D maps. Instead of presenting finished results, this Chapter, we hope, may inspire some new research interest.

There is a quite detailed *Table of Contents* and an *Index*, which may help the reader to see the scope of the book and to look for interested topics.

This book is not a mathematical treatise. However, we hope mathematicians may also find a few new tricks or some interesting applications of their abstract theory, including some contributions of Chinese scientists that are not readily available elsewhere.

Some years ago one of the authors published a book entitled *Elementary Symbolic Dynamics and Chaos in Dissipative Systems* (Hao [B1989]). So many new results and a much deeper understanding have been achieved since then that the present monograph can hardly be considered as an update of the 1989 book. We refer to that book only in a few occasions when something not directly related to symbolic dynamics is touched.

A few words about the reference convention in this book. References to the list at the end of the book are given as, e.g., Poincaré [B1899], the capital B indicating the first part of the References on "Books', or Xie [1996], addressing a paper in the second part "Papers" of the References. A few citations to sources not included in the References are given in footnotes. No efforts have been made to clarify the chronology of one or another statement. In a rapidly expanding and interdisciplinary field like *Chaos* there have been many rediscoveries of important facts. It is better to leave these to the historians of science.

Our own work on chaos has been partially supported by the Division of Mathematics and Physics, Academia Sinica (1983–1985), the Chinese Natural Science Foundation (1986–1988), and the Project on Nonlinear Science (1990–1995). In 1989 Sun Microsystems, Inc., donated a Sun Workstation to the Nonlinear Dynamics Group at the Institute of Theoretical Physics, Academia Sinica, Beijing. Wolfram Research, Inc., donated the Sun version of Mathematica 1.0 software. These were great help to our earlier research. Later on the Local Network of ITP and the State Key Laboratory on Scientific and Engineering Computation of Academia Sinica provided computing facilities. We thank all these organizations for their support.

We would also like to acknowledge the inspiring discussion and interaction with many colleagues and former students over the years. An incomplete list includes Shi-gang Chen, Ming-zhou Ding, Hai-ping Fang, Jun-xian Liu, Li-sha Lu, Shou-li Peng, Zuo-bin Wu, Fa-gen Xie, Hui-ming Xie, and Wan-zhen Zeng.

The text was typeset by the authors using LaTeX of Leslie Lamport with indispensable help from the staff of World Scientific Publishing Co. Pte. Ltd. In particular, we would like to thank Dr. K. K. Phua, the Editor-in-Chief, and Dr. Lock-Yee Wong, the editor, for their patience and advice.

Bai-lin Hao and Wei-mou Zheng

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