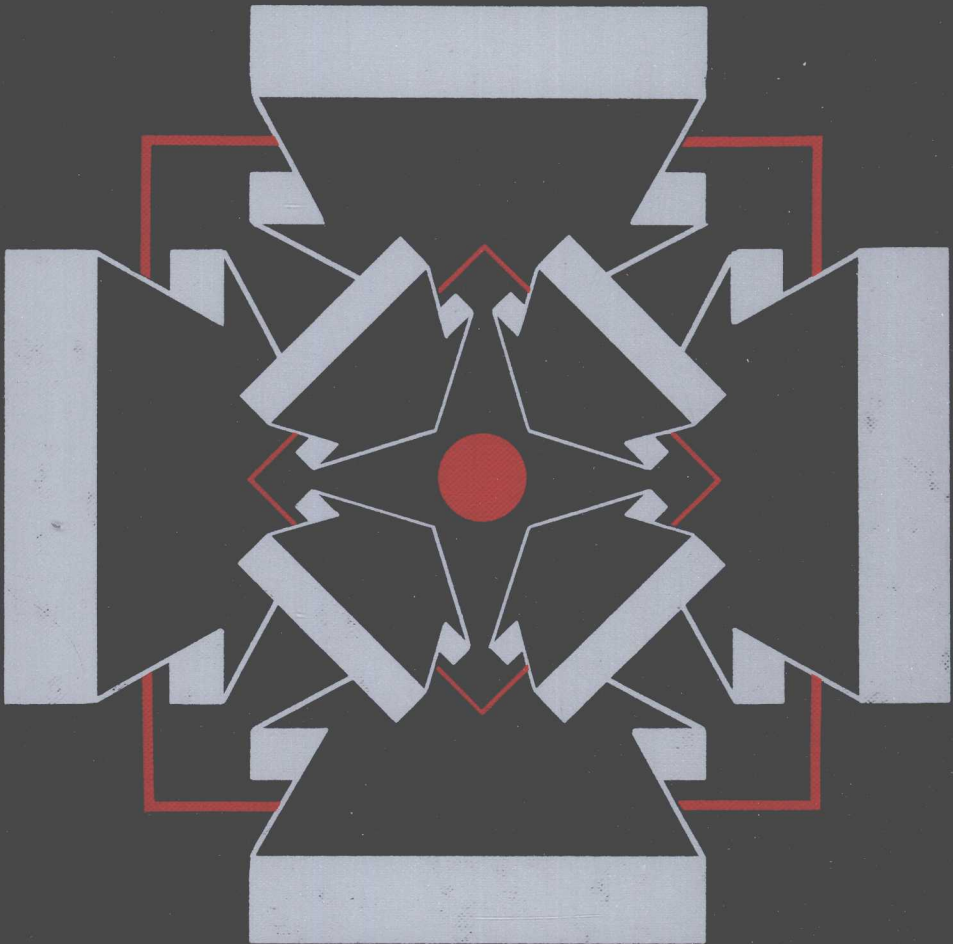


Fourth Edition

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# MODERN CONTROL SYSTEMS

Richard C. Dorf



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FOURTH EDITION

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*Modern Control  
Systems*

Richard C. Dorf

*University of California, Davis*



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## PREFACE

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Man cannot inherit the past; he has to recreate it.\* The most important and productive approach to learning is for the reader to rediscover and recreate anew the answers and methods of the past. Thus, the ideal is to present the student with a series of problems and questions and point to some of the answers that have been obtained over the past decades. The traditional method of confronting the student not with the problem but with the finished solution means depriving him or her of all excitement, to shut off the creative impulse, to reduce the adventure of humankind to a dusty heap of theorems. The issue, then, is to present some of the unanswered and important problems which we continue to confront. For it may be asserted that what we have truly learned and understood we discovered ourselves.

The purpose of this book is to present the structure of feedback control theory and to provide a sequence of exciting discoveries as the reader proceeds through the text and problems. In that this book is able to assist the student in discovering feedback control system theory and practice, it will have succeeded.

The book is organized around the concepts of control system theory as they have been developed in the frequency- and time-domain. A real attempt has been made to make the selection of topics, as well as the systems dis-

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\*A. Koestler, *The Act of Creation*, Hutchinson, London, 1964, p. 266.

cussed in the examples and problems, modern in the best sense. Therefore we have provided a discussion of sensitivity, performance indices, state variables, robotics, and computer control systems, to name a few. However, a valiant attempt has been made to retain the classical topics of control theory which have proved to be so very useful in practice.

The text is written in an integrated form so that it should be read from the first to the last chapter. However, it is not necessary to include all the sections of a given chapter in any given course, and there appears to be quite a large number of combinations of sequences of the sections for study. The book is designed for an introductory undergraduate course in control systems for engineering students. There is very little demarcation between electrical, mechanical, chemical, and industrial engineering in control system practice; therefore, this text is written without any conscious bias toward one discipline. Thus, it is hoped that this book will be equally useful for all engineering disciplines and, perhaps, assist in illustrating the unity of control engineering. The problems and examples are chosen from all fields, and the examples of the sociological, biological, ecological, and economic control systems are intended to provide the reader with an awareness of the general applicability of control theory to many facets of life.

The book is primarily concerned with linear, constant parameter control systems. This is a deliberate limitation because the author believes that for an introduction to control systems, it is wisest initially to consider linear systems. Nevertheless, several nonlinear systems are introduced and discussed where it is appropriate.

Chapter 1 provides an introduction to and basic history of control theory. Chapter 2 is concerned with developing mathematical models of these systems. With the models available, we are able to describe the characteristics of feedback control systems in Chapter 3 and illustrate why feedback is introduced in a control system. In Chapter 4 we examine the performance of control systems, and in Chapter 5 we investigate the stability of feedback systems. Chapter 6 is concerned with the  $s$ -plane representation of the characteristic equation of a system and the root locus. Chapters 7 and 8 treat the frequency response of a system and the investigation of stability using the Nyquist criterion. Chapter 9 develops the time-domain concepts in terms of the state variables of a system. Chapter 10 describes and develops several approaches to designing and compensating a control system. Finally, Chapter 11 discusses digital computer control systems and robotics.

This book is suitable for an introductory course in control systems. The text, in its first, second, and third editions, has been used for a senior level course for engineering students at over one hundred colleges and universities. Also, it has been used for a course for engineering graduate students with no previous background in control system theory.

The text presumes a reasonable familiarity with the Laplace transformation and transfer functions as developed in a first course in linear system analysis or network analysis. These concepts are discussed in Chapter 2 and

are used to develop mathematical models for control system components. Answers to selected problems are provided at the end of the book.

The fourth edition has incorporated several important developments in the field of control systems, with particular reference to robots. In addition, a new feature is the addition of a set of exercises for each chapter immediately preceding the problems. The purpose of these exercises is to permit the reader to utilize readily the concepts and methods introduced in each chapter in the solution of relatively straightforward exercises before attempting the more complex problems.

This material has been developed with the assistance of many individuals to whom I wish to express my sincere appreciation. Among those to whom I owe a particular debt of gratitude are Professors L. Gould, G. Thaler, and S. Weissenberger. I wish to acknowledge the unflagging assistance of my secretaries, Mrs. M. McKenna, Mrs. M. Mahaffey, Mrs. B. Moore, and Mrs. P. Needle. Finally, I can only partially acknowledge the encouragement and patience of my wife, Joy, who helped to make this book possible.

*Davis, California*

R.C.D.

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# CHAPTER 1

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## *Introduction to Control Systems*

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### 1.1 Introduction

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Engineering is concerned with understanding and controlling the materials and forces of nature for the benefit of mankind. Control system engineers are concerned with understanding and controlling segments of their environment, often called *systems*, in order to provide useful economic products for society. The twin goals of understanding and control are complementary because, in order to be controlled more effectively, the systems under control must be understood and modeled. Furthermore, control engineering often must consider the control of poorly understood systems such as chemical process systems. The present challenge to control engineers is the modeling and control of modern, complex, interrelated systems such as traffic-control systems, chemical processes, and economic regulation systems. However, simultaneously, the fortunate engineer has the opportunity to control many very useful and interesting industrial automation systems. Perhaps the most characteristic quality of control engineering is the opportunity to control machines, and industrial and economic processes for the benefit of society.

Control engineering is based on the foundations of feedback theory and linear system analysis, and integrates the concepts of network theory and communication theory. Therefore, control engineering is not limited to any

engineering discipline but is equally applicable for aeronautical, chemical, mechanical, environmental, civil, and electrical engineering. For example, quite often a control system includes electrical, mechanical, and chemical components. Furthermore, as the understanding of the dynamics of business, social, and political systems increases, the ability to control these systems will increase also.

A *control system* is an interconnection of components forming a system configuration that will provide a desired system response. The basis for analysis of a system is the foundation provided by linear system theory, which assumes a cause-effect relationship for the components of a system. Therefore, a component or process to be controlled can be represented by a block as shown in Fig. 1.1. The input-output relation represents the cause and effect relationship of the process, which in turn represents a processing of the input signal to provide an output signal variable, often with a power amplification. An *open-loop* control system utilizes a controller or control actuator in order to obtain the desired response as shown in Fig. 1.2.

In contrast to an open-loop control system, a closed-loop control system utilizes an additional measure of the actual output in order to compare the actual output with the desired output response. A simple *closed-loop feedback control system* is shown in Fig. 1.3. A standard definition of a feedback control system is as follows: A feedback control system is a control system that tends to maintain a prescribed relationship of one system variable to

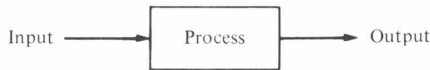


Figure 1.1. Process to be controlled.

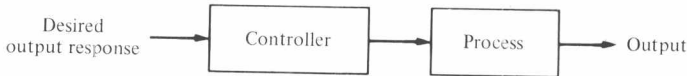


Figure 1.2. Open-loop control system.

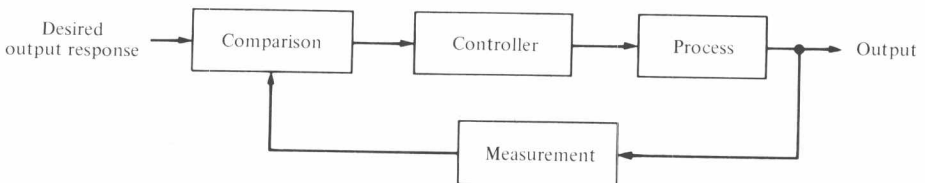


Figure 1.3. Closed-loop feedback control system.

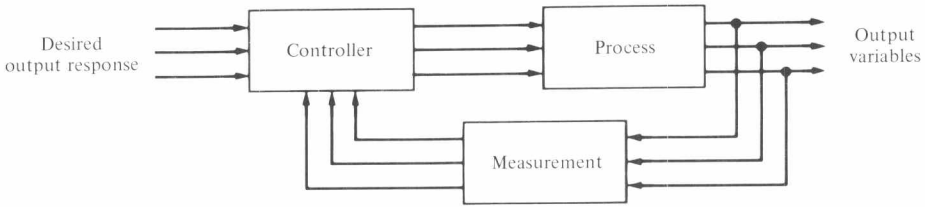


Figure 1.4. Multivariable control system.

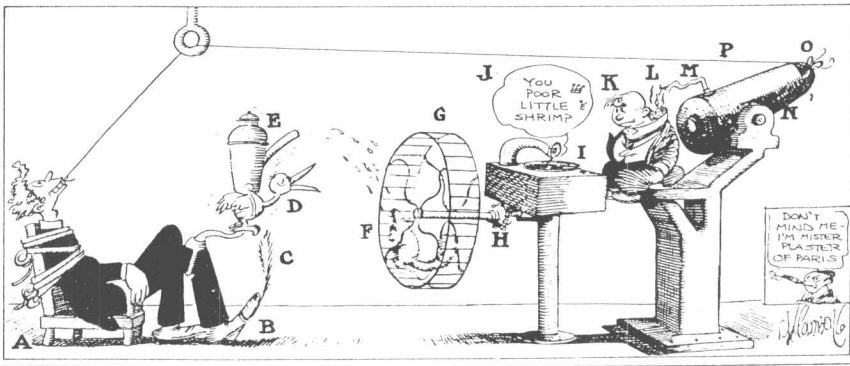


Figure 1.5. Rube Goldberg's elaborate creations were almost all closed-loop feedback systems. Goldberg called this simply, "Be Your Own Dentist." (© Rube Goldberg, permission granted by King Features Syndicate, Inc., 1979.)

another by comparing functions of these variables and using the difference as a means of control.

A feedback control system often uses a function of a prescribed relationship between the output and reference input to control the process. Often the difference between the output of the process under control and the reference input is amplified and used to control the process so that the difference is continually reduced. The feedback concept has been the foundation for control system analysis and design.

Due to the increasing complexity of the system under control and the interest in achieving optimum performance, the importance of control system engineering has grown in this decade. Furthermore, as the systems become more complex, the interrelationship of many controlled variables must be considered in the control scheme. A block diagram depicting a *multivariable control system* is shown in Fig. 1.4. A humorous example of a closed-loop feedback system is shown in Fig. 1.5.

## 1.2 History of Automatic Control

The use of feedback in order to control a system has had a fascinating history. The first applications of feedback control rest in the development of float regulator mechanisms in Greece in the period 300 to 1 B.C. [1, 2]. The water clock of Ktesibios used a float regulator (refer to Problem 1.11). An oil lamp devised by Philon in approximately 250 B.C. used a float regulator in an oil lamp for maintaining a constant level of fuel oil. Heron of Alexandria, who lived in the first century A.D., published a book entitled *Pneumatica*, which outlined several forms of water-level mechanisms using float regulators [1].

The first feedback system to be invented in modern Europe was the temperature regulator of Cornelis Drebbel (1572–1633) of Holland [1]. Dennis Papin [1647–1712] invented the first pressure regulator for steam boilers in 1681. Papin's pressure regulator was a form of safety regulator similar to a pressure-cooker valve.

The first automatic feedback controller used in an industrial process is generally agreed to be James Watt's flyball governor developed in 1769 for controlling the speed of a steam engine [1, 2]. The all-mechanical device, as shown in Fig. 1.6, measured the speed of the output shaft and utilized the movement of the flyball with speed to control the valve and therefore the amount of steam entering the engine. As the speed increases, the ball weights rise and move away from the shaft axis thus closing the valve. The flyweights

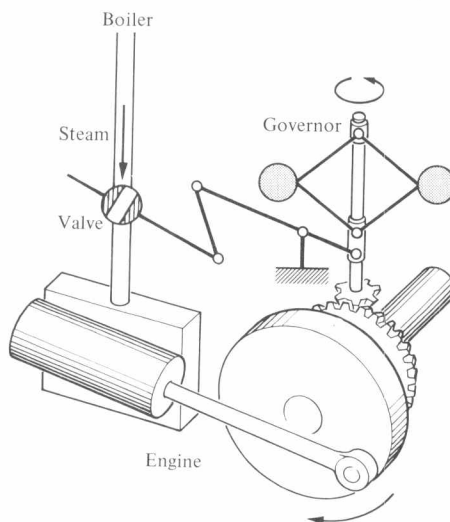


Figure 1.6. Watt flyball governor.



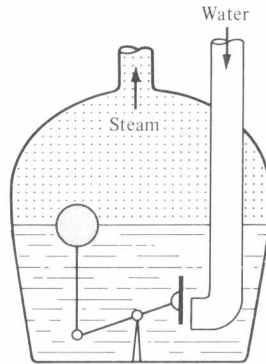


Figure 1.7. Water-level float regulator.

require power from the engine in order to turn and therefore make the speed measurement less accurate.

The first historical feedback system claimed by the Soviet Union is the water-level float regulator said to have been invented by I. Polzunov in 1765 [4]. The level regulator system is shown in Fig. 1.7. The float detects the water level and controls the valve that covers the water inlet in the boiler.

The period preceding 1868 was characterized by the development of automatic control systems by intuitive invention. Efforts to increase the accuracy of the control system led to slower attenuation of the transient oscillations and even to unstable systems. It then became imperative to develop a theory of automatic control. J. C. Maxwell formulated a mathematical theory related to control theory using a differential equation model of a governor [5]. Maxwell's study was concerned with the effect various system parameters had on the system performance. During the same period, I. A. Vyshnegradskii formulated a mathematical theory of regulators [6].

Prior to World War II, control theory and practice developed in the United States of America and Western Europe in a different manner than in the U.S.S.R. and Eastern Europe. One main impetus for the use of feedback in the United States was the development of the telephone system and electronic feedback amplifiers by Bode, Nyquist, and Black at the Bell Telephone Laboratories [7, 8, 9, 10, 12]. The frequency domain was used primarily to describe the operation of the feedback amplifiers in terms of bandwidth and other frequency variables. In contrast, the eminent mathematicians and applied mechanics in Russia inspired and dominated the field of control theory. Therefore, the Russian theory tended to utilize a time-domain formulation using differential equations.

A large impetus to the theory and practice of automatic control occurred during World War II when it became necessary to design and construct automatic airplane pilots, gun-positioning systems, radar antenna control systems, and other military systems based on the feedback control approach.