

国外数学名著系列

(影印版) 34

I. R. Shafarevich(Ed.)

Algebraic Geometry II

Cohomology of Algebraic Varieties.
Algebraic Surfaces

代数几何 II

代数簇的上同调，代数曲面



科学出版社
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北京

图字:01-2006-7386

I. R. Shafarevich(Ed.): Algebraic Geometry II: Cohomology of Algebraic Varieties. Algebraic Surfaces

© Springer-Verlag Berlin Heidelberg 1996

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图书在版编目(CIP)数据

代数几何 II:代数簇的上同调,代数曲面 = Algebraic Geometry II: Cohomology of Algebraic Varieties. Algebraic Surfaces/(俄罗斯)沙法列维奇(Shafarevich, I. R.)编著. —影印版. —北京:科学出版社,2007

(国外数学名著系列)

ISBN 978-7-03-018300-2

I. 代… II. 沙… III. ①代数几何-英文②代数簇-上同调论-英文③代数曲面-英文 IV. O187

中国版本图书馆 CIP 数据核字(2006)第 154745 号

责任编辑:范庆奎/责任印刷:安春生/封面设计:黄华斌

科学出版社 出版

北京东黄城根北街16号

邮政编码:100717

<http://www.sciencep.com>

中国科学院印刷厂印刷

科学出版社发行 各地新华书店经销

*

2007年1月第一版 开本:B5(720×1000)

2007年1月第一次印刷 印张:17 1/4

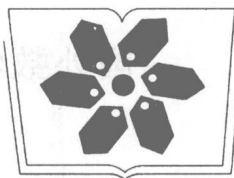
印数:1—3 500 字数:323 000

定价:50.00元

(如有印装质量问题,我社负责调换〈科印〉)

中国科学院植物研究所《植物分类学报》编辑部

(中国科学院出版社)



中国科学院科学出版基金资助出版

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来,需要数学家淡泊名利并付出更艰苦地努力。另一方面,我们也要从客观上为数学家创造更有利的发展数学事业的外部环境,这主要是加强对数学事业的支持与投资力度,使数学家有较好的工作与生活条件,其中也包括改善与加强数学的出版工作。

从出版方面来讲,除了较好较快地出版我们自己的成果外,引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说,施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书,使我国广大数学家能以较低的价格购买,特别是在边远地区工作的数学家能普遍见到这些书,无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权,一次影印了23本施普林格出版社出版的数学书,就是一件好事,也是值得继续做下去的事情。大体上分一下,这23本书中,包括基础数学书5本,应用数学书6本与计算数学书12本,其中有些书也具有交叉性质。这些书都是很新的,2000年以后出版的占绝大部分,共计16本,其余的也是1990年以后出版的。这些书可以使读者较快地了解数学某方面的前沿,例如基础数学中的数论、代数与拓扑三本,都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点,基础数学类的书以“经典”为主,应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家,例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士,曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然,23本书只能涵盖数学的一部分,所以,这项工作还应该继续做下去。更进一步,有些读者面较广的好书还应该翻译成中文出版,使之有更大的读者群。

总之,我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持,并盼望这一工作取得更大的成绩。

王元

2005年12月3日

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I. Cohomology of Algebraic Varieties

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