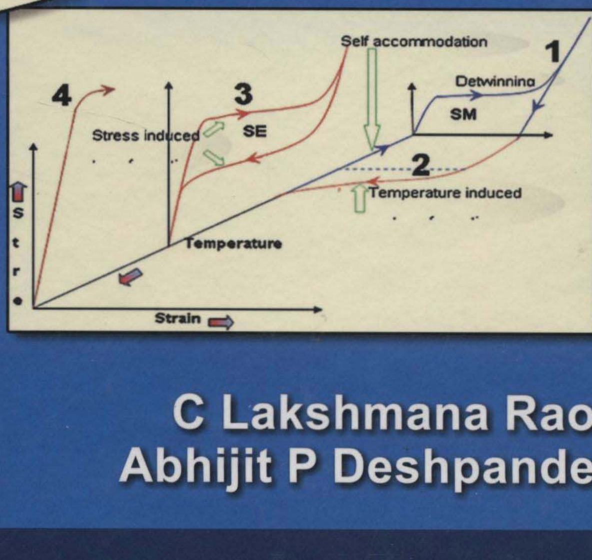
Modelling of

Engineering Materials

$$\int_{D_t} \rho(\mathbf{b}+\mathbf{b}^{em})dV_x + \int_{S_t} \mathbf{t}dA_x = \frac{d}{dt}\int_{D_t} \rho \mathbf{v} dV_x$$

$$\int_{D^t}\left[\left(\mathbf{x}\times\rho\left(\mathbf{b}+\mathbf{b}^{em}\right)\right)+\mathbf{m}^{em}\right]dV_x + \int_{S^t}(\mathbf{x}\times\mathbf{t})dA_x = \frac{d}{dt}\int_{D^t}(\mathbf{x}\times\rho\mathbf{v})dV_x$$

$$\int_{D^t}\rho\left(r+W^{em}\right)dV_x + \int_{S^t} h dA_x + \int_{D^t} \sigma \mathbf{D}\, dV_x = \frac{d}{dt}\int_{D^t}\rho\varepsilon dV_x$$



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Modelling of Engineering Materials

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Modelling of Engineering Materials

DEDICATION

The following close relatives and friends of the authors have left the mortal planes during the course of writing the current book. We dedicate this book to their inspiring memories.

To my grandfather

Krishnarao B. Deshpande

(4.8.1907 – 30.7.2008) – APD.

To my grandmother

Krovi Suryakantam

(15.7.1919 – 8.1.2009) – CLR.

To our friend and colleague

Devanathan Veeraraghavan (Dilip)

(28.9.1958 – 5.2.2009) – CLR & APD.

यस्याऽमतं तस्य मतं, मतं यस्य न वेद सः । (केनोपनिषत्)

Yasyāmatam tasya matam, matam yasya na veda saḥ (Kenopaniṣad)

For those who consider it to be not known, it is known. For those who claim to have known it, it is truly not known.

Preface

Engineers who are designing engineering systems using materials, often need a mathematical model that describes a material response, when a material is subjected to mechanical, thermal, electrical or other fields. Continuum mechanics attempts to provide the necessary mathematical framework that is useful in predicting the material response. *Continuum mechanics* is a classical as well as an emerging field that is exceedingly relevant to many researchers and practicing engineers in the fields of mechanical engineering, civil engineering, applied mechanics, chemical engineering and aerospace engineering among others.

Constitutive modeling is a topic that is part of continuum mechanics and is broadly understood by engineers as a phrase that deals with the equations that describe the response of a material sample, when it is subjected to external loads. In recent times, the term *constitutive model* is used to describe any equation that attempts to describe a material response, either during deformation or failure, independent of its origin or mathematical structure.

In the research community, *constitutive modeling* and *continuum mechanics*, involve investigations of physical mechanisms in materials and mathematical frameworks to describe them. Eventual goal of these researches is to describe macroscopic response of materials. Simulators and designers, on the other hand, are interested in using constitutive models in their simulations. These investigations are more focused on obtaining quantitative estimates of material behaviour. Reasonableness of physical mechanisms, correctness of mathematical framework, simplicity of mathematical models, ease of numerical simulations and reliability of estimates are all important aspects of modelling of engineering materials. In view of these issues involved in modeling of materials, the authors felt that a compilation and presentation of a broad review is necessary for the use of students, researchers and practicing engineers. This is attempted in the current book. The book has the following special features:

- It introduces the basic principles of continuum mechanics, so that the user is familiar with the mathematical tools that are necessary to analyze finite deformations in materials. Special care is taken to ensure that there is an engineering flavour to the topics dealt with in continuum mechanics and only the mathematical details that are necessary to appreciate the physics and engineering of a given problem, are highlighted.
- A brief review of popular linear material models, which are used in engineering, and which are derived based on infinitesimal deformation of materials, is presented in the book.
- Popular material models that are used to characterize the finite deformation of solids and fluids are described.
- Some examples of continuum characterization of failure in solids, such as modeling using plasticity theory, degradation parameter etc. are presented in this book.
- Principles behind the constitutive modeling of few modern special materials such as shape memory materials and ferroelectric materials are presented using the basic principles of continuum mechanics.
- Detailed case studies are presented which include a complete description of the material, its observed mechanical behaviour, predictions from some popular models along with a detailed discussion on a particular model.
- A brief overview of the tools that are available to solve the boundary value problems, is also given in this book.
- Detailed exercise problems, which will help students to appreciate the applications of the principles discussed are provided at the end of chapters.

The book is an outcome of the teaching of a course called *Constitutive Modelling in Continuum Mechanics*, by the authors at IIT Madras. The graduate students taking this course consist of new material modelers as well as material behaviour analysts and simulators. Majority of them, however, are involved in selection and use of material models in analysis and simulation. Therefore, main goal of the course has been to expose students with various backgrounds to basic concepts as well as tools to understand constitutive models. While teaching this course, the authors experienced the need for a book where principles of continuum mechanics are presented in a simple manner and are linked to the popular constitutive models that are used for materials. Hence, lecture notes were written to meet the course objective and these lecture notes are now compiled in the form a book.

The authors were inspired by a continuous exposure to the latest issues in the field of continuum mechanics, which was made possible through efforts by a leading expert in the field of continuum mechanics; Prof. K. R. Rajagopal, Professor of Mechanical Engineering, Texas A&M University. Prof. Rajagopal, kept the flame of interest in continuum mechanics alive at IIT Madras, through his regular involvement in workshops, seminars and discussions. The authors deeply acknowledge the inspiration provided by him to the authors, as well as to many other students and faculty at IIT Madras.

The authors place on record the contributions made by many of their faculty colleagues in the shaping of this book. Prof. Srinivasn M. Sivakumar, Department of Applied Mechanics at IIT Madras, provided us with the notes on plasticity, which formed the basis for the discussions on plasticity that is presented in Chapter 6. The authors thank him for his valuable help. Prof. Raju Sethuraman, Department of Mechanical Engineering at IIT Madras, provided the basic ideas for the review of numerical procedures, and was a constant source of inspiration for the authors in completing this book. We deeply acknowledge his encouragement and support. We acknowledge the help of Profs. Sivakumar and Sethuraman, along with Dr. Mehrdad Massoudi, U.S. Department of Energy, in formulating the contents of this book.

This book would not have been made possible but for the willing contributions of a number of M. S., Ph.D. and M.Tech students, who were working with us during their stay in IIT Madras. We also acknowledge all the students of the course over years, because class discussions and class projects were helpful towards formulating contents as well presentation of the book.

The illustrations were drawn with great enthusiasm by Mr. Jineesh George and Mr. Santhosh Kumar. The work of Dr. Rohit Vijay during dual degree project, formed the basis of the case study on asphalt that is presented in Chapter 5. Ms. K. V. Sridhanya's MS thesis formed the basis for the case study on soils that is presented in Chapter 6. Dr. S. Sathianarayanan, who worked on piezo-polymers for his Ph.D. thesis, has helped us to put together the discussion on piezoelectricity in Chapter 7. Efforts of Mr. N. Ashok Kumar, Mr. D. Pandit and Mr. M. Kishore Kumar, who worked on shape memory matrials for their theses, have helped us in compiling the material in Chapter 7. Rajesh Nair has taken the pains of going through parts of manuscript and pointing out some errors. Mr. V. Srinivasan helped us in the cover design of the book. Mr. Jose Vinoo Ananth, Mr. Mohammed Ghouse, Mr. G.G. Uday

Kumar, Mr. Suresh Kumar have also contributed in various capacities in bringing out the final form of the book.

One of the authors (CLR) has utilized his sabbatical leave that is granted by IIT Madras, towards writing the first draft of the book. We greatly acknowledge the support offered by IIT Madras for the encouraging atmosphere that it offers to pursue scholastic ambitions like writing a book.

Our publishers Ane Books Inc., were patient enough to wait from the submission of our original proposal to publish this book. We greatly appreciate their encouragement and patience in finally bringing out the final form of this book.

Last but not the least, the authors acknowledge the time spared by their family members and other friends, directly or indirectly, for encouraging the authors to pursue this project.

C. Lakshmana Rao
Abhijit P. Deshpande

Notations

Symbols style

Regular, italicized	scalar variables, components and invariants of tensors, material constants
Boldface, small	vectors
Boldface, capital and Greek	tensors
Boldface, italics	vector or tensor material constants
≡	definition
^	function
*	measurements made with reference to a moving frame of reference

Tensor operations

.	dot product involving vectors and tensors
×	cross product involving vectors and tensors
ab, **vT**	dyadic product of vectors **a** & **b**, and vector **v** & tensor **T**
A:**B**	scalar product of tensors **A** and **B** (double dot product)
$\mathbf{A}^{\mathrm{T}}$	transpose of **A**
|**a**|	magnitude of vector **a**
det **A**	determinant of **A**
tr(**A**)	trace of **A**
$\mathbf{A}^{-1}$	inverse of **A**

Derivative operations

$\dot{s}(X,t),\ \dot{v}(X,t),\ \dot{T}(X,t)$	total (material or substantial) derivative with respect to time
$\dfrac{\partial s(x,t)}{\partial t},\ \dfrac{\partial v(x,t)}{\partial t},\ \dfrac{\partial T(x,t)}{\partial t}$	partial derivative with respect to time

$\overset{\circ}{\mathbf{T}}$	rotational derivative of **T**
$\overset{\square}{\mathbf{T}}$	Jaumann derivative of **T**
$\overset{\Delta}{\mathbf{T}}$	lower convected or covariant derivative of **T**
$\overset{\nabla}{\mathbf{T}}$	upper convected or contravariant derivative of **T**
grad, div, curl	operators with respect to current configuration
Grad, Div, Curl	operators with respect to reference configuration
∇	gradient operator
∇^2	Laplace operator

<u>List of symbols: Roman</u>

A_x, A_X	areas in current and reference configuration, respectively
a	acceleration
B^r	reference configuration
b	body force
$\mathbf{b}^{em}$	electromechanical body force
B	$\mathbf{V}^2$, left Cauchy Green tensor or Finger tensor
$\mathbf{B}_t$	$\mathbf{V}_t^{\,2}$
C_{ij}	material parameter associated with strain energy density function
C	$\mathbf{U}^2$, right Cauchy Green tensor, matrix of elastic constants
C	stiffness coefficient
C^0	stiffness coefficient for biased piezoelectricity
$\mathbf{C}_t$	$\mathbf{U}_t^{\,2}$
$\mathbf{D}^v$	region (volume) in reference configuration
$\mathbf{C}^E$	electric current
D^r	Region (volume) in reference configgruation
D^t	region (volume) in current configuration
D	stretching tensor (rate of strain tensor, symmetric part of the velocity gradient tensor)
$\mathbf{D}^E$	electric displacement
e	strain
e^e, e^p	elastic and plastic strain
e_p	accumulated plastic strain, locked-in strain
$\dot{e}$	strain rate at small deformations
E	enthalpy, Young's modulus

E_r	relaxation modulus
E^*, E', E''	complex, storage and loss modulus
E_1,E_2	Burger's model parameters
$\mathbf{e}$	infinitesimal strain tensor
$\mathbf{e}^0$	biased infinitesimal strain tensor
$\mathbf{e}_i$	set of orthogonal unit base vectors,
$\dot{\mathbf{e}}$	strain rate tensor at small deformations
$\mathbf{E}$	Green strain, Electric field
$\mathbf{E}_t$	relative Green strain
$\mathbf{E}^E$	electric field
$\mathbf{E}^{E0}$	biased electric field
f	yield function
$\mathbf{f}_i$	set of orthogonal base vectors in a rotating frame
$\mathbf{f}_t$	force acting on region D^t
$\mathbf{F}$	deformation gradient
$\mathbf{F}^e$, $\mathbf{F}^p$	elastic and plastic deformation gradient
$\mathbf{F}_t$	relative deformation gradient
G	shear modulus, Doi model parameter
$\mathbf{g}$, g	acceleration due to gravity
g_{ij}, g^{ij}	metric coefficients
$\mathbf{g}_i$, $\mathbf{g}^i$	set of generalized base vectors
h	surface source of heat
$\dot{H}_t$	rate of heating
$\mathbf{H}$	displacement gradient
$\mathbf{H}_L$	linear momentum
$\mathbf{H}_A$	angular momentum
i, j, k	dummy indices
I_A, II_A, III_A	first, second and third invariants of tensor $\mathbf{A}$, respectively
$\mathbf{I}$	unit tensor
J	Jacobian associated with $\mathbf{F}$
J_c	creep compliance
K	power law model parameter
$\mathbf{L}$	velocity gradient
M	degradation parameter
m_{D^t}	mass of the body in the sub-region D^t
$\mathbf{m}$	unit tangential vector
$\mathbf{m}^{em}$	electromechanical body moment

M	total mass enclosed in a control volume D_t
$\mathbf{M}_t$	total moment
$\mathbf{n}$	unit normal vector
n	power law model parameter
N	number of cycles in cyclic plastic models
N_1, N_2	first and second normal stress difference
p	pressure, material particle
p'	effective mean stress
$\mathbf{P}$	material polarization
q	effective deviatoric stress
q^i, q_i	set of generalised coordinates
Q	electric charge, state variable in plasticity
$\mathbf{q}$	heat flux vector
$\mathbf{Q}$	orthogonal tensor, state variables in plasticity
r	volumetric source of heat
R	radius of the yield surface in the octahedral plane
$\mathbf{R}$	rotation tensor
$\mathbf{R}_t$	relative rotation tensor
s	distance, length
s_v	kinetic variable
S^r	area in reference configuration
S^t	area in current configuration
$\mathbf{s}$	1st Piola Kirchhoff traction
$\mathbf{S}$	1st Piola Kirchhoff stress
$\mathbf{S}_1$	2nd Piola Kirchhoff stress
t	current time
t^r	time at which material body takes B^r
t'	observation of time from a moving reference frame
$\mathbf{t}$	traction
u	pore pressure in soil mechanics
$\mathbf{u}$	displacement vector
$\mathbf{U}$	right stretch tensor
$\mathbf{U}_t$	relative right stretch tensor
V_x, V_X	volumes in current and reference configurations, respectively
v	volumetric strain
$\mathbf{v}$	velocity vector