

**BASIC
MATHEMATICS
FOR TRADES
AND
TECHNOLOGIES**

BASIC MATHEMATICS FOR TRADES AND TECHNOLOGIES

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Preface

Rapid growth of technology develops greater need for skilled personnel in business and industry. Competence in basic job-related mathematics is one of the most essential skills required for employment in the trades and technologies. *Basic Mathematics for Trades and Technologies* was developed to help meet this need for trained personnel.

Content

Our text provides an easy-to-understand approach to building skills in basic technical mathematics. We begin with a thorough review of arithmetic in Part I and then carefully guide the student through basic algebra in Part II. Using the foundation in basic algebra, we continue with fundamentals of geometry and elementary trigonometry in Parts III and IV, respectively.

Readability Level

To estimate the readability level of our text, we selected 100-word samples from every 50 pages and applied both the Fry and the Flesch readability formulas to the samples. We then averaged the results obtained from application of each formula to each sample passage. From this analysis we determined that the readability level of our text is what Flesch calls "standard," that is, comparable to the readability levels of daily newspapers and popular magazines. We believe this level makes our text quite suitable for just about any instructional use. It is easy enough so that students will not get bogged

down by the narrative and explanations, and yet is mature enough so that our narrative and explanations will not talk down to the students.

Flexibility of Use

Basic Mathematics for Trades and Technologies is written for use in the classroom, business and industrial training programs, or learning laboratories. Our text may be easily adapted to self-paced courses, individualized programs, or independent study.

The instructional method of the text uses simple, direct language; easy-to-find rules, definitions, and explanations; and over 600 number-coded examples with step-by-step directions for each skill presented. Using these examples, we show nearly 1000 different problems and their solutions. This frees the instructor from having to make unnecessary explanations and makes our text easy for both instructors and students to use in an unlimited number of instructional programs and learning situations.

The organization of our text also contributes to its flexibility of use. Each chapter is divided into sections that present one set of related topics at a time. Each section contains one or more sets of self-study exercises with answers and a set of assignment exercises for additional practice in the skills of the entire section. Our text contains more than 180 sets of self-study exercises, and over 100 sets of assignment exercises comprising over 4000 problems. Each chapter ends with a trial test that enables the student to review the major skills and concepts of the entire chapter. The twenty-four trial tests contain a total of over 600 problems and each problem is coded to the appropriate section in the text. With this organizational format we have achieved in our text a desirable variety of exercise types, practice materials, and review aids—making our text suitable for use with various instructional methods, teaching styles, and learning preferences.

The material is sufficient for two semesters or three quarters if a thorough presentation is intended. However, the material may be presented in one semester or two quarters if only a review or survey is intended or if chapters not needed in some programs or by some classes are omitted or treated in less depth than other chapters.

Applications

In *Basic Mathematics for Trades and Technologies*, we include an abundance of trade, technical, and other job-related applications. Included are applications in welding, bookkeeping, pipefitting, carpentry, finance, mechanics, air conditioning and refrigeration, economics, TV service, drafting, building construction, electronics, surveying, electricity, masonry, business operations, machinery, manufacturing, automotive technology, and other areas. We use a practical, learn-by-doing approach to basic technical mathematics without frills and with a minimum of theory.

Use of Calculators

Use of a basic calculator is optional for Parts I and II, Arithmetic and Algebra. In many cases it may be preferable for the student to perform the arithmetic calculations, particularly in Part I, without a calculator to assure a solid foundation in basic computational skills. In other cases it may be more desirable for the student to acquire speed and accuracy in the use of a calculator to assure efficient use later in the text and on the job.

For Parts III and IV, Geometry and Trigonometry, we recommend use of a basic calculator. In these latter parts, a basic calculator is used to take the square roots of numbers not included in the square root table and to perform the multiple calculations of geometry and trigonometry with precision and convenience. To make these computations without a calculator would be too time consuming both in the classroom and on the job.

In Parts I and II we show both noncalculator and calculator answers (in parentheses) if they differ because of precision lost through rounding. In Parts III and IV we show only the calculator answers. Here the major difference between calculator and noncalculator answers will be found in problems involving square roots of numbers not in the table. All other noncalculator answers will be approximately the same as the calculator answers.

To help the student, we include in Part I a handy explanation of how to use a basic calculator to perform a continuous series of operations in problem-solving. Throughout the text we highlight particular problems where a calculator is especially convenient and indicate the sequence of calculator operations necessary for solving the problem. We present the calculator as an aid to problem solving and not as a substitute for mathematical competence.

Field Testing

Basic Mathematics for Trades and Technologies was field tested at State Technical Institute at Memphis, Richards Manufacturing Company, and Ripley Industries, Inc., all located in or near Memphis, Tennessee. We wish to thank these institutions and the instructors for field testing our text. We especially thank Ted Davis and Jana Turner, who made valuable suggestions for improvement.

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Arithmetic

Arithmetic is a branch of mathematics that helps us solve problems involving numbers. When we calculate the gasoline mileage an automobile gets after an engine tune-up, we are using arithmetic. We are using arithmetic when we figure out how much mortar is needed to lay a brick wall or what combination of resistors is needed to complete an electronic circuit. We are also using arithmetic when we compute the cooling capacity of an air conditioner, estimate the amount of paint needed to redecorate a living room, take a temperature reading, or set the margins on a typewriter. Arithmetic gives us short-cuts for solving these and other job-related problems containing numbers.



Whole Numbers

When we study a subject for the first time or review in some detail a subject we studied some time ago, we need to begin with the basics. Often, as we examine the basics of a subject, we discover—or rediscover—many bits and pieces of useful information. In this sense arithmetic is no different from any other subject. We will begin arithmetic with a study of whole numbers and the basic operations we perform with them, including the order of these operations. It is whole numbers and their operations which form the foundation for our study of arithmetic. We will begin our study of whole numbers in this chapter with a review of how to read numbers.

1-1 READING NUMBERS

Our system of numbers uses 10 individual figures called *digits*: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A *number* is made up of one or more digits. When a number contains two or more digits, each digit must be in the right place for the number to have the value we want it to have. If we mean "ninety-eight," we must place the 9 first and the 8 second to represent 98. If we change the places of these two digits by putting the 8 first and the 9 second, we get a new value (eighty-nine) and a new number (89).

Place Value

Each place a digit can occupy in a number has a value called a *place value*. If we know the place value of each digit in a number, we can read the number and understand how much it means. Look at the chart of place values in

Billions			Millions			Thousands			Units		
Hundred Billions	(100,000,000,000's)										
Ten Billions	(10,000,000,000's)										
Billions	(1,000,000,000's)										
Hundred Millions	(100,000,000's)										
Ten Millions	(10,000,000's)										
Millions	(1,000,000's)										
Hundred Thousands	(100,000's)										
Ten Thousands	(10,000's)										
Thousands	(1,000's)										
Hundreds	(100's)										
Tens	(10's)										
Ones	(1's)										

Figure 1-1

Fig. 1-1. Notice that each place value *increases* as we move from *right to left* and that each increase is *10 times* the value of the place to the right. For example, the tens place is ten times the ones place, the hundreds place is ten times the tens place, and so on.

The place values are arranged in groups of three to make numbers easier to read. The first group of three is called *units*, the second group of three is called *thousands*, the third group is *millions*, and the fourth group is *billions*. Commas are used to mark off these groups in a number. In four-digit numbers the comma separating the units group from the thousands group is optional. Thus, both 4,575 and 4575 are correct.

Example 1-1.1 In the number 2,472,694,500 identify the place value of the digit 7.
To solve this problem we will apply the following rule.

Rule 1-1.1. Identifying place values of digits:

1. Identify the place-value *group* in which the digit is located.

2,	472,	694,	500
Billions	Millions	Thousands	Units

The 7 is in the millions group.

2. In the millions group, name the place which the digit 7 occupies.

		ten millions place	
	↓		
2,	472,	694,	500
Billions	Millions	Thousands	Units

Answer: 7 is in the ten millions place.

Self-Study Exercises 1-1.1

In the number 2,304,976,186 identify the place value of the following digits.

- | | | | | |
|------|------|------|------|------|
| 1. 3 | 2. 7 | 3. 1 | 4. 0 | 5. 2 |
|------|------|------|------|------|

In the number 8,972,069,143 identify the place value of the following digits.

- | | | | | |
|------|------|------|------|-------|
| 6. 0 | 7. 4 | 8. 7 | 9. 8 | 10. 6 |
|------|------|------|------|-------|

Answers

1. hundred millions 2. ten thousands 3. hundreds 4. ten millions 5. billions
6. hundred thousands 7. tens 8. ten millions 9. billions 10. ten thousands

Now that we have reviewed place values, we can read numbers without much difficulty. All we need to do is follow the three steps shown in the following examples.

Example 1-1.2 Read 7543026129 by writing it in words.

Rule 1-1.2. Reading numbers:

1. Starting at the right, separate the number into place-value groups with commas.

7,543,026,129

2. Name each place-value group.

7,	543,	026,	129
Billions	Millions	Thousands	Units

3. From the left, read the numbers in each group and the group name. (The group name *units* is not usually read.)

Seven *billion*, five hundred forty-three *million*, twenty-six *thousand*, one hundred twenty-nine

Note: The word "and" should *not* be used when reading whole numbers. Also, the numbers from 21 to 99 use a hyphen when they are written out as two words (forty-three, twenty-six, etc.).

Example 1-1.3 Show how 2000125 would be read by writing it in words.

1. 2,000,125 (*Separate with commas.*)

2. 2, 000, 125 (*Name groups.*)
Millions Thousands Units

3. Two *million*, one hundred twenty-five (*Read from left.*)

Note: If a place-value group contains all zeros, it usually is *not* read.

Self-Study Exercises 1-1.2

Show how these numbers would be read by writing them out as words.

- | | | | |
|----------------|----------|--------------|------------|
| 1. 6704 | 2. 89021 | 3. 662900714 | 4. 3000101 |
| 5. 15407294376 | 6. 150 | | |

Write these words as numbers. Use commas when necessary.

7. Seven billion, four hundred
8. One million, six hundred twenty-seven thousand, one hundred six
9. Fifty-eight thousand, two hundred one
10. In a telephone conversation a contractor submitted the following bid for a job, "one thousand six dollars." Write this bid in numbers.