CALCULUS

VOLUME I

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One-Variable Calculus, with an Introduction to Linear Algebra

SECOND EDITION

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PREFACE

Excerpts from the Preface to the First Edition

There seems to be no general agreement as to what should constitute a first course in calculus and analytic geometry. Some people insist that the only way to really understand calculus is to start off with a thorough treatment of the real-number system and develop the subject step by step in a logical and rigorous fashion. Others argue that calculus is primarily a tool for engineers and physicists; they believe the course should stress applications of the calculus by appeal to intuition and by extensive drill on problems which develop manipulative skills. There is much that is sound in both these points of view. Calculus is a deductive science and a branch of pure mathematics. At the same time, it is very important to remember that calculus has strong roots in physical problems and that it derives much of its power and beauty from the variety of its applications. It is possible to combine a strong theoretical development with sound training in technique; this book represents an attempt to strike a sensible balance between the two. While treating the calculus as a deductive science, the book does not neglect applications to physical problems. Proofs of all the important theorems are presented as an essential part of the growth of mathematical ideas; the proofs are often preceded by a geometric or intuitive discussion to give the student some insight into why they take a particular form. Although these intuitive discussions will satisfy readers who are not interested in detailed proofs, the complete proofs are also included for those who prefer a more rigorous presentation.

The approach in this book has been suggested by the historical and philosophical development of calculus and analytic geometry. For example, integration is treated before differentiation. Although to some this may seem unusual, it is historically correct and pedagogically sound. Moreover, it is the best way to make meaningful the true connection between the integral and the derivative.

The concept of the integral is defined first for step functions. Since the integral of a step function is merely a finite sum, integration theory in this case is extremely simple. As the student learns the properties of the integral for step functions, he gains experience in the use of the summation notation and at the same time becomes familiar with the notation for integrals. This sets the stage so that the transition from step functions to more general functions seems easy and natural.

Preface to the Second Edition

The second edition differs from the first in many respects. Linear algebra has been incorporated, the mean-value theorems and routine applications of calculus are introduced at an earlier stage, and many new and easier exercises have been added. A glance at the table of contents reveals that the book has been divided into smaller chapters, each centering on an important concept. Several sections have been rewritten and reorganized to provide better motivation and to improve the flow of ideas.

As in the first edition, a historical introduction precedes each important new concept, tracing its development from an early intuitive physical notion to its precise mathematical formulation. The student is told something of the struggles of the past and of the triumphs of the men who contributed most to the subject. Thus the student becomes an active participant in the evolution of ideas rather than a passive observer of results.

The second edition, like the first, is divided into two volumes. The first two thirds of Volume I deals with the calculus of functions of one variable, including infinite series and an introduction to differential equations. The last third of Volume I introduces linear algebra with applications to geometry and analysis. Much of this material leans heavily on the calculus for examples that illustrate the general theory. It provides a natural blending of algebra and analysis and helps pave the way for the transition from one-variable calculus to multivariable calculus, discussed in Volume II. Further development of linear algebra will occur as needed in the second edition of Volume II.

Once again I acknowledge with pleasure my debt to Professors H. F. Bohnenblust, A. Erdélyi, F. B. Fuller, K. Hoffman, G. Springer, and H. S. Zuckerman. Their influence on the first edition continued into the second. In preparing the second edition, I received additional help from Professor Basil Gordon, who suggested many improvements. Thanks are also due George Springer and William P. Ziemer, who read the final draft. The staff of the Blaisdell Publishing Company has, as always, been helpful; I appreciate their sympathetic consideration of my wishes concerning format and typography.

Finally, it gives me special pleasure to express my gratitude to my wife for the many ways she has contributed during the preparation of both editions. In grateful acknowledgment I happily dedicate this book to her.

T. M. A.

Pasadena, California
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