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Fritz Haake

# Quantum Signatures of Chaos

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## Preface to the Third Edition

Nine years have passed since I dispatched the second edition, and the book still appears to be in demand. The time may be ripe for an update.

As the perhaps most conspicuous extension, I describe the understanding of universal spectral fluctuations recently reached on the basis of periodic-orbit theory. To make the presentation of those semiclassical developments selfcontained, I decided to underpin them by a new short chapter on classical Hamiltonian mechanics. Inasmuch as the semiclassical theory not only draws inspiration from the nonlinear sigma model but actually aims at constructing that model in terms of periodic orbits, it appeared indicated to make small additions to the previous treatment within the chapter on superanalysis.

Less voluminous but as close to my heart are additions to the chapter on level dynamics which close previous gaps in that approach to spectral universality.

It was a pleasant duty to pay my respect to colleagues in our Transregio-Sonderforschungsbereich, Martin Zirnbauer, Alex Altland, Alan Huckleberry, and Peter Heinzner, by including a short account of their beautiful work on nonstandard symmetry classes.

The chapter on random matrices has not been expanded in proportion to the development of the field but now includes an up-to-date treatment of an old topic in algebra, Newton's relations, to provide a background to the Riemann-Siegel look-alike of semiclassical periodic-orbit theory.

The chapters on level clustering, localization, and dissipation are similarly preserved. I disciplined myself to just adding an occasional reference to recent work and to cutting some stuff of lesser relative importance.

There was the temptation to rewrite the introduction, to no avail. Only a few additional words here and there announce new topics taken up in the main text. So that chapter stands as a relic from the olden days when quantum chaos was just beginning to form as a field.

Encouragement and help has come from Thomas Guhr, Dominique Spehner, Martin Zirnbauer, and, as always, from Hans-Jürgen Sommers and Marek Kuś.

I owe special gratitude to Alex Altland, Peter Braun, Stefan Heusler, and Sebastian Müller. They have formed a dream team sharing search and finding, suffering and joy, row and laughter.

Essen  
August 2009

F. Haake

## Preface to the Second Edition

The warm reception of the first edition, as well as the tumultuous development of the field of quantum chaos have tempted me to rewrite this book and include some of the important progress made during the past decade.

Now we know that quantum signatures of chaos are paralleled by wave signatures. Whatever is undergoing wavy space-time variations, be it sound, electromagnetism, or quantum amplitudes, each shows exactly the same manifestations of chaos. The common origin is nonseparability of the pertinent wave equation; that latter “definition” of chaos, incidentally, also applies to classical mechanics if we see the Hamilton–Jacobi equation as the limiting case of a wave equation. At any rate, drums, concert halls, oscillating quartz blocks, microwave and optical oscillators, electrons moving ballistically or with impurity scattering through mesoscopic devices all provide evidence and data for wave or quantum chaos. All of these systems have deep analogies with billiards, much as the latter may have appeared of no more than academic interest only a decade ago. Of course, molecular, atomic, and nuclear spectroscopy also remain witnesses of chaos, while the chromodynamic innards of nucleons are beginning to attract interest as methods of treatment become available.

Of the considerable theoretical progress lately achieved, the book focuses on the deeper statistical exploitation of level dynamics, improved control of semiclassical periodic-orbit expansions, and superanalytic techniques for dealing with various types of random matrices. These three fields are beginning, independently and in conjunction, to generate an understanding of why certain spectral fluctuations in classically nonintegrable systems are universal and why there are exceptions.

Only the rudiments of periodic-orbit theory and superanalysis appeared in the first edition. More could not have been included here had I not enjoyed the privilege of individual instruction on periodic-orbit theory by Jon Keating and on superanalysis by Hans–Jürgen Sommers and Yan Fyodorov. Hans–Jürgen and Yan have even provided their lecture notes on the subject. While giving full credit and expressing my deep gratitude to these three colleagues, I must bear all blame for blunders.

Reasonable limits of time and space had to be respected and have forced me to leave out much interesting material such as chaotic scattering and the semiclassical art of getting spectra for systems with mixed phase spaces. Equally regrettably, no justice could be done here to the wealth of experiments that have now been

performed, but I am happy to see that gap filled by my much more competent colleague Hans-Jürgen Stöckmann.

Incomplete as the book must be, it now contains more material than fits into a single course in quantum chaos theory. In some technical respects, it digs deeper than general introductory courses would go. I have held on to my original intention though, to provide a self-contained presentation that might help students and researchers to enter the field or parts thereof.

The number of co-workers and colleagues from whose knowledge and work I could draw has increased considerably over the years. Having already mentioned Yan Fyodorov, Jon Keating, and Hans-Jürgen Sommers, I must also express special gratitude to my partner and friend Marek Kuś whose continuing help was equally crucial. My thanks for their invaluable influence go to Sergio Albeverio, Daniel Braun, Peter Braun, Eugene Bogomolny, Chang-qi Cao, Dominique Delande, Bruno Eckhardt, Pierre Gaspard, Sven Gnutzmann, Peter Goetsch, Siegfried Grossmann, Martin Gutzwiller, Gregor Hackenbroich, Alan Huckleberry, Micha Kolobov, Pavel Kurasov, Robert Littlejohn, Nils Lehmann, Jörg Main, Alexander Mirlin, Jan Mostowski, Alfredo Ozorio de Almeida, Piotr Peplowski, Ravi Puri, Jonathan Robbins, Kazik Rzążewski, Henning Schomerus, Carsten Seeger, Thomas Seligmann, Frank Steiner, Hans-Jürgen Stöckmann, Jürgen Vollmer, Joachim Weber, Harald Wiedemann, Christian Wiele, Günter Wunner, Dmitri Zaitsev, Kuba Zakrzewski, Martin Zirnbauer, Marek Zukowski, Wojtek Zurek, and, last but not at all least, Karol Życzkowski.

In part this book is an account of research done within the Sonderforschungsbereich “Unordnung und Große Fluktuationen” of the Deutsche Forschungsgemeinschaft. This fact needs to be gratefully acknowledged, since coherent long-term research of a large team of physicists and mathematicians could not be maintained without the generous funding we have enjoyed over the years through our Sonderforschungsbereich.

Times do change. Like many present-day science authors I chose to pick up  $\text{\LaTeX}$  and key all changes and extensions into my little machine myself. As usually happens when learning a new language, the beginning is all effort, but one eventually begins to enjoy the new mode of expressing oneself. I must thank Peter Gerwinski, Heike Haschke, and Rüdiger Oberhage for their infinite patience in getting me going.

Essen  
July 2000

F. Haake

# Preface to the First Edition

More than 60 years after its inception, quantum mechanics is still exerting fascination on every new generation of physicists. What began as the scandal of non-commuting observables and complex probability amplitudes has turned out to be the universal description of the micro-world. At no scale of energies accessible to observation have any findings emerged that suggest violation of quantum mechanics.

Lingering doubts that some people have held about the universality of quantum mechanics have recently been resolved, at least in part. We have witnessed the serious blow dealt to competing hidden-variable theories by experiments on correlations of photon pairs. Such correlations were found to be in conflict with any local deterministic theory as expressed rigorously by Bell's inequalities. – Doubts concerning the accommodation of dissipation in quantum mechanics have also been eased, in much the same way as in classical mechanics. Quantum observables can display effectively irreversible behavior when they are coupled to an appropriate environmental system containing many degrees of freedom. Even in closed quantum systems with relatively few degrees of freedom, behavior resembling damping is possible, provided the system displays chaotic motion in the classical limit.

It has become clear that the relative phases of macroscopically distinguishable states tend, in the presence of damping, to become randomized in exceedingly short times; that remains true even when the damping is so weak that it is hardly noticeable for quantities with a well-defined classical limit. Consequently, a superposition (in the quantum sense) of different readings of a macroscopic measuring device would, even if one could be prepared momentarily, escape observation due to its practically instantaneous decay. While this behavior was conjectured early in the history of quantum mechanics it is only recently that we have been able to see it explicitly in rigorous solutions for specific model systems.

There are many intricacies of the classical limit of quantum mechanics. They are by no means confined to abrupt decay processes or infinitely rapid oscillations of probability amplitudes. The classical distinction between regular and chaotic motion, for instance, makes itself felt in the semiclassical regime that is typically associated with high degrees of excitation. In that regime quantum effects like the discreteness of energy levels and interference phenomena are still discernible while the correspondence principle suggests the onset of validity of classical mechanics.



The semiclassical world, which is intermediate between the microscopic and the macroscopic, is the topic of this book. It will deal with certain universal modes of behavior, both dynamical and spectral, which indicate whether their classical counterparts are regular or chaotic. Conservative as well as dissipative systems will be treated.

The area under consideration often carries the label “quantum chaos”. It is a rapidly expanding one and therefore does not yet allow for a definite treatment. The material presented reflects subjective selections. Random-matrix theory will enjoy special emphasis. A possible alternative would have been to make current developments in periodic-orbit theory the backbone of the text. Much as I admire the latter theory for its beauty and its appeal to classical intuition, I do not understand it sufficiently well that I can trust myself to do it justice. With more learning, I might yet catch up and find out how to relate spectral fluctuations on an energy scale of a typical level spacing to classical properties. There are other regrettable omissions. Most notable among these may be the ionization of hydrogen atoms by microwaves, for which convergence of theory and experiment has been achieved recently. Also too late for inclusion is the quantum aspect of chaotic scattering, which has seen such fine progress in the months between the completion of the manuscript and the appearance of this book.

This book grew out of lectures given at the universities of Essen and Bochum. Most of the problems listed at the end of each chapter have been solved by students attending those lectures. The level aimed at was typical of a course on advanced quantum mechanics. The book accordingly assumes the reader to have a good command of the elements of quantum mechanics and statistical mechanics, as well as some background knowledge of classical mechanics. A little acquaintance with classical nonlinear dynamics would not do any harm either.

I could not have gone through with this project without the help of many colleagues and coworkers. They have posed many of the questions dealt with here and provided most of the answers. Perhaps more importantly, they have, within the theory group in Essen, sustained an atmosphere of dedication and curiosity, from which I keep drawing knowledge and stimulus. I can only hope that my young coworkers share my own experience of receiving more than one is able to give. I am especially indebted to Michael Berry, Oriol Bohigas, Giulio Casati, Boris Chirikov, Barbara Dietz, Thomas Dittrich, Mario Feingold, Shmuel Fishman, Dieter Forster, Robert Graham, Rainer Grobe, Italo Guarneri, Klaus-Dieter Harms, Michael Höhnerbach, Ralf Hübner, Felix Israilev, Marek Kuś, Georg Lenz, Maciej Lewenstein, Madan Lal Mehta, Jan Mostowski, Akhilesh Pandey, Dirk Saher, Rainer Scharf, Petr Šeba, Dima Shepelyansky, Uzy Smilansky, Hans-Jürgen Sommers, Dan Walls, and Karol Życzkowski.

Angela Lahee has obliged me by smoothening out some clumsy Teutonisms and by her careful editing of the manuscript. My secretary, Barbara Sacha, deserves a big thank you for keying version upon version of the manuscript into her computer.

My friend and untiring critic Roy Glauber has followed this work from a distance and provided invaluable advice. – I am grateful to Hermann Haken for his invitation to contribute this book to his series in synergetics, and I am all the more honored

since it can fill but a tiny corner of Haken's immense field. However, at least Chap. 8 does bear a strong relation to several other books in the series inasmuch as it touches upon adiabatic-elimination techniques and quantum stochastic processes. Moreover, that chapter represents variations on themes I learned as a young student in Stuttgart, as part of the set of ideas which has meanwhile grown to span the range of this series. The love of quantum mechanics was instilled in me by Hermann Haken and his younger colleagues, most notably Wolfgang Weidlich, as they were developing their quantum theory of the laser and thus making the first steps towards synergetics.

Essen  
January 1991

F. Haake

## Foreword to the First Edition

The interdisciplinary field of synergetics grew out of the desire to find general principles that govern the spontaneous formation of ordered structures out of microscopic chaos. Indeed, large classes of classical and quantum systems have been found in which the emergence of ordered structures is governed by just a few degrees of freedom, the so-called order parameters. But then a surprise came with the observation that a few degrees of freedom may cause complicated behavior, nowadays generally subsumed under the title “deterministic chaos” (not to be confused with microscopic chaos, where many degrees of freedom are involved). One of the fundamental problems of chaos theory is the question of whether deterministic chaos can be exhibited by quantum systems, which, at first sight, seem to show no deterministic behavior at all because of the quantization rules. To be more precise, one can formulate the question as follows: How does the transition occur from quantum mechanical properties to classical properties showing deterministic chaos?

Fritz Haake is one of the leading scientists investigating this field and he has contributed a number of important papers. I am therefore particularly happy that he agreed to write a book on this fascinating field of quantum chaos. I very much enjoyed reading the manuscript of this book, which is written in a highly lively style, and I am sure the book will appeal to many graduate students, teachers, and researchers in the field of physics. This book is an important addition to the Springer Series in Synergetics.

Stuttgart  
February 1991

H. Haken

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