OPTIMAL CONTROL ENGINEERING WITH MATLAB

RAMI A. MAHER

Engineering Tools, Techniques and Tables

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To my students over the years

PREFACE

People aspire after the optimum when working, or doing tasks, or completing duties. However, writing a book about optimal control for engineering students would ask for careful treatment. This is deliberately because the subject is so wide-ranging and technically enriched and new branches are still emerging in this field. Furthermore, the optimal theory always finds new areas of challenges in different fields. For instance, the motor control is one of the newest areas in the bio-field.

As I lectured the optimal control subject over the years, graduate engineering students often asked the same questions: *What is the difference between an optimized controller and an optimal controller? How can engineers implement optimal controllers? Is there a bridge between conventional control design methods and the optimal control approaches? Is it worth learning this subject first, before studying the robust optimal control approaches? How does studying this subject help with the final project or with future control engineering practice? How the benefit of the MATLAB facilities for solving optimal controller design problems can be exploited?* These questions made me think of writing a book addressed to graduate engineering students to satisfy their engineering inquisitiveness.

This book has been written primarily for use as a first-year-graduate-level optimal control course. It covers the deterministic part of the optimal control theory. Essential theories and explanations of the various techniques are explained in a simple manner without rigorous mathematics but also without sacrificing accuracy. A sufficient number of worked examples and problems are included to highlight the engineering design concepts. Many of these examples are worked by utilizing simple and readable MATLAB programs. Simulink models are also used on one hand to provide and check solutions, and on the other hand, to ensure understanding of the implementation aspects.

The book consists of eight chapters. Chapter 1 introduces optimal control principles, essential definitions, and concepts of optimal controller design, and a mathematical review of static optimization. The modeling of optimal control problems for some engineering applications is finally presented. In chapter 2, the design of controllers based on parameter optimization via the inward and outward approaches is considered. The performance index as a measure of the quality of the required response and the resultant forms of the closed-loop are first considered. While the inward approach is briefly introduced, the outward is thoroughly discussed, and many illustrative examples are given. Control vector parameterization and genetic algorithm are also presented. To introduce the optimal control theory from a mathematical point of view, Chapter 3 reviews the approach of calculus of variations. The reader will sense the close relation between the material of the chapter and the optimal control framework presented in the next chapters.

Chapter 4 is intended to highlight the design concepts of optimal control based on calculus of variations. The necessary and sufficient conditions of optimality of various types of problems are given and several examples, which are illustrated with the aid of MATLAB programming, are presented. Furthermore, the chapter includes the approximate technique of optimal or suboptimal state control law and the numerical solution of the Two-Point Boundary Value Problem (TBVP) using the available MATLAB special functions. The very famous optimality principle of Pontryagin is presented in chapter 5, which covers the problem of constrained input or state imposed on the control systems. It includes the solutions of optimal time problems, the minimum control-effort problems, singular problems, and the problem of state variable inequality constraints. Analytic optimal solutions are presented, as well as the numerical gradient method that is applied to solve the state variable inequality problem.

Chapter 6 introduces the dynamic programming approach for both of discrete and continuous systems, where the "principle of optimality" introduced by Bellman is thoroughly discussed and illustrated. The parametric expansion approach is considered as a possible tool for solving the Hamilton-Jacobi-Bellman (HJB) equation (the necessary condition of optimality). In addition, differential dynamic programming (DDP) is included as an approach for nonlinear systems. Nevertheless, the results for the Linear Quadratic Regulator (LQR) approach are derived via the application of the optimality principle and the topic of linear quadratic optimal control is found in chapter 7. In chapter 7, the mathematical formulation of three basic control problems is addressed: the regulating problem, the output-regulating problem, and the tracking problem. The stability of the optimal solution, equivalent cost functions, solutions based on the Hamiltonian matrix are all considered in this chapter for both continuous-time and discrete-time LQR problems. The chapter closes with a new iterative linear quadratic regulator (ILQR) approach adopted for nonlinear systems.

Chapter 8 covers the optimal control solution based on one of the newest evolutionary techniques, the genetic programming (GP). The chapter includes a brief introduction for GP and details of an enhanced algorithm to solve the optimal problem. Both input unconstrained and input constrained optimal problems are considered.

Rami A. Maher

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Chapter 1

MATHEMATICAL BACKGROUND AND OPTIMAL PROBLEM MODELING

1.1. INTRODUCTION

One of the engineering challenges is to perform tasks in a best possible way with a small amount of energy and with a minimum number of resources. Nature of things makes the meaning of the word "*best*" very ambiguous and confusing. Moreover, tasks spread widely from the manufacturing of simple objects to complex systems. For instance, to obtain a *maximum* number of specified circle pieces out of a given rectangular sheet is a simple example. On the other hand, to control complicated dynamic systems like *minimizing* the miss distance between a pursuer missile and its target is a complex one. Therefore, the way, or approach to undertake one task is not necessarily preferable for other tasks. However, for a specific system or process, a precise definition of the demands, the available resources, and the physical constraints make engineers more comfortable and hopeful about finding the best, most favorable, most advantageous or *optimal* solution. The practical realization of this optimal solution is the next stage the engineer has to accomplish. During implementation, engineer encounters often many obstacles that oblige him to postulate a near or *sub-optimal* solution. It can be reached by modifying or discarding some of the customer demands without much loss.

The extreme (minimum or maximum) of a goal function, a cost function, performance index, or satisfying certain *optimal criterion* is the *optimization* process. Intuitively, a maximization problem can be put easily as minimization one and vice versa. The alternative usage of terminology to express almost the same meaning is due to the wide range of applications. In this book, the optimization process considers the problem of finding the optimal input control for a dynamic system provided that a specific optimal criterion and a set of constraints are satisfied.

Historically, the calculus of variations has constituted the theoretical framework for solving the optimal control problems. The most important necessary and sufficient conditions were derived based on the concepts and theories of this branch. Later, several important conductive results and new approaches to tackle the problem were established. The Pontryagin's principle of maximum and Bellman principle of optimality are the most famous.

Nowadays, the optimal robust control concepts and theories make a burst through in the field where multiple objectives can be handled simultaneously.

To pass over these entire subjects, it is believed that for better understanding one should start from the beginning. For this reason in this chapter some fundamental concepts, essentials of optimization theory and optimal problem modeling are presented. Chapter two considers controllers design based on the methodology of parameter's optimization. Chapter three is devoted to explain briefly the fundamental concepts of the calculus of variations and how it relates to the optimal control problems. Chapter four explains and discusses solutions based on the calculus of variation methods. Chapters five and six respectively treats the problem based on the vision of Pontryagin and Bellman. The LQR technique, which is one of the most important results in the field, is considered in chapter seven. The last chapter covers partially one of the most recent evolutionary techniques, genetic programming GP. The optimal solution is proposed via an enhanced algorithm of GP.

1.2. OPTIMAL CONTROL IN ENGINEERING

Optimal Control theory is a mature mathematical discipline with numerous applications. In fact, the applications of optimal control theory are very widespread that includes different fields. It is not exclusive to the engineering area. To give some idea of the breadth of this field, we shall make a few sample citations. For instance, the treatment of a two-strain tuberculosis model [1] or drug treatment strategies of the HIV immunology model [2] are two examples of many applications in the biological area. Besides engineering problems, mathematicians are usually interested in financial models, social models, production and maintenance models [3], transportation or vehicle-passenger models [4], etc. Mathematicians also contribute in many different theoretical new approaches and develop modern solution methods. For example, the solution of the famous Hamilton-Jacobin-Bellman HJB equation, the solution of optimal singular and bang-bang problems, the sensitivity of the optimal solution to the variation of model parameters, etc., has taken the greatest attention [5, 6, 7, 8]. Engineers believe that tuning the controller parameters by a trail-and-error procedure (a usual engineering practice) in some applications is a good motivation to search for an optimal solution.

On the other hand, engineers do not look at the mathematical solution as an end of the story but what is more important to them is the possibility of implementing that solution in real time with the available resources. Therefore, engineers contribute positively in the developments of the optimal control field. Weighting of a control cost versus quality in the performance function is one of the fundamentals in optimal control theory, however, stating a new cost function that considers the cost of the sensors and actuators to design a special optimal controller [9] is an example of such engineering contributions. Obviously, it is hard to list or to reference all engineering applications, but it will be illustrative, to mention some abstract examples.

Optimal temperature control of a drug store is an engineering task that belongs to biological fields. Suppose that the outside temperature is higher than the required inside temperature while a demanded electrical power to drive the cooling compressor can be chosen to range from minimum to maximum values. The store size, the outside temperature,