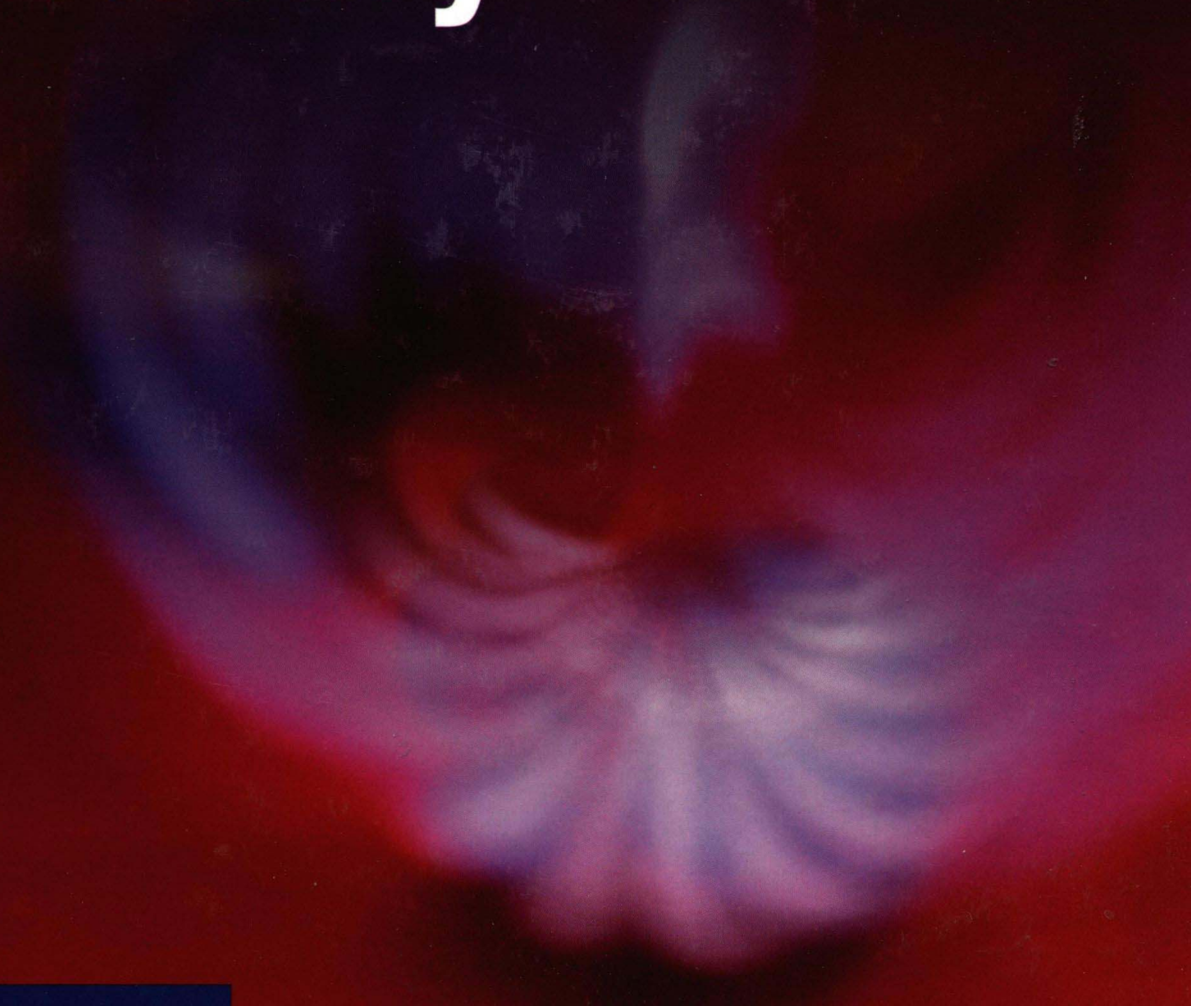
Mark M. Wilde

Quantum Information Theory



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Quantum Information Theory

Finally, here is a modern, self-contained text on quantum information theory suitable for graduate-level courses. Developing the subject "from the ground up," it covers classical results as well as major advances of the past decade.

Beginning with an extensive overview of classical information theory suitable for the non-expert, the author then turns his attention to quantum mechanics for quantum information theory, and the important protocols of teleportation, super-dense coding, and entanglement distribution. He develops all of the tools necessary for understanding important results in quantum information theory, including capacity theorems for classical, entanglement-assisted, private, and quantum communication. The book also covers important recent developments such as superadditivity of private, coherent, and Holevo information, and the superactivation of quantum capacity.

This book will be warmly welcomed by the upcoming generation of quantum information theorists and by the already established community of classical information theorists.

MARK M. WILDE is currently a Lecturer in the School of Computer Science at McGill University, Montréal and will begin in August 2013 as an Assistant Professor with a joint appointment in the Department of Physics and Astronomy and the Center for Computation and Technology at Louisiana State University, Baton Rouge.

How To Use This Book

For Students

Prerequisites for understanding the content in this book are a solid background in probability theory and linear algebra. If you are new to information theory, then there is enough background in this book to get you up to speed (Chapters 2, 10, 12, and 13). Though, classics on information theory such as Cover and Thomas (1991) and MacKay (2003) could be helpful as a reference. If you are new to quantum mechanics, then there should be enough material in this book (Part II) to give you the background necessary for understanding quantum Shannon theory. The book of Nielsen and Chuang (sometimes known as "Mike and Ike") has become the standard starting point for students in quantum information science and might be helpful as well (Nielsen & Chuang, 2000). Some of the content in that book is available in Nielsen's dissertation (Nielsen, 1998). If you are familiar with Shannon's information theory (at the level of Cover and Thomas (1991), for example), then this book should be a helpful entry point into the field of quantum Shannon theory. We build on intuition developed classically to help in establishing schemes for communication over quantum channels. If you are familiar with quantum mechanics, it might still be worthwhile to review Part II because some content there might not be part of a standard course on quantum mechanics.

The aim of this book is to develop "from the ground up" many of the major, exciting, pre- and post-millennium developments in the general area of study known as quantum Shannon theory. As such, we spend a significant amount of time on quantum mechanics for quantum information theory (Part II), we give a careful study of the important unit protocols of teleportation, superdense coding, and entanglement distribution (Part III), and we develop many of the tools necessary for understanding information transmission or compression (Part IV). Parts V and VI are the culmination of this book, where all of the tools developed come into play for understanding many of the important results in quantum Shannon theory.

For Instructors

This book could be useful for self-learning or as a reference, but one of the main goals is for it to be employed as an instructional aid for the classroom. To aid instructors in designing a course to suit their own needs, a preprint version of this book is available from `http://arxiv.org/abs/1106.1445` under a Creative Commons Attribution-NonCommercial-ShareAlike license. This means that you can modify and redistribute the preprint version of this book as you wish, as long as you attribute the author, you do not use it for commercial purposes, and you share a modification or derivative work under the same license (see `http://creativecommons.org/licenses/by-nc-sa/3.0/` for a readable summary of the terms of the license). These requirements can be waived if you obtain permission from the present author. By releasing the preprint version of this book under this license, I expect and encourage instructors to modify it for their own needs. This will allow for the addition of new exercises, new developments in the theory, and the latest open problems. It might also be a helpful starting point for a book on a related topic, such as network quantum Shannon theory.

I used an earlier version of this book in a one-semester course on quantum Shannon theory at McGill University during Winter semester 2011 (in many parts of the USA, this semester is typically called "Spring semester"). We almost went through the entire book, but it might also be possible to spread the content over two semesters instead. Here is the order in which we proceeded:

1. Introduction in Part I.
2. Quantum mechanics in Part II.
3. Unit protocols in Part III.
4. Chapter 9 on distance measures, Chapter 10 on classical information and entropy, and Chapter 11 on quantum information and entropy.
5. The first part of Chapter 13 on classical typicality and Shannon compression.
6. The first part of Chapter 14 on quantum typicality.
7. Chapter 17 on Schumacher compression.
8. Back to Chapters 13 and 14 for the method of types.
9. Chapter 18 on entanglement concentration.
10. Chapter 19 on classical communication.
11. Chapter 20 on entanglement-assisted classical communication.
12. The final explosion of results in Chapter 21 (one of which is a route to proving the achievability part of the quantum capacity theorem).

The above order is just a particular order that suited the needs for the class at McGill, but other orders are of course possible. One could sacrifice the last part of Part III on the unit resource capacity region if there is no desire to cover the quantum dynamic capacity theorem. One could also focus on going from classical communication to private classical communication to quantum communication in order to develop some more intuition behind the quantum capacity theorem.

Other Sources

There are many other sources to obtain a background in quantum Shannon theory. The standard reference has become the book of Nielsen and Chuang (2000), but it does not feature any of the post-millennium results in quantum Shannon theory. Other books that cover some aspects of quantum Shannon theory are Hayashi (2006) and Holevo (2002a). Patrick Hayden has had a significant hand as a collaborative guide for many PhD and Masters' theses in quantum Shannon theory, during his time as a postdoctoral fellow at the California Institute of Technology and as a professor at McGill University. These include the theses of Yard (2005), Abeyesinghe (2006), Savov (2008, 2012), Dupuis (2010), and Dutil (2011). All of these theses are excellent references. Naturally, Hayden also had a strong influence over the present author during the development of this book.

Acknowledgments

I began working on this book in the summer of 2008 in Los Angeles, with much time to spare in the final months of dissertation writing. I had a strong determination to review quantum Shannon theory, a beautiful area of quantum information science that Igor Devetak had taught me three years earlier at USC in fall 2005. I was carefully studying a manuscript entitled "Principles of Quantum Information Theory," a text that Igor had initiated in collaboration with Patrick Hayden and Andreas Winter. I read this manuscript many times, and many parts of it I understood well, though other parts I did not.

After a few weeks of reading and rereading, I decided "if I can write it out myself from scratch, perhaps I would then understand it!", and thus began the writing of the chapters on the packing lemma, the covering lemma, and quantum typicality. I knew that Igor's (now former) students Min-Hsiu Hsieh and Zhicheng Luo knew the topic well because they had already written several quality research papers with him, so I requested if they could meet with me weekly for an hour to review the fundamentals. They kindly agreed and helped me quite a bit in understanding the packing and covering techniques.

Not much later, after graduating, I began collaborating with Min-Hsiu on a research project that Igor had suggested to the both of us: "find the triple trade-off capacity formulas of a quantum channel." This was perhaps the best starting point for me to learn quantum Shannon theory because proving this theorem required an understanding of most everything that had already been accomplished in the area. After a month of effort, I continued to work with Min-Hsiu on this project while joining Andreas Winter's Singapore group for a two-month visit. As I learned more, I added more to the notes, and they continued to grow.

After landing a job in the DC area for January 2009, I realized that I had almost enough material for teaching a course, and so I contacted local universities in the area to see if they would be interested. Can Korman, formerly chair of the Electrical Engineering Department at George Washington University, was excited about the possibility. His enthusiasm was enough to keep me going on the notes, and so I continued to refine and add to them in my spare time in preparing for teaching. Unfortunately (or perhaps fortunately?), the course ended up being canceled. This was disheartening to me, but in the mean time, I had contacted Patrick Hayden to see if he would be interested in having me join his group at

McGill University for postdoctoral studies. Patrick Hayden and David Avis then offered me a postdoctoral fellowship, and I moved to Montréal in October 2009.

After joining, I learned a lot by collaborating and discussing with Patrick and his group members. Patrick offered me the opportunity to teach his graduate class on quantum Shannon theory while he was away on sabbatical, and this encouraged me further to persist with the notes.

I am grateful to everyone mentioned above for encouraging and supporting me during this project, and I am also grateful to everyone who provided feedback during the course of writing up. In this regard, I am especially grateful to Dave Touchette for detailed feedback on all of the chapters in the book. Dave's careful reading and spotting of errors has immensely improved the quality of the book. I am grateful to my father, Gregory E. Wilde, Sr., for feedback on earlier chapters and for advice and love throughout. I thank Ivan Savov for encouraging me, for feedback, and for believing that this is an important scholarly work. I also thank Constance Caramanolis, Raza-Ali Kazmi, John M. Schanck, Bilal Shaw, and Anna Vershynina for valuable feedback. I am grateful to Min-Hsiu Hsieh for the many research topics we have worked on together that have enhanced my knowledge of quantum Shannon theory. I thank Michael Nielsen and Victor Shoup for advice on Creative Commons licensing and Kurt Jacobs for advice on book publishing. I acknowledge funding from the MDEIE (Quebec) PSR-SIIRI international collaboration grant. I am grateful to Sarah Payne and David Tranah of Cambridge University Press for their extensive feedback on the manuscript and their outstanding support throughout the publication process.

I am indebted to my mentors who took me on as a student during my career. Todd Brun was a wonderful PhD supervisor—helpful, friendly, and encouraging of creativity and original pursuit. Igor Devetak taught me quantum Shannon theory in fall 2005 and helped me once per week during his office hours. He also invited me to join Todd's and his group, and more recently, Igor provided much encouragement and "big-picture" feedback during the writing of this book. Bart Kosko shaped me as a scholar during my early years at USC and provided helpful advice regarding the book project. Patrick Hayden has been an immense bedrock of support at McGill. His knowledge of quantum information and many other areas is unsurpassed, and he has been kind, inviting, and helpful during my time at McGill. I am also grateful to Patrick for giving me the opportunity to teach at McGill and for advice throughout the development of this book.

I thank my mother, father, sister, and brother and all of my surrounding family members for being a source of love and support. Finally, I am indebted to my wife Christabelle and her family for warmth and love. I dedicate this book to the memory of my grandparents Joseph and Rose McMahon, and Norbert Jay and Mary Wilde. *Lux aeterna luceat eis, Domine.*

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